Conditional Heteroscedasticity and Stock Market Returns: Empirical Evidence from Morocco and BVRM

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Using data from Casablanca (Morocco) and Bourse Régionale des Valeurs Mobilières (BVRM) stock markets, this paper investigates and compares different distribution density and forecast methodology of three generalised autoregressive conditional heteroscedasticity (GARCH) models for Morocco and BVRM indices. The symmetric GARCH and asymmetric Glosten Jagannathan and Runkle (GJR) version of GARCH (GJR-GARCH) and Exponential GARCH methodology are employed to investigate the effect of stock return volatility in both stock markets using Gaussian, Student-t and Generalised Error distribution densities. The study further examines the forecasting ability of each GARCH model using alternative densities. In both markets, the EGARCH results show that negative shocks will have a greater impact on future volatility than positive shocks of the same magnitude, confirming the existence of leverage effect. However, for both markets, the GJR estimates imply that positive instead of negative shocks will have a higher next period conditional variance. This means that positive instead of negative shocks would have greater effects on next period volatility. Regarding forecasting evaluation, the results reveal that the symmetric GARCH model coupled with fatter-tail distributions present a better out-of-sample forecast for both stock markets.

INTRODUCTION

Empirical studies have shown that stock returns tend to follow non-normal distribution density (Hsu et al., 1974; Hagerman, 1978; Lau *et al*., 1990; Kim & Kon, 1994). These studies indicate that the distribution is either tilted to the left or to the right when the kurtosis of time series of stock returns is greater than normal and the variance of the stock returns is heteroscedastic. This heteroscedasticity in the error variance, also described as risk or uncertainty by financial analysts, has become an important issue in modern finance theory. Consequently, Engle (1982) applied an econometric technique known as Autoregressive Conditional Heteroscedasticity (ARCH) to model the time varying variances of United Kingdom inflation. Many researchers have subsequently applied the linear ARCH technique to model economic and financial time series. Nonetheless, the linear ARCH (q) model requires a long lag length of q in many of its applications. Bollerslev (1986), in an attempt to resolve this empirical weakness of the ARCH, introduced a more flexible lag structure of the ARCH referred to as the Generalised Autoregressive Conditional Heteroscedasticity (GARCH). Some empirical works have shown that the first order lag length of the GARCH is adequate to model the long memory processes of time varying variance (French et al., 1987; Franses and Van Dijk, 1996).

Black (1976) shows that the rise and fall in stock prices has an asymmetric effect on volatility. Large-size negative returns were observed to increase volatility more than do positive returns of equal magnitude. This attribute in financial time series is known as leverage effect. The standard GARCH is found inadequate to model the dynamics of this leverage effect. Therefore, the Exponential GARCH and Threshold GARCH (also known as GJR, so named after its proponents) were respectively developed by Nelson (1991) and Glosten et al. (1993) to account for this asymmetric volatility response.

Since the introduction of ARCH/GARCH, GJR-GARCH and EGARCH models (Engle, 1982; Bollerslev, 1986; Glosten et al, 1993 and Nelson, 1991), studies have largely been conducted on symmetric and asymmetric GARCH models. Nevertheless, less effort has been made towards comparing alternative density forecast models. And though, Nor and Shamiri (2007) compared alternative density forecast models in Malaysia and Singapore, similar studies on Emerging African Stock markets appear non-existent. One essential feature of high-frequency financial time series of stock returns is that they are normally characterised by fat-tailed distribution. Finance literature has established that the kurtosis of most financial asset returns is greater than 3 (Simkowitz & Beedles, 1980; Kon, 1984). This suggests that extreme values are much more likely to be observed in stock market returns with fat-tailed distribution than with the normal distribution. Again, though finance literature provided sound evidence of high kurtosis of stock market returns, the state of the symmetrical distribution is still obscure.

Research studies on economic and financial time series have long highlighted that stock returns exhibit heavy-tailed distribution probability. One possible explanation is that the conditional variance may be heteroscedastic. However, while the GARCH model can successfully be used to control excess kurtosis of stock returns, it cannot cope with the skewness of the distribution of stock market returns. Therefore, forecast estimates from GARCH can be expected to be biased for a skewed time series. Some econometric studies have proposed alternative non-linear models (for instance, the exponential GARCH (EGARCH) model, introduced by Nelson (1991)) which can take into account the skewed distribution. Meanwhile, recent econometric software also, have embedded in them alternative distribution densities for GARCH models (i.e. normal *vis-a-vis* non-normal). Stock market returns distribution has tails that are heavier than implied by the GARCH process with Gaussian. In modelling financial time series such as stock returns therefore, one must not only assume the Gaussian white noise but also independently identical distribution (*i.i.d*) white noise process with a heavy-tailed distribution.

Against this background, this study fills the gap by introducing alternative density distribution methodology of symmetric and asymmetric GARCH models for Morocco and BVRM (this is the stock market for all West African Francophone countries based in La Cote D'Ivoire) stock returns. The performance of GARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1) models are compared with the introduction of different distribution densities (Gaussian, Student-t and GED).The study is inspired by the acknowledgement of the importance attached to the accurate volatility measurement and forecast in a wide range of financial applications and the seemingly non-existential empirical evidence available to date for Morocco and BVRM markets. In addition, the paper contributes to existing literature in three ways. First, data set from emerging African stock markets are used, where such kind of study has not been conducted previously. Second, both symmetric and asymmetric GARCH models (i.e. GARCH versus GJR and EGARCH) are applied. The latter captures the time series properties of skewness, kurtosis and volatility clustering, and also the leverage effect. Third, the performance of GARCH-type models is compared with the introduction of three different distribution densities (i.e. Gaussian versus non-normal) for modelling and forecasting the stock returns volatility of Morocco and BVRM stock markets.

The next section presents the empirical models. Data description and method used in this study are offered in section three. The fourth section presents the results and analyses and the conclusions are presented in the final section.

EMPIRICAL MODELS

Two moment (i.e. mean and variance) equations are used to define the ARCH/GARCH models. The return process, r_t, is captured by the mean equation encapsulating the conditional mean, μ, which might encompass terms of autoregressive(AR) and moving average(MA) and error term, ε_t, that follows a conditional normal distribution with mean of zero and variance, σ^2. Furthermore, the information set available to investors up to time t_{-1} is represented by, Ωt_{-1}, thus,

$$r_t = \mu + \varepsilon_t \qquad (1)$$

Where, $[\varepsilon_t \Omega_{t-1} \approx N(0, \sigma_t^2)]$, $[\sigma_t^2 = h_t]$ (2)

The conditional variance h_t is modelled using symmetric and asymmetric GARCH models with the introduction of three different distribution densities (i.e. Gaussian, Student-t and GED).

Arch Model

Engle's (1982) seminal work proposed to model time varying conditional heteroscedasticity using past error term to estimate the series variance as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \qquad (3)$$

Garch Model

Bollerslev (1986) proposed the GARCH model which suggest that time varying heteroscedasticity is a function of both past innovations and past conditional variance (i.e. past volatility). The GARCH model represents an infinite order ARCH model express as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, h_{t-j} \qquad (4)$$

Where α_0, α_i and β_j are non-negative constants.

Exponential Garch (EGARCH) Model

The exponential GARCH model was introduced by Nelson (1991) to capture the asymmetric (or 'directional') response of volatility. Nelson and Cao (1992) critiqued the imposition of non-negativity constraints on the parameters; α_i and β_j in the linear GARCH model as being too restrictive, but in the EGARCH model there is no such restriction. The conditional variance, h_t, in the EGARCH model is an asymmetric function of lagged disturbances as follows:

$$\ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, g(Z_{t-i}) + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) \qquad (5)$$

Where, $\left[Z_t = \frac{\varepsilon_t}{\sqrt{h_t}}\right]$ is the standardised residual series.

The value of $g(Z_t)$ is a function of both the magnitude and sign of Z_t and is defined as follows:

$$g(Z_t) = \theta Z_t + \gamma[|Z_t| - E|Z_t|] \qquad (6)$$

Since the log of the conditional variance is modelled, the leverage effect is exponential, rather than quadratic and even if the parameters are negative, the conditional variance will be positive. The hypothesis that $g < 0$ is used to test the presence of the leverage effect. The impact is asymmetric if $g \neq 0$. If the relationship between returns and volatility is negative, g will be negative. The EGARCH model allows positive and negative shocks to have a distinct impact on volatility. It also allows large shocks to have a greater impact on volatility than the standard GARCH model.

The GJR-Garch Model

Glosten, Jagannathan and Runkle (1993) introduced the GJR-GARCH model to augment the standard GARCH with an additional ARCH term conditional on the sign of the past innovation which is expressed as:

$$h_t = \alpha_0 + \sum_{i=1}^{q}\left(\alpha_i \varepsilon_{t-i}^2 + \lambda_i \varepsilon_{t-i}^2 I_{t-1}\right) + \sum_{j=1}^{p}\beta_j h_{t-j} \tag{7}$$

Where λ_1 measures the asymmetric (or leverage) effect and I_t is a dummy variable which is equal to 1 when ε_t is negative. In the GJR (1, 1) model, good news, $\varepsilon_{t-1} > 0$ and bad news, $\varepsilon_{t-1} < 0$ possess differential effects on the conditional variance. Good news has an impact of α_1, while bad news has an impact of $\alpha_1 + \lambda_1$. If $\lambda_1 > 0$, bad news increases volatility and this in turn means that there is a leverage effect for the AR (1)-order. If $\lambda_1 \neq 0$, the news impact is asymmetric.

DATA AND METHODS

Data Description

The daily stock price indices data used in this study are obtained from Standard & Poor/International Finance Corporation Emerging Market Database (S&P/IFC EMDB). This source remains widely used as it provides well organised and comprehensive source of stock price data, readily accessible and reliable data on emerging equity markets than most other sources. For example, S&P/IFC EMDB was the first database, from 1975, to track comprehensive information and statistics on emerging stock market indices. The S&P/IFC Global indices, used in this study, do not impose restrictions on foreign ownership and include sufficient number of stocks in individual market indices without imposing float or artificial industry-composition models on markets. Besides, the S&P/IFC database is attractive because they have been adjusted for all capital changes including the effects of corporate restructuring such as merger, acquisition, and spin offs/demerger as well as being free from data backfilling and survivorship bias.

The daily return, r_t, consists of transformed daily closing index price, P_t, measured in local currency. Our measurements include the Casablanca (Morocco) Stock Exchange's All-Share Index and Bourse Régionale des Valeurs Mobilières (BVRM) Composite Index. The stock price indices are transformed into their returns in order to obtain stationary series which is computed as:

$$r_t = \ln\left(p_t / p_{t-1} \right) * 100 \tag{8}$$

Where r_t is the market return, p_t and p_{t-1} are natural log returns of contemporaneous and one period lagged equity price indices, respectively. Natural log is preferred as it computes continuous compound returns.

Table 1 below gives further particulars of the data used in this research including the types of the stock indices used, the time period of the data for each market (and hence sample observations), and currency of denomination. The indices used in this study are the benchmark indices in their respective stock markets.

TABLE 1
STOCK MARKET DATA PROFILE

	Method of compiling data	Index of Name	Period of data	No of Obs	Currency	Source of Data
Morocco	Weighted index market capitalization	SEM All Share Index	1997–2014	4,674	MD	S&P/IFC EMDB
BVRM	Weighted index market capitalization	BVRM All Share Index	1998–2014	4, 251	CFA	S&P/IFC EMDB

TABLE 2
DESCRIPTIVE STATISTICS FOR DAILY RETURNS

	Mean	Std. Dev.	Skewness	Kurtosis	J. Bera	Q-Stat	ACF (100)	PACF (100)
Morocco	0.0003	0.0076	0.5874	44.6339	337772.9***	315.90***	0.103	0.105
BVRM	0.0002	0.0094	-0.1482	84.7744	1184180***	142.91***	0.010	0.015

J. Bera is the Jarque-Bera test for normality, Q-stat refers to Ljung-Box test for autocorrelation.

*** denotes statistical significance at 1%. ** denotes statistical significance at 5%

The descriptive statistics in Table 2 indicate that both markets produce positive mean returns. However, the mean returns for Morocco is slightly higher than that of BVRM. However, the non-conditional variance as measured by the standard deviation for Morocco is lower than that of BVRM. The returns distribution for Morocco is positively skewed while that of BVRM is negatively skewed. The null hypothesis for skewness that conforms to a normal distribution with coefficients of zero is rejected by both indices. The returns for both indices exhibit fat tail as seen in the significant kurtosis well above the normal value of 3. The high value of J.Bera test for normality decisively rejects the hypothesis of a normal distribution at 1 per cent significance level. Ljung-Box Q test statistic (Q-Stat) rejects the null hypothesis of no autocorrelation at 1 per cent level for all numbers of lags (100) considered as shown by ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) results in Table 2. The preceding statistics legitimize the use of autoregressive conditional heteroscedastic models.

The statistical results indicate that both indices display similar characteristics. For instance they both have positive mean returns, skewed, found to display non-normal distribution and exhibit autoregression. These stylized features are similar to the existing empirical literature from the developing markets (Kim, 2003; Ng, 2000) and developed markets (Fama, 1976; Kim & Kon, 1994). Further, as return series revealed high value of kurtosis, it can be expected that a fatter-tailed distribution density such as the Student-t or GED should provide a more accurate results than the Gaussian (Normal) distribution.

METHODS

The GARCH models are estimated using maximum likelihood estimation (MLE) process. This allows the mean and variance processes to be jointly estimated. The MLE has numerous optimal properties in estimating parameters and these include sufficiency (i.e. complete information about the parameter of importance contained in its MLE estimator); consistency (true parameter value that generated the data recovered asymptotically, i.e. data of sufficiently large samples); efficiency (lowest possible variance of parameter estimates achieved asymptotically. Furthermore, many methods of inference in statistics and econometrics are developed based on MLE, such as chi-square test, modelling of random effects, inference with missing data and model selection criteria such as Akaike information criterion and Schwarz criterion.

ML estimation presupposes that the error distribution is Gaussian; however, evidence shows that the error exhibits non-normal distribution densities, for example, Nelson, (1991). The choice of the underlying distribution for the error term is crucial if the volatility model is used in risk modelling. As it is expected that the problems pose by skewness and kurtosis by the residuals of conditional heteroscedasticity models will be reduced when appropriate distribution density is used, our study considers and evaluate the three most commonly used densities, the Gaussian, Student-t and Generalised Error Distribution (GED).

GAUSSIAN

The Gaussian, also known as the normal distribution, is the commonly used model when estimating GARCH models. For a stochastic process, the log-likelihood function for the normal distribution is calculated as:

$$L_{gaussian} = -\tfrac{1}{2}\sum_{t=1}^{T}\left(\ln[2\pi] + \ln\left[\sigma_t^2\right] + z_t\right) \tag{9}$$

Where T is the number of observations.

STUDENT'S-T DISTRIBUTION

For a student-t distribution, the log-likelihood is computed as:

$$L_{stu-t} = \ln\left(\Gamma\left[\frac{v+1}{2}\right]\right) - \ln\left(\Gamma\frac{v}{2}\right) - \tfrac{1}{2}\ln(\pi[v-2]) - \tfrac{1}{2}\sum_{t=1}^{T}\left(\ln\sigma_t^2 + [1+v]\ln\left[1+\frac{z_t^2}{v-2}\right]\right) \tag{10}$$

Where v is degrees of freedom, $2<v=\infty$ and $\Gamma(\cdot)$ is the gamma function.

GENERALISED ERROR DISTRIBUTION (GED)

In applied finance, such as, asset pricing, option pricing, portfolio selection and VaR, skewness and kurtosis are very important. The GED is an error distribution that represents a generalised form of the Gaussian, possesses a natural multivariate form, has a parametric kurtosis that is unbounded above and has special cases that are identical to the normal and property which controls the skewness. Thus, choosing the appropriate distribution density that can model these two moments is important, hence, the GED log-likelihood function of a normalised random error is computed as:

$$L_{GED} = \sum_{t=1}^{T}\left(\ln\left[\tfrac{v}{\lambda_v}\right] - 0.5\left|\frac{z_t}{\lambda_v}\right|^v - \left[1+v^{-1}\right]\ln 2 - \ln\Gamma\left[\tfrac{1}{v}\right]0.5\ln\left[\sigma_t^2\right]\right) \tag{11}$$

Where $\lambda_v = \sqrt{\dfrac{\Gamma\left(\frac{1}{v}2^{-2/v}\right)}{\Gamma\left(\frac{3}{v}\right)}}$ (12)

The order of the GARCH process can be determined by computing Q-statistic from the squared residuals and the Engle (1982) LM test is applied to test for the ARCH effect in the residuals. The GARCH models in this study are compared by using various goodness-of-fit diagnostics such as Akaike information criterion, Schwarz Bayesian information criterion and log-likelihood.

FORECAST EVALUATION

The one-step-ahead forecast of the conditional variance for the GARCH, EGARCH and GJR is obtained by updating equations (4), (5) and (7) by one period as,

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_t \quad (13)$$

$$\ln(h_{t+1}) = \alpha_0 + \alpha_1 g(Z_t) + \beta_1 \ln(h_t) \quad (14)$$

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \lambda_1 \varepsilon_t^2 I_t + \beta_1 h_t \quad (15)$$

Similarly, j-step-ahead forecast on the conditional variance can be obtained by updating equations (10), (11) and (12) by j-periods as,

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \beta_1 h_{t+j-1} \quad (16)$$

$$\ln(h_{t+j}) = \alpha_0 + \alpha_1 g(Z_{t+j-1}) + \beta_1 \ln(h_{t+j-1}) \quad (17)$$

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \lambda_1 \varepsilon_{t+j-1}^2 I_{t+j-1} + \beta_1 h_{t+j-1} \quad (18)$$

However, it is rather difficult to obtain the j-step-ahead forecasts than the one-period-ahead forecasts assumed in this study although it is possible to obtain the j-step-ahead forecasts of the conditional heteroscedasticity recursively.

In order to evaluate the forecasting performance of the GARCH, EGARCH and GJR models, forecasting tests encompassing different distribution densities are performed. The model that minimises the loss function under these evaluation criteria is preferred. To assess the performance of the asymmetric GARCH models in forecasting the conditional variance, we compute four statistical measures of fit as follows;

Mean Absolute Error (MAE) – This is represented as:

$$MAE = \frac{1}{h}\sum_{t=s}^{s+h}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| \quad (19)$$

Where h is the number of steps ahead (i.e. number of forecast data points), s the sample size, $\hat{\sigma}^2$ is the forecasted variance and σ^2 is the conditional variance computed from equations (4) and (5).

Root Mean Square Error (RMSE) is represented as:

$$RMSE = \sqrt{\frac{1}{h}\sum_{t=s}^{s+h}\left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2} \quad (20)$$

The Mean Absolute Percentage Error (MAPE) is represented as:

$$MAPE = \frac{1}{h}\sum_{t=s}^{s+h}\left|(\hat{\sigma}_t^2 - \sigma_t^2) \Big/ \sigma_t^2\right| \quad (21)$$

Theil Inequality Coefficient (TIC) is represented as:

$$TIC = \frac{\sqrt{MSE}}{\sqrt{\frac{1}{h}\sum_{t=s}^{s+h}\sigma^2 + \frac{1}{h}\sum_{t=s}^{s+h}\hat{\sigma}^2}} \quad (22)$$

To compute daily forecast and in order to evaluate the forecasting performance of each model, we simply split the respective time series in half between the in-sample period, $t = 1,..., T$ and the out-of-sample period, $t = T,..., h$. We further estimate each model over the first part of the sample and then apply these results to forecast the conditional variance (volatility) over the second part of the sample period.

EMPIRICAL RESULTS AND ANALYSIS

This section presents and analyse our results of the estimated models. Tables 3, 4 and 5 presents the results for the estimated parameters for GARCH, EGARCH and GJR models respectively, while some useful in-sample diagnostics statistics are reported in Tables 6, 7 and 8.

TABLE 3
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GARCH MODEL

	Morocco			**BVRM**		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.1256 (7.9943)***	0.0014 (0.0980)	9.66e-07 (8.15e-05)	0.0002 (1.8385)*	3.35E-06 (0.0667)	7.49E-07 (0.0122)
α_0	0.7360 (1.6216)	0.2968 (0.7746)	0.3574 (0.7539)	1.71E-05 (22.1737)***	0.0018 (0.0038)	2.49E-06 (13.6627)***
α_1	-0.0014 (-5.6043)***	-0.0002 (-37.5106)***	-0.0003 (-5.8944)***	0.1156 (14.9045)***	430.7788 (0.0038)	0.1722 (14.7611)***
β_1	0.5966 (1.9908)**	-0.1034 (-0.0726)	0.2420 (0.2405)	0.6590 (43.3089)***	0.7628 (62.0184)***	0.7490 (70.1465)***
$\alpha_1 + \beta_1$	0.5952	-0.1036	0.2417	0.7746	431.5416	0.9212

TABLE 4
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	Morocco			**BVRM**		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	4.60e-05 (1.1642)	8.01e-08 (0.1877)	-4.16e-06 (-0.0024)	0.0003 (5.8427)***	1.95E-07 (0.0035)	-3.72E-07 (-0.0203)
α_0	0.0361 (0.4291)	-2.8206 (-0.4599)	-1.0253 (-11.0104)***	-1.7456 (-27.9209)***	-0.9245 (-7.1526)***	-4.8053 (-10.8730)***
α_1	-0.4423 (-4.3966)***	0.0578 (0.2542)	0.1130 (13.2862)***	0.2058 (25.5352)***	1.5187 (1.9461)*	0.1141 (8.9201)***
β_1	0.7360 (20.1983)***	0.2187 (6.4135)***	0.6377 (19.0690)***	0.8292 (134.5562)***	0.8985 (93.3368)***	0.5258 (11.9541)***
g	-0.2318 (-4.2060)***	-0.1447 (-0.2543)	-0.0878 (-11.0296)***	0.0227 (-4.8926)***	-0.1599 (-1.5362)	-0.1564 (-13.1036)***

TABLE 5
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

	Morocco			**BVRM**		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.1059 (5.5936)***	0.0002 (0.1532)	-0.2597 (-6.1274)***	0.0003 (2.0844)**	7.70E-08 (0.0015)	1.80E-06 (0.0315)
α_0	1.0776 (3.3381)***	0.4042 (0.1619)	1.0124 (3.5338)***	1.64E-05 (22.1012)***	0.0018 (0.0027)	1.60E-06 (11.3279)***
α_1	0.0466 (3.1877)***	0.0530 (0.1614)	0.0520 (2.2521)**	0.1336 (14.2756)***	668.0680 (0.0027)	0.1536 (12.4584)***
β_1	0.3975 (4.2715)***	-0.0057 (- 3.2657)***	0.5496 (4.3340)***	0.6726 (45.7276)***	0.7925 (74.0363)***	0.7993 (85.2516)***
λ_1	-0.0585 (-4.4001)***	-0.0557 (-0.1622)	-0.0679 (-2.9677)***	-0.0494 (-4.9192)***	-276.8113 (-0.0027)	-0.0006 (-0.0441)

The statistics reported in the tables above show that news impact is asymmetric in both stock markets as the asymmetric coefficients for all densities are unequal to zero.

The sum of the lagged error and the lagged conditional variance of the symmetrical GARCH model for both indices are far from the expected value of 1 (i.e. unity) regardless of the distribution density except BVRM with student-t density which pose unusually high α term of which we cannot explain statistically (i.e. it is just an anomaly). This implies that the current shocks to the conditional variance will have less impact on future volatility (*see* Coffie, 2015). In both Morocco and BVRM, the leverage effect term, g, in the EGARCH has the correct sign for all distribution densities and is significant with Gaussian and GED at 1 percent level. This means that in both markets, negative shocks will have a greater impact on future volatility than positive shocks of the same magnitude, confirming the existence of the leverage effect. The GED-EGARCH model (Nelson, 1991) replaces the traditional use of conditionally normal error distribution assumption of GARCH models with the assumption of innovations that follow generalised error distribution. Hence, the presence of leverage effect suggests that investors in these markets are to be rewarded for taking up additional leverage risks. Therefore, investors and fund managers should go beyond the simple mean-variance approach when allocating portfolios for these markets. Instead, they should explore information about volatility, information asymmetry, correlation, skewness and kurtosis. Required rate of return is expected to be high in these markets due to compensation for additional leverage risk which places additional burden on indigenous companies seeking to raise finance from the domestic capital markets.

The use of GJR with normal and non-normal distribution appears justified to model the asymmetric characteristics of morocco whiles in BVRM, the Gaussian seems appropriate. Furthermore, the asymmetric coefficients with Gaussian and GED densities for Morocco are statistically significant at standard levels whiles with BVRM, the Gaussian density show 1 per cent level of significance. For both markets, the coefficient estimates of the GJR are negative, suggesting that positive instead of negative shocks imply a higher next period conditional variance of the same sign. This means that negative shocks would have no greater effects on volatility than positive shocks as expected. Instead positive shocks would have greater effect on volatility as the asymmetric term, λ, is less than zero for all density distributions. This evidence invalidates the GJR proposition that bad news has greater impact on volatility than good news. Therefore, like Wan et al. (2014), the evidence in Morocco and BVRM shows that both markets exhibit a reverse volatility asymmetry, controverting the widely accepted theory of volatility asymmetry (i.e. negative returns induce a higher return volatility than positive returns). Mainly, this reverse volatility asymmetry can be attributed to higher trading volume associated with momentum stocks (i.e. price rising stocks) as investors from Morocco and BVRM countries are known to rush for such stocks than their contrarian counterparts and this leads to the arousal of higher volatility for positive returns than negative returns. Hence, positive return-volatility correlation is observed in both stock markets.

The estimated GARCH parameters for both indices of the asymmetric models (i.e. EGARCH & GJR) indicate that the ARCH (α_1) and GARCH (β_1) terms are generally significant at standard level. The coefficient of the symmetric ARCH (α_1) term for both markets are mainly significant at standard level while that of the GARCH (β_1) term is significant with all densities for BVRM.

TABLE 6
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GARCH MODEL

	Morocco			**BVRM**		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	0.6357 (1.000)	0.6775 (1.000)	0.6798 (1.000)	1.9597 (1.000)	0.2604 (1.000)	0.3826 (1.000)
ARCH(2)	0.0461 (0.9772)	0.0693 (0.9659)	0.0715 (0.9649)	0.6629 (0.5154)	0.0112 (0.9889)	0.0205 (0.9797)
AIC	3.0684	0.6125	0.4196	-6.7858	-7.6451	-7.5426
SBIC	3.0740	0.6195	0.4266	-6.7797	-7.6376	-7.5351
Log-Like	-7035	-1400	-958	14423	16251	16033

$Q^2(20)$ are the Ljung-Box statistic at lag 20 of the squared standardised residuals. ARCH (2) refers to the Engle (1982) LM test for the presence of ARCH effect at lag 2. *P*-values are given in parentheses. AIC, SBIC and Log-Like are Akaike information criterion, Schwartz Bayesian information criterion and Log-Likelihood value respectively.

TABLE 7
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	Morocco			BVRM		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	1.0787 (1.000)	0.6727 (1.000)	0.6500 (1.000)	1.8550 (1.000)	0.3963 (1.000)	443.98 (0.000)
ARCH(2)	0.4545 (0.7967)	0.0706 (0.9653)	0.0681 (0.9665)	0.1332 (0.8709)	0.0353 (0.9653)	278.186 (0.000)
AIC	2.6532	-3.8547	-0.5846	-6.7603	-7.6435	-7.6501
SBIC	2.6602	-3.8463	-0.5762	-6.7528	-7.6345	-7.6411
Log-Like	-6081	8849	1347	14371	16248	16262

TABLE 8
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

	Morocco			BVRM		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	0.6555 (1.000)	0.6792 (1.000)	1.3700 (1.000)	1.5736 (1.000)	0.2407 (1.000)	0.3358 (1.000)
ARCH(2)	0.0597 (0.9706)	0.0714 (0.9649)	0.6329 (0.7287)	0.4954 (0.6094)	0.0099 (0.9901)	0.0193 (0.9809)
AIC	3.0815	-2.5084	2.9568	-6.7867	-7.6470	-7.5849
SBIC	3.0885	-2.5000	2.9652	-6.7792	-7.6380	-7.5759
Log-Like	-7064	5760	-6777	14427	16256	16124

Turning to distribution densities (Tables 6, 7 & 8); the fatter tails (Student-t and GED) distributions clearly outperform the Gaussian. For instance, the log-likelihood function strongly increases when fatter tailed distribution densities are used for both indices. Furthermore, using the non-normal densities of Student-t and GED produces lower AIC and SBIC than the Gaussian. From the preceding evidence, all the three GARCH models perform well with non-normal distribution densities. All models appear effective by describing the dynamics of the series as shown by the Ljung-Box statistics for the squared standardised residuals with lag 20 which are all non- significant for both indices. The LM test for the presence of ARCH at lag 2, indicate that conditional heteroscedasticity are removed for all three GARCH models regardless of the distribution density which are all non-significant.

The comparison between models with each distribution density indicates that, giving the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for Morocco, clearly outperforms EGARCH with Gaussian and GED as well as GARCH and GJR models. Furthermore, from the results, GJR with student-t provides a better in-sample estimation for BVRM than with Gaussian and GED and clearly outperforms symmetric GARCH and EGARCH models.

TABLE 9
FORECAST PERFORMANCE-COMPARATIVE DISTRIBUTION DENSITY

Gaussian

	Morocco			**BVRM**		
Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	0.292479	0.273953	0.174511	0.004497	0.004504	0.004517
RMSE	1.281024	1.280752	1.284478	0.009378	0.009378	0.009379
MAPE	4.554697	4.561448	4.597685	87.44634	89.05093	91.95668
TIC	0.908472	0.921161	0.999961	0.975764	0.972431	0.966967

Student-t

	Morocco			**BVRM**		
Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	0.292479	0.174657	0.174485	0.004455	0.004454	0.004454
RMSE	1.281024	1.284466	1.284480	0.009381	0.009381	0.009381
MAPE	4.554697	4.597632	4.597695	82.20057	82.21151	82.21111
TIC	0.908472	0.999832	0.999985	0.999635	0.999992	0.999979

GED

	Morocco			**BVRM**		
Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	0.174469	0.426068	0.174472	0.004454	0.004454	0.004454
RMSE	1.284482	1.329788	1.284482	0.009381	0.009381	0.009381
MAPE	4.597701	4.702422	4.597703	82.20926	82.20575	82.21301
TIC	0.999999	0.861133	0.999997	0.999918	0.999804	0.999961

TABLE 10
RANKING PERFORMANCE FORECAST

Gaussian

	Morocco			**BVRM**		
Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	3	2	1	1	2	3
RMSE	2	1	3	1	1	2
MAPE	1	2	3	1	2	3
TIC	1	2	3	3	2	1
Total	7	7	10	6	7	9

Student-t

	Morocco			BVRM		
Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	3	2	1	2	1	1
RMSE	1	2	3	1	1	1
MAPE	1	2	3	1	3	2
TIC	1	2	3	1	3	2
Total	6	8	10	5	8	6

GED

Model	GARCH	GJR	EGARCH	GARCH	GJR	EGARCH
MAE	1	3	2	1	1	1
RMSE	1	3	1	1	1	1
MAPE	1	3	2	2	1	3
TIC	3	1	2	2	1	3
Total	6	10	7	6	4	8

TABLE 11
SUMMARY OF BEST PERFORMING MODEL

	Morocco	BVRM
Gaussian	GARCH/GJR	GARCH
Student-t	GARCH	GARCH
GED	GARCH	GJR

Table 10 ranks the GARCH models when evaluated against each other with the introduction of the three different distribution densities for the disturbance term. The evidence in tables 9 and 10 indicate that the symmetric GARCH model clearly outperform the GJR and EGARCH in both markets. Furthermore, Table 11 indicates that the symmetric GARCH model provides the best out-of-sample forecast followed by GJR in both stock markets. This contradicts the evidence found in Malaysia and Singapore where asymmetric GARCH models clearly outperform the symmetric GARCH (Nor and Shamiri, 2007). The findings also show that forecasting with heavy-tailed distribution densities yield no significant reduction of the forecast error than when normal distribution is assumed. However, it appears that the symmetric GARCH model with fatter-tailed distribution have a slight tendency over normal distribution to produce superior forecast.

CONCLUSION

Over the last three decades many academics and analysts have paid particular attention to stock market volatility since it can be used to measure and forecast in a wide range of financial applications including portfolio selection, value at risk, asset pricing, hedging strategies and option pricing. This paper aimed to model and forecast the performance of the symmetric GARCH model and asymmetric GARCH (i.e. GJR and EGARCH) models with the introduction of different distribution densities for Morocco and BVRM stock markets.

The statistical results point towards the fact that the current shocks to the conditional variance will have less impact on future volatility in both markets. The results show that the leverage effect exists in both markets. However, the evidence from GJR for both markets reveal a reverse volatility asymmetry, contradicting the widely accepted observation of volatility asymmetry, where negative returns induce a higher return volatility than positive returns.

The comparison between models with each distribution density indicates that, giving the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for Morocco whiles the GJR with student-t provides a better in-sample estimation for BVRM. Regarding forecasting evaluation, the results reveal that the symmetric GARCH model coupled with fatter-tailed distribution presents a better out-of-sample forecast for both stock markets.

Finally, there are areas where further studies might be useful. For example, future research should focus on modelling and forecasting GARCH models with high frequency trading (i.e. intra-day) data. Further research may also consider exploring variety of models including other conditional variance models such as APARCH and long memory models such as FIEGARCH, FIAPARCH and CGARCH in order to allow a superior insight into the dynamics of these two markets. Lastly, similar study should be conducted in other African stock markets in order to provide a wider insight into how GARCH models are capable of modelling volatility in African stock markets.

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