Reformulation of Maximal Available Coverage Location Problem: A Technical Note

Sanjay Mayadunne
Georgia Gwinnett College

Hari K Rajagopalan
Francis Marion University

Cem Saydam
The University of North Carolina at Charlotte

We reformulate the maximum availability location problem (MALP) with fewer variables and constraints. Our computational experiments demonstrate that the reformulated MALP solves much faster and provides improved solutions under certain conditions.

INTRODUCTION

This technical note presents a revised formulation of the novel maximum availability location problem (MALP) and documents its advantages. Hence, we focus on the important developments which preceded MALP. For a more detailed literature review, we refer the reader to ReVelle et al. (C. ReVelle, Eiselt, & Daskin, 2008) for a comprehensive review of location modeling, and to Brotcorne et al.'s (Brotcorne, Laporte, & Semet, 2003) review of recent developments in ambulance location problems.

The two earliest models where the Set Covering Location Problem (SCLP) (Toregas, Swain, ReVelle, & Bergman, 1971), which minimized the total cost of ambulances needed to cover a region and the Maximum Covering Location Problem (MCLP) (Church & ReVelle, 1974) which maximized coverage given a set of ambulances. Both these models were deterministic in nature and did not account for the probability that the ambulances will not be available.

One of the two models which did address the unavailability of ambulances was the Maximum Expected Coverage Location Problem (MEXCLP) (Daskin, 1983). By making assumptions that ambulances are busy with the same probability and operate independently, the MEXCLP maximizes the expected coverage given a set of ambulances. About the same time, ReVelle and Hogan (C. ReVelle & Hogan, 1989) addressed this issue by proposing MALP which maximized coverage while guaranteeing coverage with a certain reliability. In this technical note we reformulate MALP with a fewer numbers of constraints and variables and show that it solves faster and provides improved solutions. In section 2 we review MALP in detail and in section 3, we introduce r-MALP. In section 4 we present computational statistics, and results from simulated data and historical data from Charlotte Mecklenburg. Section 5 contains the conclusions.

MAXIMUM AVAILABLE LOCATION PROBLEM

ReVelle and Hogan (C. ReVelle & Hogan, 1989) formulate the Maximum Availability Location Problem (MALP I and MALP II) to maximize the population the servers can cover (within a target response time) with a reliability of α. Let,

$n =$ number of nodes in the system
$m =$ number of ambulances available
$h_j =$ demand at node j
$\bar{t} =$ average service time

$$x_i = \begin{cases} 1 \text{ of servers positioned in node } i \\ 0 \text{ if not} \end{cases}$$

$$y_{jb} = \begin{cases} 1 \text{ if } b \text{ servers cover node } j \\ 0 \text{ if not} \end{cases}$$

$$a_{ij} = \begin{cases} 1 \text{ if node } j \text{ is covered by server at node } i \\ 0 \text{ if not} \end{cases}$$

Then the average busy probability of an ambulance can be estimated by

$$\rho = \frac{\bar{t}\sum_{j=1}^{n} h_j}{24\sum_{i=1}^{n} x_i} = \frac{\bar{t}\sum_{j=1}^{n} h_j}{24\, m} \quad (1)$$

With this definition of busy fraction, a chance constraint on service availability can be written in order to determine the service requirements of the demand areas. Chance constraints formulated by Charnes and Cooper (1959) can be used. The chance constraint is

$$1 - \rho^{\sum_{i=1}^{n} a_{ij}x_i} \geq \alpha \quad (2)$$

Where α is the desired reliability in coverage. Therefore the number of servers needed to cover a node so that we can have α reliable coverage is

$$b = \left\lceil \frac{\log(1-\alpha)}{\log p} \right\rceil \quad (3)$$

Therefore each demand area will require at least b servers in order to attain the required level of coverage with α reliability. The objective function in the MALP is to maximize the total demand covered by at least an α level of reliability

Maximize

$$\sum_{j=1}^{n} h_j y_{jb} \quad (4)$$

Subject to

$$\sum_{i=1}^{n} a_{ij}x_i \geq \sum_{k=1}^{b} y_{jk} \qquad \forall j \quad (5)$$

$$y_{jk} \leq y_{j,k-1} \qquad \forall j \quad (6)$$

$$\sum_{i=1}^{n} x_i \leq m \quad (7)$$

$x_i, y_{jk} \epsilon \{0,1\} \qquad \forall j$ (8)

The objective function (4) maximizes demand covered by b servers which guarantees α reliability. Constraint (5) counts the number of nodes covered by b servers and constraint (6) ensures that a node is first covered once before it is covered twice and so on. Constraint (7) ensures that the total number ambulances are not greater than the number of ambulances available.

REFORMULATED MAXIMUM AVAILABLE LOCATION PROBLEM (R-MALP)

In r-MALP we modify the following variables from MALP. We change

$y_{jb} = \begin{cases} 1 \text{ if } b \text{ servers cover node } j \\ 0 \text{ if not} \end{cases}$ to $y_j = \begin{cases} 1 \text{ if node } j \text{ is covered at least b times} \\ 0 \text{ if not} \end{cases}$

and redefine

$x_i = \begin{cases} 1 \text{ of servers positioned in node } i \\ 0 \text{ if not} \end{cases}$ to $x_i = \text{number of servers located at node i}$

With these modifications MALP is reformulated to

Maximize

$$\sum_{j=1}^{n} h_j y_j \tag{9}$$

Subject to

$$\sum_{i=1}^{n} a_{ij} x_i \geq b y_j \qquad \forall j \tag{10}$$

$$\sum_{i=1}^{n} x_i \leq m \tag{11}$$

$$y_j \epsilon \{0,1\} \qquad \forall j \tag{12}$$

The objective function (9) still maximizes the demand locations covered by b servers which guarantees α reliability. Constraint (10) ensures that node j is deemed covered ($y_j = 1$) only if it is within coverage distance of the required number of vehicles which is denoted by b. By changing the definition of variables we have eliminated the need for constraint (6). Further by changing the right hand side value of constraint (10) we have eliminated the need for variables $y_{jk}, k = 1 \; to \; b$.

These changes result in a more compact and robust model. In a problem size with b servers and j demand nodes the MALP has $(jb + 1)$ constraints and $j(2 + b)$ variables while the r-MALP has $(j + 1)$ constraints and $(3j)$ variables thus reducing the number of constraints and variables by $j(b - 1)$. Table 1 below shows the difference between MALP and r-MALP for a problem size of 100 demand nodes and 100 server nodes. As can be seen from Table 1 as the number of vehicles needed to guarantee coverage with α reliability (b) increases the difference between the number of variables and constraints used in the two models increases significantly.

TABLE 1
COMPARISON BETWEEN MALP AND r-MALP FOR VARYING VALUES OF *b*

b	*MALP*		*r-MALP*	
	Variables	*Constraints*	*Variables*	*Constraints*
1	300	101	300	101
2	400	201	300	101
3	500	301	300	101
4	600	401	300	101
5	700	501	300	101

This shows the robustness of the r-MALP model as it can solve for multiple levels of server availability without increasing the problem size. The usefulness of the r-MALP model in this regard is evident when solving problems where ambulance availability is limited. If the number of ambulances available is large enough to saturate the region then the problem is trivial. However in a practical setting we typically deal with limited ambulance availability and thus have to evaluate a large number of potential solutions. The fact that the r-MALP is less constrained is an advantage since it will lead to faster solution times. In Table 2 we consider the impact of increasing demand nodes on the two formulations. As can be seen from Table 2 as the number of available vehicles increases the number of variables and constraints increase for MALP but for the reformulated r-MALP they remain constant. In instances where we have a predetermined level of reliability ($b = 3$) and wish to increase the number of demand nodes. The difference between the number of variables /constraints used in the MALP and the number of variables/constraints used in the r-MALP increases exponentially as the number of demand nodes is increased.

One of the issues that arise when using spatial information in optimization models is the error caused by aggregation. Emergency calls arising from a zone are aggregated into a single demand node. Francis et al. (Francis, Lowe, Rayco, & Tamir, 2009) explain that, using a larger number of demand nodes will decrease model error but will increase modeling and computing cost. By using the r-MALP we are better able to decrease model error while keeping computing cost relatively low.

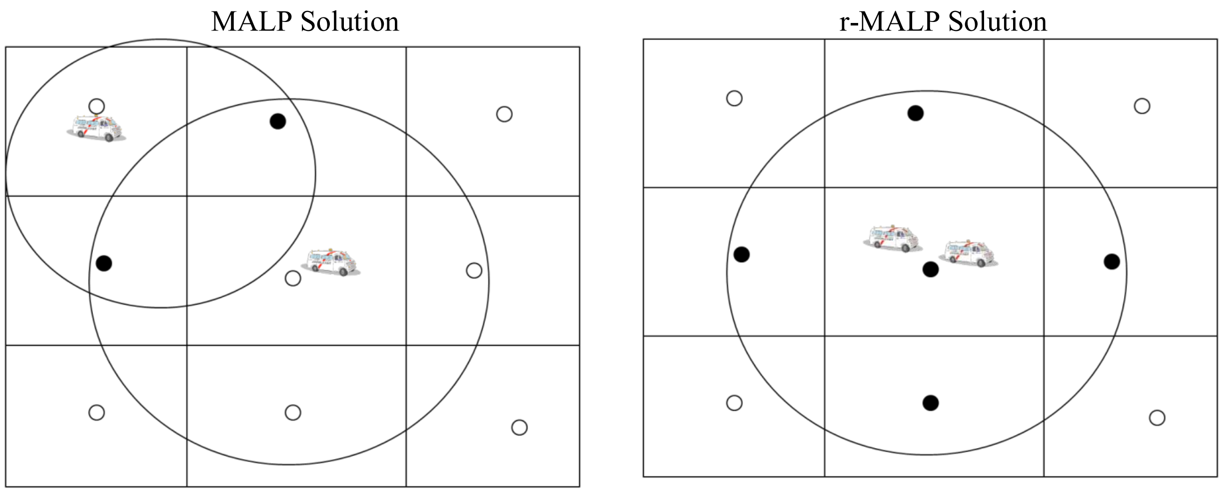
TABLE 2
COMPARISON BETWEEN MALP AND r-MALP FOR VARYING VALUES OF *j*

j	*MALP*		*r-MALP*	
	Variables	*Constraints*	*Variables*	*Constraints*
25	125	76	75	26
50	250	151	150	51
100	500	301	300	101
200	1000	601	600	201
400	2000	1201	900	401

The ability of the r-MALP to provide a superior solution can be shown by the following example. Consider an area divided into 9 nodes($3X3$). An ambulance can cover demand originating from the zone it is located in and demand originating from the four adjacent zones. We set the minimum number of servers required to achieve the desired reliability at 2 and solve the problem of locating 2 servers so that coverage is maximized. An ambulance placed in the center zone covers 5 zones while an ambulance placed in any other zone will at most, cover 3 zones. Therefore the optimal solution would be to place both available ambulances in the center and provide coverage with the desired reliability to 5 zones. The results given by both models are illustrated in Figure 1.

The MALP model defines x_i as a binary variable. As a result it allows placing of one or zero ambulances in any given zone. By changing variable x_i from a binary to an integer variable the r-MALP allows the placing of multiple servers in a single zone. Thus the solution provides coverage to 5 zones by placing both servers in zone 5. The MALP solution can provide at best coverage to 2 zones by placing one server in in the center zone and the second server in any other zone. The difference between the optimal solution values given by the two models is magnified as the desired level of reliability is increased.

FIGURE 1
COMPARISON OF MALP AND r-MALP

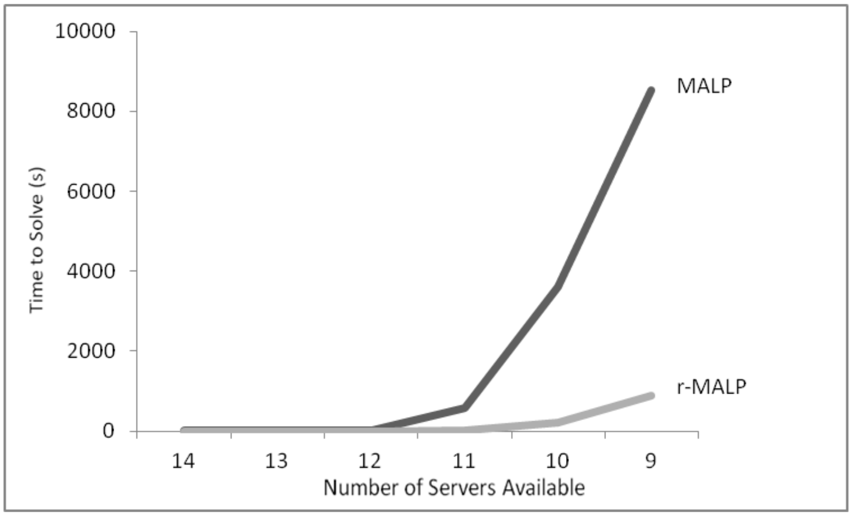


COMPUTATIONAL EXPERIMENTS

In order to compare the original MALP and the revised model we first generated a region which is 1024 sq miles in size and is divided in to 256 zones. Each zone is a square, 4 sq miles in size (2 miles by 2 miles). We randomly generated uniformly distributed call (demand) rates for each of the zones. Ten different sets of data were generated. We applied both models to the simulated data in order to compare the performance of each at different levels of server availability and reliability. The experiments, were conducted on a Dell PC Pentium IV 3.4 MHz with 2 GB RAM. The linear programming solver used was LINDO.

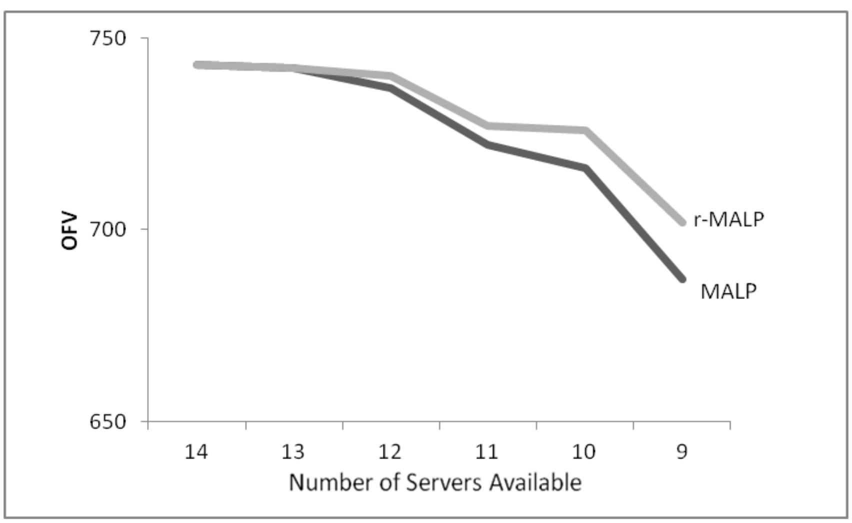
The results shown in Figure 2 indicate that the time to solve for the original MALP increases exponentially as the number of available servers decrease. The revised MALP utilizes the same number of variables and constraints for all levels of b therefore the average time to solve remains consistent. As mentioned above if a large number of ambulances are available then achieving blanket coverage becomes trivial. However when the number of available ambulances is limited then the problem becomes increasingly complex. Therefore for problems with a larger number of zones and limited resources using the revised MALP would result in a significant saving of time without any loss of optimality.

FIGURE 2
IMPACT OF REDUCING THE NUMBER OF AVAILABLE AMBULANCES ON SOLUTION TIME



We now examine the improvement in objective function values that the revised MALP provides. Figure 3 indicates that as the number of ambulances available reduces we are able to achieve an increasingly higher level of coverage using r-MALP when compared the coverage obtained using MALP. The r-MALP model allows us to provide enhanced coverage to high demand areas by placing multiple ambulances in one zone. As a result when resources are limited it provides a more optimal allocation.

FIGURE 3
IMPACT OF REDUCING THE NUMBER OF AVAILABLE AMBULANCES ON THE OBJECTIVE FUNCTION VALUE



These results are more pronounced when we examine the effect of an increasing level of reliability (α) as shown in figure 4 and 5. Increasing the desired reliability will increase the minimum number of ambulances required (b) to cover a zone. As shown above the variables /constraints used in the r-MALP remain constant as b increases. As a result the r-MALP displays consistent solution times across a range of values for α. It is therefore much better equipped to handle a large problem space which requires a high level of reliability. Figure 5 demonstrates the superiority of the r-MALP solution when α increases. Using the r-MALP we are better equipped to avoid sub-optimal solutions in cases of limited ambulance availability and high reliability requirements.

FIGURE 4
IMPACT OF INCREASING α ON SOLUTION TIME

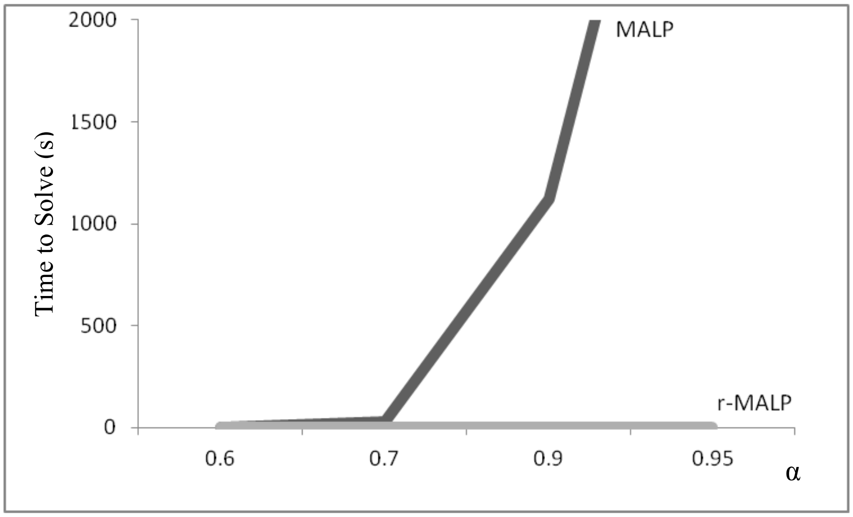
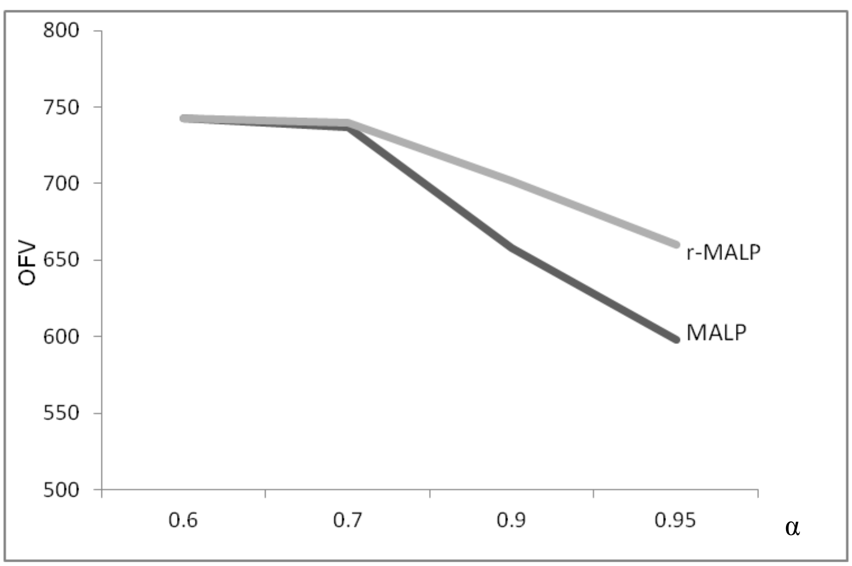


FIGURE 5
IMPACT OF INCREASING α ON THE OBJECTIVE FUNCTION VALUE



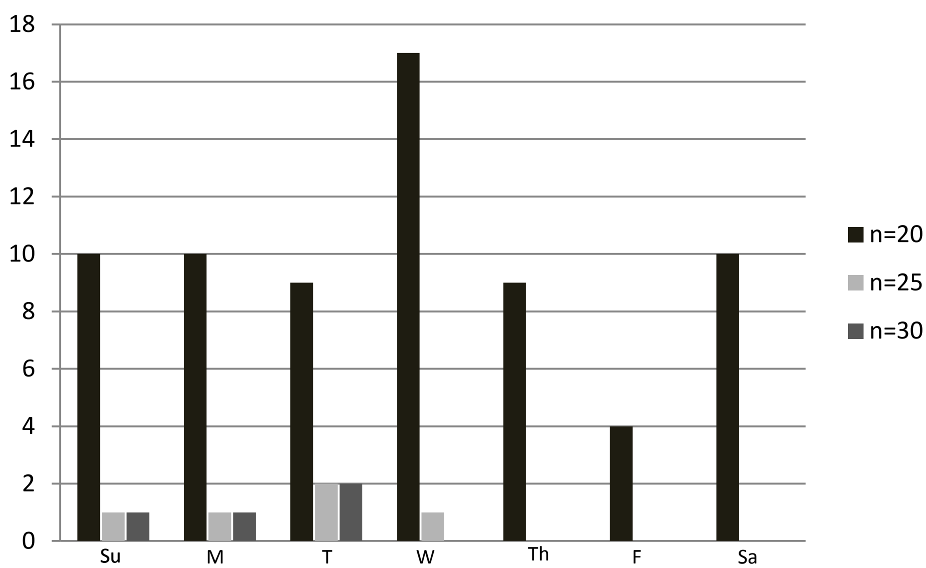
RESULTS FOR MECKLENBURG COUNTY DATA

We applied both models to Mecklenburg County (essentially, Charlotte and environs), North Carolina, a region of approximately 540 square miles with a population of 801,137 in 2004, which has grown to 919,628 as of 2010. In 2004, the county received a total of 77,292 calls for assistance, 62,092 of which were classified as medical emergencies. We superimposed a 2 x 2 sq. mile grid in order to aggregate the call data, resulting in 168 demand nodes. A key assumption was that ambulances could be located at any node with the exception of those on the outer edges of the grid. We ran both models on call data for each day of the week. We varied the parameters for reliability and number of ambulances available to compare the performance of each model.

The revised MALP outperformed the MALP for every problem set instance in our analysis. The difference in time taken to solve was more pronounced when the complexity of the problem increases. For example when the number of available ambulances is restricted to 20 and a reliability of 80% is desired the r-MALP solves to optimality in an average of 15 seconds while the MALP does not reach an optimal solution after 4 hours in 6 out of the 7 problem instances. When the problem is more constrained (reliability is increased to 90%) the r-MALP provides an optimal solution in less than 3.5 minutes in 5 out of the 7 problem instances while the MALP solution is suboptimal after 4 hours in all 7 problem instances.

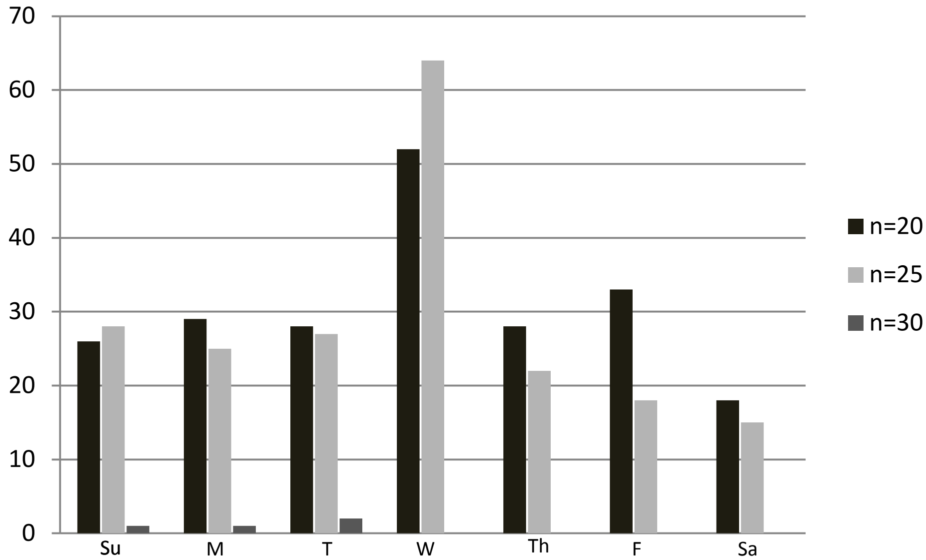
The ability of the r-MALP to provide a solution with superior coverage is illustrated in figures 6 and 7. Figure 6 shows the difference in objective function values for r-MALP and MALP when utilizing 20, 25 and 30 ambulances and a reliability of 80% is desired. The difference in optimal solutions is higher when the number of available ambulance is restricted. For all 3 levels of available ambulances the r-MALP provides at least as good a solution as the MALP. The impact of r-MALP is pronounced when the system is busy (either smaller number of servers or when higher reliability is required). Figure 7 gives the results a reliability level of 90%. Requiring higher levels of reliability with the same set of ambulances increases how busy the ambulances are going to be.

FIGURE 6
DIFFERENCE IN COVERAGE BETWEEN r-MALP AND MALP FOR α=80%



As expected the difference in optimal solutions is more pronounced as α is increased to 90%. When the number of servers increases to 30 and the system has more servers than necessary the difference between the two models drop. The r-MALP solution provides coverage to as much as 64 more calls when compared to the MALP solution. On average the r-MALP covers 26.5 more calls when the number of ambulances available is 20 and 25.

FIGURE 7
DIFFERENCE IN COVERAGE BETWEEN r-MALP AND MALP FOR α=90%



CONCLUSIONS

The contribution of this technical note is twofold. First, the reformulation of MALP allows the problem to be solved with a fewer number of variables and constraints leading to considerable savings in solution time. The efficiency of the reformulated MALP when compared to the original model is pronounced when it is applied to complex problems (problems with a large number of zones requiring a high level of reliability). Second the reformulated model provides a superior solution. As a result a larger number of calls are covered with the desired level of reliability. We have shown the improvements in efficiency, solution time and objective function value by applying both models to simulated and real data from Charlotte Mecklenburg. This is an important development since the MALP has subsequently been extended in models such as Marianov and ReVelle's (Marianov & ReVelle, 1996) Q-MALP. Here the minimum number of servers required is calculated by "treating the arrival and service times as a queuing system. The Q-MALP was further extended by Ghani (Ghani, 2012) where two types of servers (Advanced Life Support and Basic Life Support) are considered. Rajagopalan, Saydam et al. (Rajagopalan, Saydam, Setzler, & Sharer, 2012) formulated the Dynamic Available Coverage Location Model (DACL) where the objective function is to minimize the number of servers required to ensure a predetermined level of coverage with reliability α. In both the MALP extensions and the DACL the revisions to the MALP can be used to reduce solution times and improve objective function values.

REFERENCES

Brotcorne, L., Laporte, G., & Semet, F. (2003). Ambulance location and relocation models. *European Journal of Operational Research, 147*(3), 451-463.
Charnes, A., & Cooper, W. (1959). Chance Constrained Programming. *Management Science, 6*, 73-79.
Church, R. L., & ReVelle, C. (1974). The maximal covering location problem. *Papers of Regional Science, 32*(1), 101-118.
Daskin, M. S. (1983). A maximal expected covering location model: Formulation, properties, and heuristic solution. *Transportation Science, 17*(1), 48-69.

Francis, R. L., Lowe, T. J., Rayco, M. B., & Tamir, A. (2009). Aggregation error for location models: survey and analysis. *Annals of Operations Research, 167*(1), 171-208. doi: 10.1007/s10479-008-0344-z

Ghani, N. A. (2012). *Multi-server Queuing Maximum Availability Location Problem with Stochastic Travel Times.* Paper presented at the World Congress on Engineering London.

Marianov, V., & ReVelle, C. (1996). The Queueing Maximal availability location problem: A model for the siting of emergency vehicles. *European Journal of Operational Research, 93*(1), 110-120.

Rajagopalan, H. K., Saydam, C., Setzler, H., & Sharer, E. (2012). Decision Making for Emergency Medical Services. In M. P. Johnson (Ed.), *Community-Based Operations Research* (pp. 275-296). New York: Springer.

ReVelle, C., Eiselt, H. A., & Daskin, M. S. (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research, 184*(3), 817-848.

ReVelle, C., & Hogan, K. (1989). The maximum availability location problem. *Transportation Science, 23*(3), 192-200.

Toregas, C., Swain, R., ReVelle, C., & Bergman, L. (1971). The location of emergency service facilities. *Operations Research, 19*(6), 1363-1373.