Unreliable Inference for Business and Economic Event Studies Based on Variance Dummy Variable in a GARCH Regression Model

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GARCH models with dummy variables included in variance equations have been often used in various business and economic event studies to detect temporary volatility structure break. However, this methodology can be questionable if the event window is relatively short. In this paper, through simulation studies, we show that with a short event window, this GARCH-dummy methodology may afford unreliable statistical inferences in that variance deflation can be overstated. On the other hand, variance inflation can be understated. Moreover, based on results of our simulation studies, we provide some practical advices on applying the GARCH-dummy methodology in event studies.

INTRODUCTION

Because of its simplicity and better estimation in many cases, GARCH-type models proposed by Bollerslev (1986) are frequently adopted in event studies to identify structure break in time series data due to specific business and economic events. For instance, some financial researchers, e.g. Campbell et al. (1997), propose using event studies to determine whether a particular financial event will cause significantly abnormal returns on underlying securities. Binder (1998) indicates that the most popular event study methodology is to introduce event dummy variables into various regression models. Thus, inferences for effects of events are based on estimation for coefficients of the dummy variables, e.g. Nikkinen et al. (2007).

Because a GARCH model specifies both the mean equation and the variance equation, researchers are afforded options of classifying a specific event's impact as either mean structure break or volatility structure break by introducing dummy variables into the mean equation or the variance equation. Due to the fact that regularity conditions are hard to verify for GARCH models, asymptotic distribution properties of maximum likelihood estimates for GARCH models have not been well understood. The situation may become more complicated if some additional explanatory variables are included in a GARCH model.

Some empirical studies tend to ignore the problem and simply assume that regularity conditions hold. However, some scholars have recently started to report various concerns on including dummy variables into GARCH models. Doornik and Ooms (2008) point out that including a dummy variable as a regressor into mean equation may cause a multimodality problem. Specifically, the maximum likelihood estimate (MLE, hereafter) of the coefficient of the dummy regressor may no longer be unique. In the meantime, given the fact that dummy variables in variance equations also enjoy popularity, a limited number of studies, e.g. Hillerbrand (2005), have been published to stress specific issues related to application of including variance dummy variables into GARCH models to business and economic event studies.

In this paper, we attempt to investigate distribution properties of MLE for the variance dummy variable coefficients through simulation studies. Especially, the aim is to gain a better understanding of under what circumstances the variance dummy variable technique can be problematic, and what the problems is. The paper is organized as follows: Section II gives a brief introduction to regression based event study methodology. Section III investigates whether multimodality is a problem for the variance dummy variable. In Section IV, we conducted two simulation studies to identify some important distribution properties of MLE for variance dummy variable coefficients. Finally, some discussions and conclusions are presented in Section V.

EVENT STUDY METHODOLOGY BASED ON REGRESSION MODEL

It has been a long history since the first event study conducted by Dolley (1933), in which he examined the effect of stock splits on stock price. Today, event studies are carried out in almost every research field in business and economics. For instance, economists measure effect of economic or political events, e.g. major regulatory change or 911 terrorist attack, by investigating the abnormal performance of some economic indices. Meanwhile, financial researchers may be interested in the impact of quarterly earnings announcements on companies' daily common stock returns.

Numerous econometric methods have been developed for event studies. However, the standard format of those methods has not changed much over time. It is still based on the classical event study conducted by Fama et al. (1969). The essence of the event study methodology lies in measuring abnormal data observations around the event time point, namely event window, and determining statistical significance of the abnormal data through statistical hypothesis testing. Comprehensive reviews of classical event study methodology are covered in various useful references including Campbell et al. (1997).

The most commonly applied event study model is the dummy variable regression with dummy variables corresponding to abnormal data observations covered by some event windows. The simplest regression based event study model is given as

FIGURE 1
REGRESSION EQUATION WITH DUMMY VARIABLE

$$y_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i x_{i,t} + \gamma d_t + \varepsilon_t, \varepsilon_t \sim N(0,h),$$

where x's are lagged endogenous or/and exogenous variables, dummy variable d_t □is specified for a particular event window $\{s_1, s_2\}$ ($d_t = 1$ if $s_1 \le t \le s_2; = 0$, otherwise). Then, inference for the event effect is based on comparison between two hypotheses: H$_0$: $\gamma = 0$ *vs.* H$_1$: $\gamma \ne 0$. If strong statistical evidence exists to reject the null hypothesis, study may conclude that effect of the event on underlying business or economic time series y_t is substantial. However, this methodology bears two important assumptions: 1). the event induces mean structure break; 2) the variance structure is consistent through time. However, both of the two assumptions can be sometimes questionable. In much of the research concerning business and/or economic time series data, modeling mean structures as essentially stable is a preferred practice, and the mean structures are determined by long-term factors with aberrations assumed into the variance and/or distribution structure. Especially in various financial researches, that is consistent with efficient market hypothesis by Malkiel (1987).

Engle (1982) and Bollerslev (1986) reveal that many business and economic time series have conditionally dependent volatility structure. In particular, Bollerslev (1986) proposes a GARCH model which utilizes the property of conditionally dependent volatility and provides more accurate forecasts than traditional regression models. Unlike classical regression models which assume that volatility is time invariant and constant, GARCH-type models consider that one-step-ahead conditional variance be dependent on current available information. As a result, we first have the mean structure model

FIGURE 2
MEAN STRUCTURE MODEL

$$y_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i x_{i,t} + \varepsilon_t, \varepsilon_t \left| \mathrm{I}_{t-1} \sim N(0, h_t) \right.,$$

where x's are lagged endogenous and/or exogenous variables; I_{t-1} is the information accumulated up to time t-1. Then, the GARCH (q, p) regression model further specifies a conditional variance regression model

FIGURE 3
VARIANCE STRUCTURE MODEL

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_i \varepsilon_{t-j}^2 + \sum_{k=1}^{p} \beta_k h_{t-k}.$$

In FIGURE 3, α_i and β_k are subject to the constraints: $\sum_{j=1}^{q} \alpha_i + \sum_{k=1}^{p} \beta_k < 1$ and $\alpha_i, \beta_k > 0$. It has been shown that GARCH (1, 1) is already sufficient for most business and economic time series data, e.g. Chong et al. (1999), French et al. (1987) and Franses and Van Dijk (1996).

Because the GARCH model specifies the volatility structure and has very simple form in many cases, it has been widely applied in event studies. With GARCH model, researchers can assume that an event causes either mean structure break or volatility structure break by introducing dummy variables into the mean equation in FIGURE 2 or the variance equation in FIGURE 3. Adding the dummy variables into the variance equation is especially meaningful for economic researchers. That is because many economic time series are better considered having stable long term mean structure, which should not be easily affected by some temporary events. In that case, temporary volatility structure break had better to be considered. Numerous event studies using the variance dummy variable methodology have been published. Moreover, those studies consider both variance inflation ($\gamma > 0$) problems and variance deflation ($\gamma < 0$) problems, e.g. Edison and Reinhart (2001), Bologna and Cavallo (2002) and Mazouz and Bowe (2009).

MAXIMUM LIKELIHOOD ESTIMATION FOR VARIANCE DUMMY VARIABLE COEFFICIENT

Doornik and Ooms (2008) point out that including a dummy variable as a regressor into a mean equation with GARCH residual structure may cause a multimodality problem. Specifically, the maximum likelihood estimate (MLE) of the dummy regressor coefficient may no longer be unique. In this study, we instead consider a GARCH-Dummy model in the following form

FIGURE 4
GARCH (*p*, *q*) MODEL

$$y_t = \xi_0 + \sum_{i=1}^{l} \xi_i x_i + \varepsilon_t, \varepsilon_t \left| \mathrm{I}_{t-1} \sim N(0, h_t) \right.,$$

$$h_t = \alpha_0 + \sum_{j=1}^{p} \alpha_i \varepsilon_{t-j}^2 + \sum_{k=1}^{q} \beta_k h_{t-k} + \gamma d_t,$$

where d_t is defined as the dummy variable for a particular event window $\{s_1, s_2\}$ ($d_t = 1$ if $s_1 \le t \le s_2$; $= 0$ otherwise). The log likelihood function for the GARCH model in FIGURE 4 is given by

FIGURE 5
LIKELIHOOD FUNCTION

$$l(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left(\log(h_t) + \frac{\varepsilon_t^2}{h_t} \right).$$

In this paper, we particularly focus on the GARCH (1, 1) conditional variance structure

FIGURE 6
GARCH (1, 1) VARIANCE STRUCTURE

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma d_t,$$

which is plausible in that numerous studies have demonstrated that GARCH (1, 1) model is sufficient for many business and economic time series.

Assuming ε_0 and h_0 are given, we rewrite the function in FIGURE 6 into

FIGURE 7
MODIFIED GARCH (1, 1) VARIANCE STRUCTURE

$$h_t = \alpha_0 \frac{1 - \beta_1^t}{1 - \beta_1} + \alpha_1 \sum_{j=1}^{t} \beta_1^{j-1} \varepsilon_{t-j}^2 + \beta_1^t h_0 + \gamma \sum_{j=1}^{t} \beta_1^{t-j} d_j$$

with $\alpha_0 + \beta_1 < 1$ and $\alpha_1, \beta_1 > 0$. The first derivative of h_t with respect to γ is simply written as $\frac{\partial h_t}{\partial \gamma} = \sum_{j=s_1}^{t} \beta_1^{t-j}$ when $s_1 \le t \le s_2$; $= \sum_{j=s_1}^{s_2} \beta_1^{t-j}$; $=0$ otherwise. Then, the conditional score function with respect to γ is

FIGURE 8
SCORE FUNCTION

$$\frac{\partial l(\theta)}{\partial \gamma} = -\frac{1}{2} \left(\sum_{t=s_1}^{s_2} \frac{h_t - \varepsilon_t^2}{h_t^2} \sum_{j=s_1}^{t} \beta_1^{t-j} + \sum_{t=s_2+1}^{T} \frac{h_t - \varepsilon_t^2}{h_t^2} \sum_{j=s_1}^{s_2} \beta_1^{t-j} \right)$$

The MLE $\hat{\gamma}$ at first satisfies $\left. \frac{\partial l(\theta)}{\partial \gamma} \right|_{\gamma = \hat{\gamma}} = 0$.

From FUGURE 7 and FIGURE 8, it is inferred that if the solution $\hat{\gamma}$ does exist for given s_1, s_2 and T, the solution is unique since h_t is monotonic function of γ. As a result, the multimodality problem will not be a concern for the variance dummy variable. In other words, MLE of the variance dummy variable coefficient does not have misleading mathematical properties as the mean dummy variable coefficient does.

MONTE CARLO SIMULATION

In this section, we attempt to gain a better understanding of distribution properties of MLE for the variance dummy variable coefficient in a GARCH model. Especially, the focus is on detection of temporary volatility structure break in business and economic event studies.

The first study proposed in this paper is aimed at understanding distribution properties of *t*-test statistic for MLE for the variance dummy variable coefficient γ, when the null hypothesis in an event study is correct, i.e. $\gamma = 0$.

Time series data satisfying the null hypothesis are simulated according to the following model

FIGURE 9
GARCH (1, 1) SIMULATION MODEL

$$
\begin{aligned}
y_t &= 1 + \varepsilon_t, \\
\varepsilon_t &= \lambda h_t^{1/2}, \lambda \sim N(0,1), \\
h_t &= 1 + 0.1\varepsilon_{t-1}^2 + 0.6h_{t-1} + 0d_t,
\end{aligned}
$$

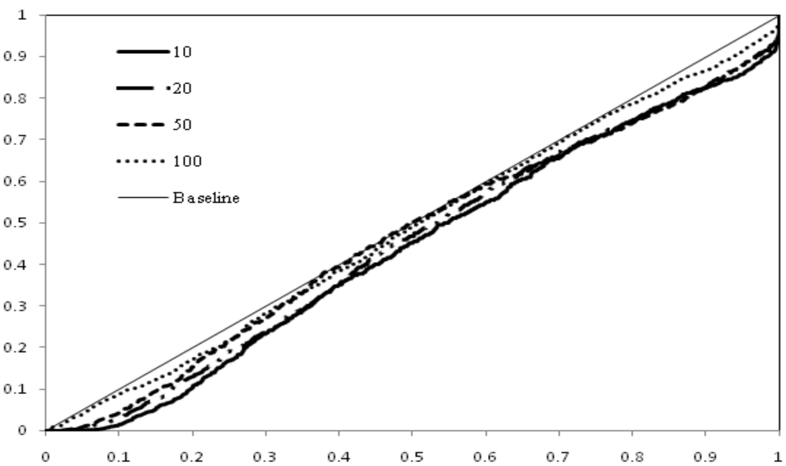
where □α_0, α_1, and β_1 values are specified such that the model is well in the interior of the parameter space. Similar simulation parameter settings were also adopted by Lumsdaine (1995). The initial values, ε_0 and h_0, are simply assigned a value of 1. Lee and Hansen (1994) show that the dependence of model results on initial values is asymptotically negligible according to the property of GARCH (1, 1) model likelihood they identified. Certainly, different choices of the initial values can be proposed here.

A data sample of size 5,500 was repeatedly simulated for 1,000 times. The first 500 observations in each data sample were eliminated from the following study in order to minimize the influence of initial value setting on estimation results. Moreover, asymptotic properties should be less a problem with such a large sample size.

Next, the model in FIGURE 9 was estimated on each of the 1000 simulated data samples through maximum likelihood method. We studied four event windows including 10, 20, 50 and 100 observations respectively. FIGURE 10 is Normal Q-Q plot of the *t*-test statistic distribution associated with the estimated dummy variable coefficient corresponding to each of the above four event windows.

FIGURE 10 shows that the empirical distribution of *t*-test statistic for estimated coefficient of the variance dummy variable under null hypothesis has heavier tail to the left and lighter tail to the right, i.e., more negatively skewed than standard distribution. Moreover, the departure from normality is more significant for a relatively short event window, e.g., fewer than 100. This interesting property implies inflated Type I error rate when estimated γ is negative.

FIGURE 10
Q-Q PLOTS OF *t*-TEST STATISTIC DISTRIBUTIONS



To better understand the relationship between Type I error rate and event window size, we repeatedly generated 1,000 random sample of size 5,000 and calculated Type I error rate as the relative proportion of samples supporting rejection decision on null hypothesis at 5% significance level for each event window size from 10 to 200. The results are presented in FIGURE 11.

FIGURE 11
SCATTER PLOT OF TYPE I ERROR RATES

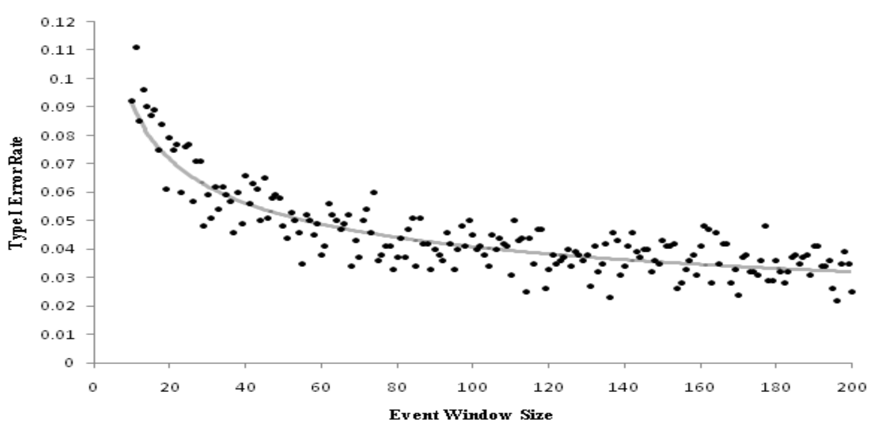
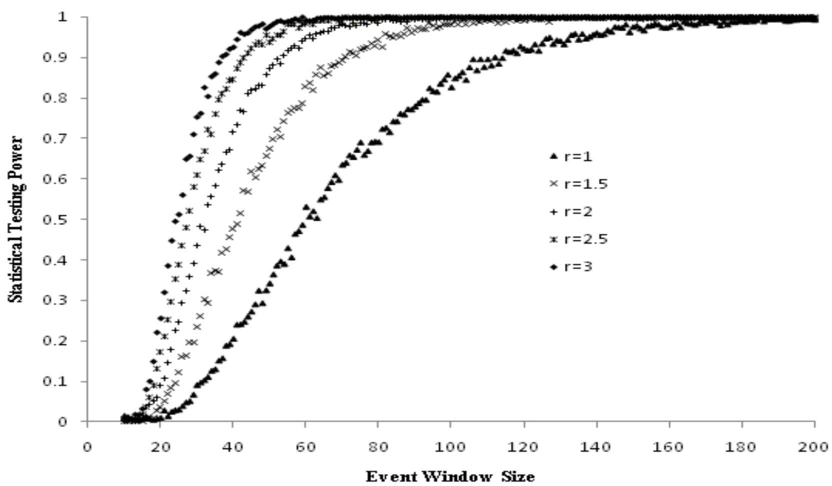


FIGURE 11 reveals a strong relationship between Type I error rate and event window size. Especially, it is seen that for an event window size approximately smaller than 100, empirical Type I error rate is more likely inflated against the nominal level at 0.05. This result is particularly meaningful for some event studies involving the detection of variance deflation. e.g. Mazouz and Bowe (2009). That is variance deflation can be overstated if an event study defines a relatively small event window size.

The second study focuses on how both scale of variance inflation and event window size can affect maximum likelihood estimate for the variance dummy variable coefficient. Similar to the first study, one thousand random GARCH (1, 1) series in FIGURE 9 of size 5,000 were generated for each window size from 10 to 200, except that this process was repeated for different selected γ values rather than value of 0, i.e. $\gamma = 1$, 1.5, 2, 2.5 and 3 respectively. The simulated Type II error rates, which are calculated as relative proportion of samples supporting no rejection decision on null hypothesis at 5% significance level, are used to create plot of statistical testing power shown in FIGURE 12, i.e. Type II error rate subtracted from one.

FIGURE 12
SCATTER PLOT OF STATISTICAL TESTING POWER



The low statistical testing power shows that the GARCH model often underestimates variance inflation. Given 0.8 as a convenient standard for adequacy of testing power, FIGURE 12 implies event window size of at least 100 is needed in order that reliable inference is obtained when variance inflation does appear.

DISCUSSION AND CONCLUSION

The GARCH (1, 1) model with dummy variables included in variance equation has been a popular methodology applied to various business and economic event studies to detect time series structure break. Notably, conclusion on effects of a business or economic event depends largely on statistical inferences based on MLEs for coefficients of the variance dummy variables.

Motivated by other scholars' work on different concerns about including dummy variables in GRACH models, e.g., Hillerbrand (2005) and Doornik and Ooms (2008), we conducted some simulation studies in this paper, which revealed important distribution properties of MLE for the variance dummy variable coefficient in a GARCH (1, 1) model. Moreover, those properties may lead to misleading inference for business and economic event studies.

The unreliable inference can go in two separate directions. At first, our simulation results show that distribution of *t*-test statistic for the variance dummy variable coefficient is more negatively skewed than normal distribution under null hypothesis. As a result, variance deflation can possibly be overstated. On the other hand, if variance inflation is present, the GARCH model tends to understate its significance. Consequently, if this GARCH-Dummy methodology is used, hypothesis testing power for determining the impact of business and economic events can be limited. Therefore, when conducting event studies, researchers need to be aware of the possibility of those unreliable inferences resulted from including dummy variables into GARCH variance equations. In particular, the concern of unreliable inference becomes crucial for some event studies focusing on relatively short event windows. In regular regression model, it is even allowable that dummy variable is used to identify a single data point. However, our study results indicate that the size of an event window is a critical factor determining the reliability of statistical inferences, if the GARCH-Dummy methodology is applied. It is suggested that an event window including at least 100 data points be defined in order to ensure reliable statistical inference in most cases. On the other hand, if short event window is necessary in an event study, some discussion on the possibility of unreliable inference is recommended when the study obtains slightly significant result for the detection of variance deflation or slightly insignificant result for the detection of variance inflation.

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