Should New Products Use Advertising and Price Promotions Simultaneously

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This research aims to study how advertising and price promotions interact as consumers with advertising information increases. A rational expectations model shows that an advertising threshold of quality indication exists. Before reaching the threshold, consumers depend more on prices to infer quality as advertising information diffuses. At this stage, advertising and price promotions offset each other’s strengths so that they should not use simultaneously. After advertising coverage crosses threshold, consumers gradually tend to use advertising information to infer product quality. At this stage, advertising can reduce negative effects from price promotions and makes price promotions more effective at generating sales.

INTRODUCTION

The success of new products depends heavily on a company's strategies for marketing mix activities such as advertising and price promotions. The association between advertising and prices are based on two broad economic theories. One predicts that advertising reduces the elasticity of price because firms use advertising to create brand loyalty that makes consumers less sensitive to prices (Bain, 1956; Comanor and Wilson, 1974; Lambin, 1976; Popkowski-Leszczyc and Rao, 1989; Boulding et al., 1994; Sethuraman and Tellis, 2002). In contrast, the other theory regards advertising as a source of information about choices. From this viewpoint, advertising increases price elasticity by allowing consumers to comparison shop (Stigler, 1961; Nelson, 1974; Grossman and Shapiro, 1984; Steiner, 1993; Shankar and Bolton, 2004; Erdem et al., 2008). Unless a reconciliatory theory is provided, it is difficult for such conflicting results to offer practical guidelines.

Price can serve as a signal of quality; the quality-assuring price must be large enough to make the value of repeated sales exceed the one-time gain from cheating, so the firms will never find it optimal to cheat consumers. Any individual who observes a firm attempting to sell a product for less than the quality-assuring price would infer the product is a low-quality product (Klein and Leffler, 1981; Bagwell
Evidence for a robust price-quality effect has been extensively recognized in empirical studies (McConnell, 1968; Monroe, 1973; Rao and Monroe, 1989; Dodds et al., 1991). Considering the price-quality relationship, price promotions simultaneously have two opposing effects on sales. One increases the sales quantity due to the downward sloping demand curve, which is referred to as the positive effect brought about by price promotions. The other depresses the sales quantity, due to consumers’ misinterpretation of the price cuts as reductions in quality, which is referred to as the negative effect brought about by price promotions.

A product with less negative effects caused by price promotions has a greater chance of being selected by retailers as a “loss leader” (Albion, 1983). Retailers use loss leaders to build store traffic on the assumption that consumers will purchase other products in addition to the promoted products. Although price promotions stimulate consumers’ incentive to purchase, they can also impose negative effects on consumer evaluation of discounted products (Kalyanaram and Winer, 1995; Mela et al., 1998; Jedidi et al., 1999; Erdem et al., 2008). Byzalov and Shachar (2004) show that advertising increases consumers’ tendency to purchase the promoted product because the advertising reduces the risk that consumers face. In this study, a rational expectations model shows that an advertising threshold can determine if advertising reduces negative effects from price promotions. Before reaching the threshold, consumers depend more on prices to infer product quality as advertising information diffuses. At this stage, advertising decreases price sensitivity so that price promotions cannot bring desired results. After advertising coverage crosses the threshold, consumers gradually tend to use advertising information to infer product quality (Clark et al., 2009). At this stage, advertising increases price sensitivity and makes price promotions an effective way at generating sales. The logic of U-shape relationship between advertising and price sensitivity can help retailers design efficient promotion strategies.

The logic of this study are demonstrated by a rational expectations model. This model was initially proposed by Muth (1961) and Lucas (1972), who developed the concept of equilibrium, where individuals utilize statistical relationships between endogenous and exogenous variables. Grossman (1976) analyzed an economy in which traders hold diverse information in regard to the return on assets and claimed investors can infer the asset’s return through the price alone because all information is contained therein. Grossman and Stiglitz (1980) proposed that the market must contain a source of noise preventing agents from obtaining all the information from the price, creating an incentive to expend resources upon obtaining information. Hellwig (1980) argued that since noise comes from the supply side, the price cannot provide sufficient information, and so, simply observing price cannot provide enough information to predict the asset’s return. Admati (1985) provided generalized solutions for the rational expectations equilibrium models developed by Hellwig (1980). Finally, noisy rational expectations equilibrium has been applied to various issues in financial economics (Diamond, 1985; Diamond and Verrecchia, 1991; Easley and O’Hara, 2004; Ozdenoren and Yuan, 2008).

**MODEL**

Consider a product held by a risk-neutral retailer that faces a finite number of consumers. The retailer has monopolistic power that enables it to reduce retail prices by increasing retail supply. The assumption that a retailer can have market power is not unrealistic (Dobson, 2005). Let \( \lambda \in (0,1) \) be the fraction of consumers whose expectations regarding quality are based both on manufacturer advertising and retail price, and \( 1 - \lambda \) be the fraction of consumers who infer quality only by observing the retail price.

Suppose that the value of quality conveyed by manufacturer advertising is

\[
\alpha = q + \varepsilon
\]

where \( \alpha \) denotes the advertising information received by consumers. It expresses true quality \( q \) but is perturbed by noise \( \varepsilon \). Assume \( \varepsilon \) is normally distributed with a mean of zero and a variance of \( \sigma_{\alpha} \), and
\( q \) is normally distributed with a mean of \( \bar{q} > 0 \) and a variance of \( \sigma_q \). The demand function of consumers that infer quality only by observing the retail price is

\[
z_d(p) = \gamma \text{Var}(q|p)^{-1} \left( E(q|p) - p \right)
\] (2)

And the demand function of consumers that infer quality by advertising and retail price is

\[
z_d(a, p) = \gamma \text{Var}(q|a, p)^{-1} \left( E(q|a, p) - p \right)
\] (3)

where \( z_d \) denotes demand quantities, \( \gamma > 0 \) is the level of risk tolerance, and \( p \) denotes the retail price. Suppose that \( (q, a, p) \) has a joint normal distribution, the mean and variance of quality perceived by consumers who can observe advertising take the following forms

\[
E(q|a, p) = h_0 + h_1a + h_2p
\] (4)
\[
\text{Var}(q|a, p) = v_0
\] (5)

The mean and variance of quality perceived by consumers who can only observe retail price are

\[
E(q|p) = c_0 + c_1p
\] (6)
\[
\text{Var}(q|p) = d_0
\] (7)

The retailer launches price promotions by altering supply quantity, the retail supply is denoted by \( z_s \), which has normal distribution with a mean of \( \bar{z} > 0 \) and a variance of \( \sigma_z \). The retail supply variance \( \sigma_s \) represents the intensity of price promotions, meaning that the retailer can accumulate a certain amount of goods and then release them on one occasion while providing discounts.

**Advertising Coverage and Prices**

We let \( \bar{z}_d \) denote the average demand quantity, which is the weighted average of informed consumers and uninformed consumers’ quantity:

\[
\bar{z}_d = \lambda z_d(a, p) + (1 - \lambda) z_d(p)
\] (8)

Substituting Equation 2 and 3 into Equation 8, we have

\[
\bar{z}_d = \lambda \gamma \left[ \frac{E(q|a, p) - p}{\text{Var}(q|a, p)} \right] + (1 - \lambda) \gamma \left[ \frac{E(q|p) - p}{\text{Var}(q|p)} \right]
\] (9)

Then we substitute Equation 4 ~ 7 into Equation 9

\[
\bar{z}_d = \lambda \gamma \left[ \frac{h_0 + h_1a + h_2p - p}{v_0} \right] + (1 - \lambda) \gamma \left[ \frac{c_0 + c_1p - p}{d_0} \right]
\]

\[
= \left[ \frac{\lambda \gamma h_0}{v_0} + \frac{(1 - \lambda) \gamma c_0}{d_0} \right] + \left[ \frac{\lambda \gamma h_1}{v_0} a + \left( \frac{\lambda \gamma h_2}{v_0} + \frac{(1 - \lambda) \gamma c_1}{d_0} \right) p \right] - \frac{\gamma (\lambda p + (1 - \lambda) p)}{v_0 d_0}
\] (10)

Let \( \bar{z}_d = z_s \), we can obtain the equilibrium price. Quality-related information can be conveyed by the retail price, but it is perturbed by noise from the retail supply. The amount of quality information a consumer acquires is determined, in part, through the retail price. The retail price, in turn, depends on the
amount of quality information that consumers acquire. The equilibrium retail price and information acquisition have to be solved simultaneously, as each one affects the other.

**Lemma 1.** A rational expectations price \( p \), for a given value of \( \lambda \), is a linear function of advertising information \( a \) and retail supply \( z_s \).

\[
p = b_0 + b_1 a - b_2 z_s
\]  
(11)

where

\[
b_0 = \frac{v_0 d_0}{(\lambda d_0 + (1-\lambda)v_0)} \left[ \frac{1}{\sigma_q} - q + \frac{\gamma \lambda}{\gamma^2 \lambda^2 + \sigma_a \sigma_a} \right]
\]

\[
b_1 = \frac{v_0 d_0}{(\lambda d_0 + (1-\lambda)v_0)} \frac{\lambda}{\sigma_a} \left[ 1 + \frac{\gamma^2 \lambda}{\gamma^2 \lambda^2 + \sigma_a \sigma_a} \right]
\]

\[
b_2 = \frac{v_0 d_0}{(\lambda d_0 + (1-\lambda)v_0)} \left[ \frac{1}{\gamma} \right] \left[ 1 + \frac{\gamma^2 \lambda}{\gamma^2 \lambda^2 + \sigma_a \sigma_a} \right]
\]

\[
v_0 = \left[ \sigma_q^{-1} + \sigma_a^{-1} + (\sigma_a + \sigma_s \sigma_a^2 (\gamma \lambda)^{-2})^{-1} \right]^{-1}
\]

\[
d_0 = \left[ \sigma_q^{-1} + (\sigma_a + \sigma_s \sigma_a^2 (\gamma \lambda)^{-2})^{-1} \right]^{-1}
\]

Proof: Substituting Equations 4~7 into Equation 10, we have

\[
b_0 = \frac{\lambda h_0 d_0 + (1-\lambda)c_0 v_0}{\lambda(1-h_2)d_0 + (1-\lambda)(1-c_1)v_0}
\]  
(12a)

\[
b_1 = \frac{\lambda h_0 d_0}{\lambda(1-h_2)d_0 + (1-\lambda)(1-c_1)v_0}
\]  
(12b)

\[
b_2 = \frac{d_0 v_0}{\lambda(1-h_2)d_0 + (1-\lambda)(1-c_1)v_0} \gamma
\]  
(12c)

The vector \((q, a, p)\) is normally distributed with the variance-covariance matrix \(\Sigma\):

\[
\Sigma = \begin{bmatrix}
\sigma_q & \sigma_q & \sigma q b_1 \\
\sigma_q & \sigma_q + \sigma_a & \sigma q b_1 \\
\sigma q b_1 & \sigma q b_1 & \sigma q b_1 + b_1^2 (\sigma_q + \sigma_a) + b_2^2 \sigma_s
\end{bmatrix}
\]

Following the techniques in Admati (1985) and Kuo (1992) and Chen (2010), we have Lemma 1. Then we show that an increase in the intensity of price promotions causes negative effects on expected price. Moreover, advertising can reduces these negative effects after advertising coverage crosses a threshold.

**Proposition 1.** An increase in the intensity of price promotions \( \sigma_s \) brings negative effects on expected price due to an increase in conditional risk over quality. However, threshold \( \lambda^* = \sqrt{\sigma_q / \gamma} \) exists such that as \( \lambda > \lambda^* \), an increase in advertising coverage \( \lambda \) can reduce the negative effects from \( \sigma_s \).

Proof: First we need to show that \( \partial E(p) / \partial \sigma_s < 0 \). The expected price can be obtained from Equation 11:
Equation 13 can be rewritten as follows:

\[
E(p) = \bar{q} - \frac{d_0 v_0}{(\lambda d_0 + (1 - \lambda) v_0) \gamma} \tag{13a}
\]

Differentiating \( v_0^{-1} \) and \( d_0^{-1} \) with respect to \( \sigma_s \) gives

\[
\frac{\partial v_0^{-1}}{\partial \sigma_s} = \frac{\partial d_0^{-1}}{\partial \sigma_s} = -\left[ \sigma_a + \sigma_s \sigma_a^2 (\gamma \lambda)^{-2} \right]^{-2} \sigma_a^2 (\gamma \lambda)^{-2} < 0 \tag{14}
\]

The first argument of Proposition 1 is

\[
\frac{\partial E(p)}{\partial \sigma_s} = \left[ \lambda v_0^{-1} + (1 - \lambda) d_0^{-1} \right]^{-2} \left[ \lambda \frac{\partial v_0^{-1}}{\partial \sigma_s} + (1 - \lambda) \frac{\partial d_0^{-1}}{\partial \sigma_s} \right] \gamma^{-1} \bar{z} < 0 \tag{15}
\]

Then, we denote the absolute value of \(|\partial E(p) / \partial \sigma_s|\) as the negative effects caused by price promotions.

To accomplish the proof, we need to show that \( \frac{\partial [\partial E(p) / \partial \sigma_s]}{\partial \lambda} < 0 \) as \( \lambda > \lambda^* \), please see Appendix A.

Proposition 1 indicates that, as \( \lambda > \lambda^* \), negative effects from price promotions can be reduced by advertising. As more consumers observe advertising, the uncertainty faced by consumers declines. Figure 1 shows the threshold \( \lambda^* \) ensures that advertising decreases the negative effect caused by price promotions. In such cases, practitioners would find advertising increases price sensitivity and makes price promotions effective. Proposition 1 can explain the result in Proposition 2.

**FIGURE 1**

THE NONLINEAR ASSOCIATION BETWEEN ADVERTISING COVERAGE AND THE NEGATIVE EFFECT CAUSED BY PRICE PROMOTIONS

Differentiating Equation 10 with respect to advertising coverage gives:
\[
\frac{\partial \varepsilon_d}{\partial p} = \frac{\lambda \gamma (h_2 - 1)d_0 + (1 - \lambda)\gamma(c_1 - 1)v_0}{v_0d_0}
\]  

(16)

Let \[|\frac{\partial \varepsilon_d}{\partial p}|\] denote the effective of price promotions, if one parameter leads to greater \[|\frac{\partial \varepsilon_d}{\partial p}|\], it represents that this parameter makes price promotions more effective at generating sales.

**Proposition 2.** Threshold \[\lambda^* = \sqrt{\sigma_s \sigma_a / \gamma}\] exists such that as \[\lambda > \lambda^*\], an increase in advertising coverage \[\lambda\] makes price promotions more effective at generating sales.

Proof: Please see Appendix B.

Proposition 2 ensures that advertising make price promotions an effective way to induce sales as \[\lambda > \lambda^*\]. The explanation for this result is provided in Proposition 1, which indicates that, after \[\lambda\] crosses \[\lambda^*\], an increase in \[\lambda\] reduces the negative effects from price promotions and keeps positive effects. The risk reduction role of advertising lets consumers’ demand be more elastic, so price promotions could bring desired effects. This relationship is supported by theoretical and empirical studies (Stigler, 1961; Nelson, 1974; Albion, 1983; Steiner, 1993; Eskin and Baron, 1977; Wittink, 1977; Moriarty, 1983; Shankar and Bolton, 2004; Erdem et al., 2008). However, Proposition 2 might not hold as \[\lambda < \lambda^*\]. Because advertising might increase the negative effects of price promotions rather than decrease them at this stage, the reason is described as follows. In the second part of the last line of Equation 10, we can see that the coefficient of \[a\] is \[\lambda \gamma h_2 / v_0\], indicating the specific weight at which consumers use advertising to infer quality. The coefficient of \[p\] is \[(\lambda \gamma (h_2 - 1)d_0 + (1 - \lambda)\gamma(c_1 - 1)v_0) / v_0d_0\], indicating the specific weight at which consumers use price to infer quality. We let the quality indication ratio of advertising to prices be

\[
\Delta = \frac{\lambda \gamma h_1}{v_0} \frac{1}{\lambda \gamma h_2 / v_0 + (1 - \lambda)c_1 / d_0} = \frac{\lambda d_0 h_1}{\lambda h_2 d_0 + (1 - \lambda)c_1 v_0}
\]  

(17)

Higher \[\Delta\] indicates that consumers depend more on advertising information than on prices to infer quality. Does \[\Delta\] become higher as advertising coverage \[\lambda\] increases? The answer is not necessarily positive. We use a numerical analysis to illustrate the effect of advertising on the means of inferring quality. Figure 1 shows that, as \[\lambda\] is still low, an increase in \[\lambda\] decreases \[\Delta\] rather than increases it.
FIGURE 2
EFFECT OF ADVERTISING ON QUALITY INDICATION RATIO OF ADVERTISING TO PRICE

Figure 2 indicates that advertising might encourage consumers to use price as a quality indicator, and explains why Proposition 1 and Proposition 2 cannot hold as $\lambda < \lambda^*$. An increase in the proportion of informed consumers $\lambda$ has two effects on the means of inferring quality: one is direct and the other is indirect. The direct effect is straightforward, since the proportion of informed consumers increases, average consumer depends more on advertising information to judge quality. Contrarily, the indirect effect makes average consumer depend less on advertising information to infer quality. Because more advertising information makes prices aggregate and reveal more information. Therefore, informative price encourages consumers to depend more on price to infer quality. As advertising coverage $\lambda$ is still low ($\lambda < \lambda^*$), the indirect effect may dominate the direct one, then the following circumstance happens. Consumers who want to buy the advertised product go to a retail store. They see price promotions and wonder why the price would decline if the product quality were as good as it were claimed to be by advertising. They cannot determine whether the price promotion reflects low quality, or are merely the result of increased retail supply. Under this circumstance, the negative effects of price promotion can be amplified by advertising, which could decrease price sensitivity and make price promotions an ineffective way to induce sales. This relationship is also supported by many researches (Bain, 1956; Comanor and Wilson, 1974; Lambin, 1976; Popkowski-Leszczyc and Rao, 1989; Boulding et al., 1994; Sethuraman and Tellis, 2002). The nonlinear relationship between information diffusion on price sensitivity is discussed in other studies (Vanhonacker, 1989; Simon, 1989; Parker, 1992; Parker and Neelamegham, 1997), which show price elasticity first declines and then ultimately increases over the life cycle of products.

CONCLUSIONS

The U-shape relationship between advertising and price sensitivity has two implications. The first is that a firm should keep the price stable when products are still in the introductory stage. At this stage ($\lambda < \lambda^*$), advertising information encourages consumers to depend more on price to judge quality. Price promotions cannot generate sales effectively because its negative effects might be amplified by an increase in advertising coverage. For a new product, advertising and price promotions offset each other’s
strengths so that they should not be used simultaneously. After advertising coverage crosses the threshold, advertising and price promotions can be used simultaneously. Because advertising can reduce the negative effects from price promotions, and makes price promotions more effective at generating sales. The second implication is that antitrust policies should reconsider the legal status of price-stabilization practices like resale price maintenance (Chen and Chen; 2007, 2010). Resale price maintenance is illegal in most countries, except under rule of reason in the United States. From the perspective of rational expectations model, resale price maintenance on products which are at introductory stage could be exempted from prohibition. Because the number of informed consumers is low in the introductory stage, the prohibition of resale price maintenance could reduce the strength of manufacturer advertising.

APPENDIX A

The effect of advertising on the negative effects is

\[
\frac{\partial E(p)}{\partial \sigma_s} = \left[ \frac{\partial v_0^{-1}}{\partial \sigma_s} + \lambda \cdot \frac{\partial v_0^{-1}}{\partial \sigma_s} - \frac{\partial d_0^{-1}}{\partial \sigma_s} - \lambda \cdot \frac{\partial d_0^{-1}}{\partial \sigma_s} \right] \left[ \lambda \cdot v_0^{-1} + (1-\lambda) \cdot d_0^{-1} \right]^{\frac{1}{2}} \gamma \cdot \zeta^{-1} \\
2 \left[ \frac{\lambda \cdot \partial v_0^{-1}}{\partial \sigma_s} + (1-\lambda) \cdot \frac{\partial d_0^{-1}}{\partial \sigma_s} \right] \left[ \lambda \cdot v_0^{-1} + (1-\lambda) \cdot d_0^{-1} \right] \left[ v_0^{-1} + \lambda \cdot \frac{\partial v_0^{-1}}{\partial \sigma_s} + \frac{\partial d_0^{-1}}{\partial \sigma_s} - d_0^{-1} - \frac{\partial d_0^{-1}}{\partial \sigma_s} \right]^{\frac{1}{2}} \gamma \cdot \zeta^{-1}
\]

(A1)

If the value in Equation A1 is negative, then advertising can reduce the negative effects. First we examine the second term in the right-hand side of Equation A1. From Equation 14, we know that \( \left[ \lambda \cdot \frac{\partial v_0^{-1}}{\partial \sigma_s} + (1-\lambda) \cdot \frac{\partial d_0^{-1}}{\partial \sigma_s} \right] < 0 \). Then we differentiate \( v_0^{-1} \) and \( d_0^{-1} \) with respect to \( \lambda \), which yields the following expression

\[
\frac{\partial v_0^{-1}}{\partial \lambda} = \frac{\partial d_0^{-1}}{\partial \lambda} = 2 \left[ \sigma_a + \sigma_s \sigma_a^2 (\gamma \lambda)^{-2} \right]^{-2} \sigma_s \sigma_a^2 \gamma^{-2} \lambda^{-3} > 0
\]

Thus we can conclude that the second term in the right-hand side of Equation A1 is negative, since we have shown that \( \frac{\partial v_0^{-1}}{\partial \sigma_s} = \frac{\partial d_0^{-1}}{\partial \sigma_s} \) in Equation 14. Therefore, \( \frac{\partial d_0^{-1}}{\partial \sigma_s} \) is the only term whose sign remains undetermined in the first term of the right-hand side of Equation A1

\[
\frac{\partial d_0^{-1}}{\partial \sigma_s} = 2 \sigma_a^2 \gamma^{-2} \lambda \left( \sigma_a (\gamma \lambda)^2 + \sigma_s \sigma_a^2 \right) \left[ \sigma_a (\gamma \lambda)^2 - \sigma_s \sigma_a^2 \right] \left( \sigma_a (\gamma \lambda)^2 + \sigma_s \sigma_a^2 \right)^{-1}
\]

We can see that if \( \lambda > \lambda^* = \frac{\sqrt{\sigma_a \sigma_a}}{\gamma} \), one has \( \sigma_a (\gamma \lambda)^2 - \sigma_s \sigma_a^2 > 0 \) and \( \frac{\partial d_0^{-1}}{\partial \sigma_s} > 0 \), then it ensures that Equation A1 is negative.

Q.E.D.
APPENDIX B

We need to show $\partial|\hat{z}_d|/\partial p/\partial \lambda > 0$ as $\lambda > \lambda^*$. In Equation 12c, we already have:

$$b_2 = \frac{d_0v_0}{(\lambda(1-h_2)d_0 + (1-\lambda)(1-c_1)v_0)\gamma} \quad (12c)$$

Substitute Equation 12c into Equation 16, $\partial|\hat{z}_d|/\partial p/\partial \lambda$ becomes

$$\frac{\partial|\hat{z}_d|/\partial p}{\partial \lambda} = \frac{\partial b_2^{-1}}{\partial \lambda} = \partial \left[ \frac{\lambda d_0 + (1-\lambda)v_0}{v_0d_0\gamma^{-1}} \frac{\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda}{\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda} \right] / \partial \lambda$$

In Appendix A, we already prove that $\partial v_0^{-1}/\partial \lambda = \partial d_0^{-1}/\partial \lambda > 0$, so we have

$$\partial(\lambda d_0 + (1-\lambda)v_0)/\partial \lambda = \partial(\lambda v_0^{-1} + (1-\lambda)d_0^{-1})/\partial \lambda > 0$$

In order to accomplish proof, we still need to check if following derivative is positive

$$\frac{\partial(\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda)}{\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda}$$

$$= \frac{2\lambda\gamma^2(\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda) - (\gamma^2\lambda^2 + \sigma_s\sigma_a)(2\gamma^2\lambda + \gamma^2)}{(\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda)^2}$$

$$= \frac{\lambda^2\gamma^4 - \gamma^2\sigma_s\sigma_a}{(\gamma^2\lambda^2 + \sigma_s\sigma_a + \gamma^2\lambda)^2}$$

We can conclude that $\partial|\hat{z}_d|/\partial p/\partial \lambda > 0$ as $\lambda > \lambda^* = \frac{\sqrt{\sigma_s\sigma_a}}{\gamma}$. Q.E.D.

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