Modelling and Forecasting the Conditional Heteroscedasticity of Stock Returns Using Asymmetric Models: Empirical Evidence from Ghana and Nigeria

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This paper examines and evaluates the performance of asymmetric first order generalised autoregressive conditional heteroscedasticity (GARCH 1, 1) models for Ghana and Nigeria stock market returns. We employed the Glosten Jagannathan and Runkle (GJR) version of GARCH (GJR-GARCH) and Exponential GARCH (EGARCH) methodology to investigate the leverage effect of return volatility in Ghana and Nigeria stock markets using Gaussian, Student-t and Generalised Error Distribution (GED) densities. Our evidence shows that both the GJR and EGARCH models capture the leverage effect of Ghana's Stock Exchange Broad Market Index (GSEI) returns, which indicates that negative shocks imply a higher next period conditional variance than positive shocks of the same magnitude. However, our results in Nigeria show that the market exhibits a reverse volatility asymmetry in contravention of the widely accepted theory. According to the different measures used to evaluate the volatility forecast performance, the EGARCH provides the best out-of-sample forecast for the Ghanaian stock market, while the GJR gives a better estimation for the Nigerian stock market.

INTRODUCTION

Over three decades financial econometricians have attempted to model and forecast high frequency financial time series. A seminal paper in this area of research can be credited to Engle (1982) who proposed the standard Autoregressive Conditional Heteroscedasticity (ARCH) to model and forecast time-varying conditional volatility using information on the past error term. Bollerslev (1986) extended Engle's (1982) study by generalising the ARCH model popularly known as GARCH which suggests that time-varying volatility is a function of both past innovations and past volatility. The standard ARCH/GARCH model can model three important characteristics of financial time series, i.e. leptokurtosis, skewness and volatility clustering. However, evidence shows that the standard model is unable to capture the dynamics of a fourth important feature of financial time series known as leverage effect, i.e. large negative returns appear to increase volatility more than do positive returns of the same magnitude (Nelson, 1991; Glosten et al., 1993). This also implies that volatility is higher after negative shocks than after positive shocks of the same magnitude. In other words, returns are considered to have asymmetric impact on volatility. Black (1976) find that fluctuation in stock prices is negatively related to volatility and this implies that as stock prices fall leverage of firms' increases, which causes uncertainty and therefore generates more volatility. Moreover, the standard ARCH/GARCH models cannot model this asymmetric behaviour of stock returns, i.e. leverage effect because they model the conditional variance as a function of past values of disturbance term, however, do not take into consideration the sign (either negative or positive) of the value of the past error. In order to account for this asymmetric response of volatility to such shocks we evaluate the performance of two widely applied asymmetric GARCH models –GJR-GARCH (Glosten et al. 1993), and EGARCH (Nelson, 1991).

Turning to empirical tests, Chiang and Doong (2001) investigate time series behaviour of stock returns by constructing a meta-analysis including Malaysia, Philippines, Singapore, South Korea, Thailand, US and Japan using the daily stock price indices for these participating countries' stock markets spanning January 1988 to June 1988. Their results show that higher levels of volatility are significantly correlated with higher average returns. A further review of the results using a Threshold Autoregressive-GARCH (1, 1) unearthed the fact that the size and significance level of the GARCH effect becomes smaller in weekly returns. They also reject the hypothesis of no asymmetric effect at a high significance level. Nor and Shamiri (2007) scrutinize the modelling of high-frequency data by examining the evaluations of three GARCH (1,1) models (i.e. GARCH, EGARCH and GJR-GARCH) using daily price data from Singapore's Strait Times Index (STI) and Malaysia's Kuala Lumpur Composite Index (KLCI) over a fourteen year period, from 2 January 1991 and ending on 31 December 2004. This was based on using three key distribution densities, Gaussian normal, Student-t and Generalized Error Distributions (GED). The study was premised on the shortcomings of traditional regression tools for modelling high-frequency (weekly, daily or intra-daily) data such as the assumptions that are usually detached from reality that only the mean response changes with covariates. Their results show that asymmetric GJR-GARCH and EGARCH exhibit superior forecasting performance to symmetric GARCH. It was further found that the AR(1)-GJR model provides the best out-of-sample forecast for the Malaysian stock market, while AR(1)-EGARCH provides a better estimation for the Singaporean stock market.

Elyasiani and Mansur (1998) use the stocks of traded commercial banks on the American stock exchange from the period of the study from January 1970 to December 1992 to investigate the effect of interest rate and its volatility on the bank stock return using GARCH-in-Mean (GARCH-M). Their results show that long-term interest rate has a negative and significant impact on the bank stock return. They further find that interest rate volatility is an important determinant of the bank stock return volatility and bank stock risk premium. French et al (1987) study shows that the expected market risk premium is positively related to unpredictable volatility of stock returns. They employ several estimates of the relation between the expected risk premium and the predicted volatility of New York Stock Exchange common stocks over the 1928 to 1984 period. Chong et al (1991) use the rate of returns from the daily stock market indices of the Kuala Lumpur stock exchange including composite index, tins index, plantations index, properties index and finance index to evaluate stationary GARCH, unconstrained GARCH, non-negative GARCH, GARCH-M, exponential GARCH and integrated GARCH ability to forecast stock market volatility. They found out that exponential GARCH, though its statistics did not have the best fit, proved most effective in forecasting stock market volatility. Tan et al (2012) in their macro analysis of interest rate volatility on stock market return in which they applied two separate GARCH(1,1) models and data from the FBM Kuala Lumpur Composite Index and 3 months deposit yields with similar data from the Singaporean market, asserted that interest rate volatility has a strong positive relationship with stock market volatility. They also found that there exists an insignificant inverse relationship between interest rate volatility and stock market return.

Ortiz and Arjona (2001) employ several versions of the GARCH model to examine the time series characteristics of six major Latin American markets comprising Argentina, Brazil, Chile, Colombia, Mexico and Venezuela, over a five-year period from 1989-1994. Weekly returns data were expressed in US dollars and the respective local currencies. Given the differences in the stochastic characteristics of the markets, they went on to test different conditional heteroscedasticity volatility characteristics for each market. They employed different GARCH models for the different markets including GARCH(1,1), GARCH(1,3), GARCH(2,2), GARCH(1,3), EGARCH(1,1),GARCH(1,2), and EGARCH(3,3). Their results show that both the dollar and local currency returns of all the six markets studied were time dependent, heteroscedastic, and asymmetric, with both right and left skewness apparent, but leptokurtic in all cases. They further found that no single GARCH model was able to describe the stock returns volatility in these markets. Instead, they reported that ‘different GARCH models are more appropriate for each country’, and that ‘the best models seem adequate’.

Leon (2007) examines the relationship between stock market returns and volatility on the Bourse Réegionale des Valeurs Mobilieres (BRVM), which is the Regional Financial Exchange for the eight French-speaking West African countries that form the West African Economic and Monetary Union. Using weekly returns over the period January 1999 to July 2005, and EGARCH-in-Mean model, he finds a positive but insignificant relationship between conditional market returns and conditional volatility. Both results are inconsistent with developed market evidence of significant positive relations document in French *et al*. (1987) and Campbell and Hentschel (1992), and the leverage effect document by Nelson (1991). Coffie and Chukwulobelu (2014) investigate volatility persistence by comparing evidence from selected emerging African and developed Western markets, taking into account the rate of volatility decay. Generalised Autoregressive Conditional Heteroscedasticity (GARCH) and GARCH-in-mean (GARCH-M) models are employed to estimate volatility persistence and risk premium for these markets. Their results show that volatility persists in the four emerging African markets and the five developed markets. Furthermore, Coffie (2015) examines volatility persistence in Southern and East African stock markets taking into account the rate of volatility decay. Generalised Autoregressive Conditional Heteroscedasticity (GARCH) and GARCH-in-mean (GARCH-M) models are employed to estimate volatility persistence and risk premium for these markets. The results presented suggest that there is volatility persistence in emerging Southern and East African stock markets. Further empirical estimates showed that the rate of volatility decay varies considerably among the markets, for example, volatility in Mauritius diminishes to half of its original size within seven hours, while it takes almost eight months for volatility in Zambia to taper off to half of its original size. The study concludes that volatility risk exists in emerging Southern and East African stock markets and investors would require compensation for bearing this type of risk.

While there has been extensive research on symmetric and asymmetric GARCH models in the academic literature since the introduction of ARCH/GARCH, GJR-GARCH and EGARCH (Engle, 1982; Bollerslev, 1986; Glosten et al, 1993 and Nelson, 1991), less effort has been made towards comparing alternative density forecast models. Although Nor and Shamiri (2007) compared alternative density forecast models in Malaysia and Singapore, nothing is found on Emerging African stock markets. A prominent feature of high-frequency financial time series of stock returns is that they are frequently characterised by fat-tailed distribution. It is established in the finance literature as a matter of fact that the kurtosis of most financial asset returns is greater than 3 (Simkowitz & Beedles, 1980; Kon, 1984). This suggests that extreme values are much more likely to be observed in financial asset returns than the normal distribution. Whereas the high kurtosis of financial asset returns is established by sound evidence in the finance literature, the state of the symmetrical distribution is still opaque. Mittnik and Paolella (2001) demonstrate that a fat-tailed distribution density is essential to model East Asian currencies' daily exchange rate against the US dollar.

Against this background, this paper fills the gap by making a rigorous comparison of alternative density forecast methodology of asymmetric GARCH models for Ghana and Nigeria stock market returns (the two most developed West African stock markets with a track record of sufficient market data). GJR-GARCH (1, 1) and EGARCH (1, 1) models are employed with the introduction of different distribution densities (Gaussian, Student-t and GED).

The next section presents data description and methodology used in this study. The third section presents our results and analyses and the conclusions are presented in the final section.

DATA AND METHODOLOGY

Data Description

The daily stock price indices data used in this study are obtained from Standard & Poor/International Finance Corporation Emerging Market Database (S&P/IFC EMDB). This source is used largely because it is a very organised and comprehensive source of stock price data, providing more readily accessible and reliable data on emerging equity markets than most other sources. For example, S&P/IFC EMDB was the first database, from 1975, to track comprehensive information and statistics on emerging stock market indices. The S&P/IFC global indices, used in this study, do not impose restrictions on foreign ownership and include sufficient number of stocks in individual market indices without imposing float or artificial industry-composition models on markets. Besides, the S&P/IFC database is attractive because the data have been adjusted for all capital changes as well as the effects of corporate restructuring such as merger, acquisition and spin offs/demerger as well as being free from data backfilling and survivorship bias.

The daily return r_t consists of the transformed daily closing index price P_t measured in local currency. Our measurements include Ghana's Stock Exchange All Index (GSEI) and Nigeria's Stock Exchange All Share Index (NSEI). The stock price indices are transformed into their returns in order to obtain stationary series as:

$$r_t = \ln\left(p_t \big/ p_{t-1} \right) * 100 \tag{1}$$

where r_t is the market return, p_t and p_{t-1} are natural log returns of contemporaneous and one period lagged equity price indices, respectively. Natural lognormal is preferred as it computes continuous compound returns.

Table 1 below provides further details of the data used in this research including the types of stock indices used, the time period of the data for each market (and hence sample observations), and currency of denomination. The indices used in this study are the benchmark indices in their respective stock markets.

TABLE 1
STOCK MARKET DATA PROFILE

Country	Type of Index	Name of Index	Currency	Period of data	No of Observ.	Source of Data
Ghana	Weighted index market capitalization	GSE All Share Index (GSEI)	Cedi	1996 – 2013	4589	S & P/IFC BMI
Nigeria	Weighted index market capitalization	NSE All Share Index(NSEI)	Naira	1995 – 2013	4719	S & P/IFC BMI

TABLE 2
DESCRIPTIVE STATISTICS FOR DAILY RETURNS

	Mean	Std. Dev.	Skewness	Kurtosis	J. Bera	Q-Stat	ARCH(5)	ACF(100)	PACF(100)
GSEI	0.0513	0.973	4.150	119.098	2590436**	360**	-0.002**	-0.008**	-0.015**
NSEI	0.0596	1.039	-0.017	6.125	1920**	1564**	0.125**	-0.022**	-0.007**

J. Bera is the Jarque-Bera test for normality, Q-stat refers to Ljung-Box test for autocorrelation. ARCH (5) refers to the Engle (1982) LM test for the presence of ARCH effect at lag 5. ACF and PACF refer to autocorrelation and partial autocorrelation function respectively at lag 100.
** and * denote 1 and 5 per cent significant levels.

The descriptive statistics in Table 2 show that both indices generate positive mean returns. However, the mean returns for NSEI are slightly higher than the GSEI but this could be due to the six months difference in data. Furthermore, the non-conditional variance as measured by the standard deviation for NSEI is higher than that of GSEI. The returns distribution for GSEI is positively skewed and that of NSEI is negatively skewed. The null hypothesis for skewness that conforms to a normal distribution with coefficients of zero has been rejected for both indices. The returns for both indices exhibit fat tail as seen in the significant kurtosis well above the normal value of 3. The high value of the Jarque-Bera (JB) test for normality rejects determinedly the hypothesis of a normal distribution at 1 per cent significance level. Engle (1982) LM test indicates the presence of Autoregressive Conditional Heteroscedasticity (ARCH) processes in the conditional variance of both series. This legitimizes the use of ARCH/GARCH type models. Moreover, for both indices, the Ljung-Box Q test statistic rejects the null hypothesis of no autocorrelation at 1 per cent levels for all numbers of lags (100) considered as shown by ACF and PACF results in Table 2.

The statistical results indicate that both indices display similar characteristics. For instance they are both skewed, found to display non-normal distribution and exhibit serial correlation. These stylized features are consistent with existing empirical evidence from developing markets (Kim, 2003; Ng, 2000) and developed markets (Fama, 1976; Kim & Kon, 1994). Besides, as return series displayed high value of kurtosis, it can be expected that a fatter-tailed distribution density such as the Student-t or GED should provide more accurate results than the Gaussian (Normal) distribution.

Empirical Models

First (mean) and second (variance) moment equations are used to define the GARCH models. The return process, r_t, is captured by the mean equation which is made up of the conditional mean, μ, which might encompass terms of autoregressive(AR) and moving average(MA) and error term, ε_t, that follows a conditional normal distribution with mean of zero and variance, σ^2. Additionally, the information set available to investors up to time t_{-1} is represented by, Ωt_{-1}, thus,

$$r_t = \mu + \varepsilon_t \tag{2}$$

where

$$\varepsilon_t \middle| \Omega_{t-1} \approx N\left(0, \sigma_t^2\right) \tag{3}$$

The conditional variance σ^2 is modelled using asymmetric GARCH (1,1) models of GJR and EGARCH with the introduction of three different distribution densities.

The GJR GARCH Model

The Threshold-GARCH and Threshold-ARCH were independently introduced by Glosten et al. (1993) and Zakoian (1994). The Threshold-GARCH model applied in this study is also known as GJR-GARCH (after its proponents). This version of Threshold-GARCH is different from the Zakoian (1994) model which models the conditional standard deviation instead of conditional variance. The GJR-GARCH simply augments the standard GARCH with an additional ARCH term conditional on the sign of the past innovation and thus specified as:

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \lambda_1 \varepsilon_{t-1}^2 d_{t-1} \tag{4}$$

Where λ_1 measures the asymmetric (or leverage) effect and d_t is a dummy variable which is equal to 1 when ε_t is negative. In the TGARCH (1, 1) model, good news, $\varepsilon_{t-1}>0$, and bad news, $\varepsilon_{t-1}<0$, have differential effects on the conditional variance. Good news has an impact of α_1, while bad news has an impact of $\alpha_1+\lambda_1$. If $\lambda_1>0$, bad news increases volatility and this in turn means that there is a leverage effect for the AR (1)-order. If $\lambda_1\neq 0$, the news impact is asymmetric.

Exponential GARCH (EGARCH) Model

Nelson (1991) introduced the exponential GARCH model to further capture the asymmetric (or 'directional') response of volatility. The EGARCH model used in this study is modelled as:

$$\log\left(\sigma_t^2\right)=\alpha_0+\beta_1\log\left(\sigma_{t-1}^2\right)+\alpha_1\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|+\lambda_1\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{5}$$

Since the log of the conditional variance is modelled, the leverage effect is exponential rather than quadratic, and even if the parameters are negative, the conditional variance will be positive. The hypothesis that $\lambda_1<0$ is used to test the presence of the leverage effect. The impact is asymmetric if $\lambda_1\neq 0$. If the relationship between returns and volatility is negative, λ will be negative. The EGARCH model allows positive and negative shocks to have a distinct impact on volatility. It also allows large shocks to have a greater impact on volatility than the standard GARCH model.

Empirical Methods

GARCH models are estimated using the maximum likelihood estimation (MLE) process. The MLE has numerous optimal properties in estimating parameters and these include sufficiency (i.e. complete information about the parameter of importance contained in its MLE estimator); consistency (true parameter value that generated the data recovered asymptotically, i.e. data of sufficiently large samples); efficiency (lowest possible variance of parameter estimates achieved asymptotically). Furthermore, many methods of inference in statistics and econometrics are developed based on MLE, such as chi-square test, modelling of random effects, inference with missing data and model selection criteria such as Akaike information criterion and Schwarz criterion.

ML estimation assumes that the error distribution is Gaussian; however, evidence shows that the error exhibits non-normal distribution densities, for example, Nelson, (1991).The choice of the underlying distribution for the error term is crucial if the volatility model is used in risk modelling. As it is expected that the problems posed by skewness and kurtosis due to the residuals of conditional heteroscedasticity models being reduced when appropriate distribution density is used, our study considers and evaluates the three most commonly used densities, the Gaussian, Student-t and Generalised Error Distribution (GED).

Gaussian

The Gaussian, also known as the normal distribution, is the most widely used model when estimating GARCH models. For a stochastic process, the log-likelihood function for the normal distribution is expressed as:

$$L_{gaussian}=-\tfrac{1}{2}\sum_{t=1}^{T}\left(\ln[2\pi]+\ln\left[\sigma_t^2\right]+z_t\right) \tag{6}$$

where T is the number of observations.

Student-T Distribution

For a Student-t distribution, the log-likelihood is:

$$L_{stu-t}=\ln\left(\Gamma\left[\frac{v+1}{2}\right]\right)-\ln\left(\Gamma\frac{v}{2}\right)-\tfrac{1}{2}\ln\left(\pi[v-2]\right)-\tfrac{1}{2}\sum_{t=1}^{T}\left(\ln\sigma_t^2+[1+v]\ln\left[1+\frac{z_t^2}{v-2}\right]\right) \tag{7}$$

where v is degrees of freedom, 2<v=∞ and $\Gamma(\bullet)$ is the gamma function.

Generalised Error Distribution (GED)

In applied finance, such as asset pricing, option pricing, portfolio selection and VaR, skewness and kurtosis are very important. The GED is an error distribution that represents a generalised form of the Gaussian, possesses a natural multivariate form, has a parametric kurtosis that is unbounded above and has special cases that are identical to the normal and property which controls the skewness. Thus, choosing the appropriate distribution density that can model these two moments is important, hence, the GED log-likelihood function of a normalised random error is expressed as:

$$L_{GED} = \sum_{t=1}^{T}\left(\ln\left[{}^{v}\!/_{\lambda_v}\right] - 0.5\left|\frac{z_t}{\lambda_v}\right|^{v} - \left[1+v^{-1}\right]\ln 2 - \ln\Gamma\left[{}^{1}\!/_{v}\right]0.5\ln\left[\sigma_t^2\right]\right) \tag{8}$$

where $$\lambda_v = \sqrt{\frac{\Gamma\left({}^{1}\!/_{v2^{-2/v}}\right)}{\Gamma\left({}^{3}\!/_{v}\right)}} \tag{9}$$

Forecast Evaluation

The one-step-ahead forecast of the conditional variance for the GJR and EGARCH is obtained by updating equations (4) and (5) by one period as,

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1\varepsilon_t^2 + \lambda_1\varepsilon_t^2 d_t + \beta_1\sigma_t^2 \tag{10}$$

$$\log\left(\sigma_{t+1}^2\right) = \alpha_0 + \beta_1\log\left(\sigma_t^2\right) + \alpha_1\left|\frac{\varepsilon_t}{\sigma_t}\right| + \lambda_1\frac{\varepsilon_t}{\sigma_t} \tag{11}$$

Similarly, we can obtain j-step-ahead forecast of the conditional variance for the GJR and EGARCH by updating equation (10) and (11) by j periods,

$$\sigma_{t+j}^2 = \alpha_0 + \beta_1\sigma_{t+j-1}^2 + \alpha_1\varepsilon_{t+j-1}^2 + \lambda_1\varepsilon_{t+j-1}^2 d_{t+j-1} \tag{12}$$

$$\log\left(\sigma_{t+j}^2\right) = \alpha_0 + \beta_1\log\left(\sigma_{t+j-1}^2\right) + \alpha_1\left|\frac{\varepsilon_{t+j-1}}{\sigma_{t+j-1}}\right| + \lambda_1\frac{\varepsilon_{t+j-1}}{\sigma_{t+j-1}} \tag{13}$$

We can therefore take the conditional expectation of equation (12) and (13) as,

$$E_t\sigma_{t+j}^2 = \alpha_0 + \beta_1 E_t\sigma_{t+j-1}^2 + \alpha_1 E_t\varepsilon_{t+j-1}^2 + \lambda_1 E_t\varepsilon_{t+j-1}^2 d_{t+j-1} \tag{14}$$

$$E_t\log\left(\sigma_{t+j}^2\right) = \alpha_0 + \beta_1 E_t\log\left(\sigma_{t+j-1}^2\right) + \alpha_1 E_t\left|\frac{\varepsilon_{t+j-1}}{\sigma_{t+j-1}}\right| + \lambda_1 E_t\frac{\varepsilon_{t+j-1}}{\sigma_{t+j-1}} \tag{15}$$

However, it is rather difficult to obtain the *j*-step-ahead forecasts than the one-period-ahead forecasts assumed in this study although it is possible to obtain the *j*-step-ahead forecasts of the conditional heteroscedasticity recursively.

In order to evaluate the forecasting performance of the GJR and EGARCH models, forecasting tests encompassing different distribution densities are performed. The model that minimises the loss function under these evaluation criteria is preferred. To assess the performance of the asymmetric GARCH models in forecasting the conditional variance, we compute four statistical measures of fit as follows;

Mean Absolute Error (MAE) – This is represented as:

$$MAE = \frac{1}{h}\sum_{t=s}^{s+h}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| \tag{16}$$

where h is the number of steps ahead (i.e. number of forecast data points), s the sample size, $\hat{\sigma}^2$ is the forecasted variance and σ^2 is the conditional variance computed from equations (4) and (5).

Root Mean Square Error (RMSE) is represented as:

$$RMSE = \sqrt{\frac{1}{h}\sum_{t=s}^{s+h}\left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2} \tag{17}$$

The Mean Absolute Percentage Error (MAPE) is represented as:

$$MAPE = \frac{1}{h}\sum_{t=s}^{s+h}\left|(\hat{\sigma}_t^2 - \sigma_t^2) \big/ \sigma_t^2\right| \tag{18}$$

Theil Inequality Coefficient (TIC) is represented as:

$$TIC = \frac{\sqrt{MSE}}{\sqrt{1/h\sum_{t=s}^{s+h}\sigma^2 + 1/h\sum_{t=s}^{s+h}\hat{\sigma}^2}} \tag{19}$$

To calculate daily forecast and in order to evaluate the forecasting performance of each model, we simply split the respective time series in half between the in-sample period, $t = 1,...,T$ and the out-of-sample period, $t = T,...,h$. We further estimate each model over the first part of the sample and then apply these results to forecast the conditional variance (volatility) over the second part of the sample period.

EMPIRICAL RESULTS AND ANALYSES

This section presents and analyses our results of the estimated models. Tables 3 and 4 present the results for the estimated parameters for GJR and EGARCH models respectively, while some useful in-sample diagnostics statistics are reported in Tables 5 and 6.

TABLE 3
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

	GHANA			NIGERIA		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.0586 (3.829)**	-7.86e-05 (-0.217)	-0.0009 (-0.042)	0.0255 (3.061)**	0.0315 (7.041)**	0.0164 (3.838)**
α_0	0.6013 (2.197)*	5.56e-06 (24.320)**	0.6105 (12.639)**	0.0040 (2.390)*	9.95e-05 1.006)	0.0003 (1.573)
α_1	-0.0031 (-9.190)**	2.3539 (30.784)**	-0.0017 (-50.564)**	0.2269 (11.116)**	0.2767 (13.227)**	0.2620 (13.015)**
β_1	0.5897 (3.390)**	0.0381 (53.347)**	0.2508 (4.193)**	0.8231 (65.726)**	0.8091 (91.966)**	0.8156 (89.650)**
λ_1	0.0668 (2.175)*	6.9110 (19.297)**	-0.0059 (-8.701)**	-0.0450 (-2.085)*	-0.0475 (-2.235)*	-0.0457 (-2.091)*

TABLE 4
ESTIMATED STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	GHANA			NIGERIA		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
μ	0.0495 (3.448)**	-7.65e-05 (-0.151)	-0.0003 (-0.035)	0.0277 (3.094)**	0.0334 (6.589)**	0.0240 (5.114)**
α_0	-0.0658 (-0.336)	-0.1414 (-20.534)**	-0.1923 (-20.658)**	-0.3103 (-13.378)**	-0.3380 (-22.186)**	-0.3277 (-21.100)**
α_1	0.0018 (0.037)	-0.0250 (5.433)**	0.0557 (17.246)**	0.3958 (13.706)**	0.4456 (19.749)**	0.4272 (19.126)**
β_1	-0.1511 (-0.241)	0.9836 (2263)**	0.8876 (153.303)**	0.9613 (135.480)**	0.9750 (262.707)**	0.9716 (265.256)**
λ_1	0.0483 (1.194)	-0.0227 (-5.434)**	-0.0210 (-7.375)**	0.0361 (2.926)**	0.0331 (2.978)**	0.0318 (2.715)**

The statistics reported support the use of asymmetric GARCH models in modelling the stock returns volatility of GSEI and NSEI. The evidence shows that news impact is asymmetric in both stock markets as $\lambda \neq 0$ for all densities with statistically significant coefficients at either 1 or 5 per cent level.

The asymmetric effect, λ, for GJR and EGARCH models produced mixed results for both indices and all densities. In Ghana, the leverage effect term, λ, in the GJR model has coefficient greater than zero with respect to Gaussian and Student-t, while the leverage coefficient term, λ, in EGARCH is less than zero with Student-t and GED. Furthermore, in Nigeria, the leverage effect term, λ, in GJR has coefficients less than zero, while all the coefficients of the leverage effect term, λ, in EGARCH are greater than zero, for all densities. The results show that both the GJR and EGARCH models capture the existence of leverage effect in GSEI stock returns. This means that in Ghana, negative shocks imply a greater next period volatility effect (i.e. conditional variance) than positive shocks of the same magnitude, indicating the existence of leverage effect in GSEI returns. Moreover, in Nigeria, the evidence shows that negative shocks would have no greater effects on volatility than positive shocks no matter the density assumption. Instead positive news would have greater effect on volatility as the asymmetric term coefficient, λ, in GJR is less than zero and greater than zero in EGARCH with α_1>0 in both cases regardless of density distribution. This evidence controverts the GJR and EGARCH propositions that bad news has greater impact on volatility than good news (i.e. leverage effect).

The estimated parameters for GJR indicate that the coefficients of ARCH (α_1) and GARCH (β_1) in the conditional variance equation are statistically significant at 1 per cent level for all distribution densities for both GSEI and NSEI. Similarly, the estimated coefficients of ARCH (α_1) and GARCH (β_1) for EGARCH for all distribution densities are statistically significant at 1 percent level for NSEI. Moreover, for GSEI, the ARCH (α_1) and GARCH (β_1) coefficients under EGARCH are statistically significant at 1 per cent level with non-normal densities, while the ARCH (α_1) is significant at 5 per cent level but the GARCH (β_1) coefficient is non-significant with Gaussian distribution. Furthermore, the leverage, λ, for GJR is statistically significant at standard levels for both markets, while the, λ, coefficient in EGARCH for both indices is also statistically significant for all distribution densities with the exception of Gaussian for Ghana which exhibits non-significance.

TABLE 5
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY GJR-GARCH MODEL

	GHANA			NIGERIA		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	3.0993 (1.000)	3.2691 (1.000)	3.2614 (1.000)	44.0350 (0.001)	7.8892 (0.993)	13.4210 (0.859)
ARCH(5)	0.3717 (0.996)	0.5291 (0.991)	0.6346 (0.986)	23.8666 (0.000)	3.3783 (0.6419)	5.6460 (0.3422)
AIC	2.8692	-1.1558	1.5858	2.3975	2.3218	2.3189
SBIC	2.8762	-1.1488	1.5943	2.4043	2.3301	2.3271
Log-Like	-6578	2657	-3633	-5472	-5472	-5465

$Q^2(20)$ are the Ljung-Box statistic at lag 20 of the squared standardised residuals. ARCH(5) refers to the Engle (1982) LM test for the presence of ARCH effect at lag 2. P-values are given in parentheses. AIC, SBIC and Log-Like are Akaike information criterion, Schwartz Bayesian information criterion and Log-Likelihood value respectively.

TABLE 6
DIAGNOSTICS STATISTICS-COMPARATIVE DISTRIBUTION DENSITY EGARCH MODEL

	GHANA			NIGERIA		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED
$Q^2(20)$	3.3479 (1.000)	0.1451 (1.000)	3.3938 (1.000)	34.6470 (0.022)	23.7750 (0.252)	29.0270 (0.087)
ARCH(5)	21.4424 (0.001)	0.0291 (1.000)	0.5821 (0.9888)	21.4424 (0.001)	11.3763 (0.044)	14.5018 (0.013)
AIC	2.7834	-3.1180	0.9374	2.3829	2.3171	2.3122
SBIC	2.7904	-3.1096	0.9458	2.3898	2.3253	2.3204
Log-Like	-6382	7162	-2144	-5617	-5461	-5450

Turning to distribution densities (Tables 5 and 6); the fatter tails (Student-t and GED) distributions clearly outperform the Gaussian for GJR and EGARCH models. For example, the log-likelihood function strongly increases when using non-normal distribution densities. Furthermore, using the non-normal densities of Student-t and GED produces lower AIC and SBIC than the Gaussian with GJR and EGARCH, for both the GSEI and NSEI. From the preceding evidence, GJR and EGARCH models perform well with non-normal distribution densities. Indeed, all models seem to do a good job by describing the dynamics of the series as shown by the Ljung-Box statistics for the squared standardised residuals with lag 20 which are all non-significant at 1 and 5 per cent levels for both indices with the exception of Gaussian distribution for NSEI. The LM test for the presence of ARCH at lag 5, indicates that conditional heteroscedasticity that was present when the test was performed on pure returns as in Table 2 is removed for GJR for all densities for Ghana and Student-t and GED for Nigeria which are all non-significant at 1 per cent level, but remains for Gaussian. Moreover, the EGARCH model with Student-t and GED distributions indicates that the conditional heteroscedasticity is successfully removed for Ghana but remains for Gaussian as well as for all density distributions for Nigeria.

The comparison between models with each distribution density indicates that, given the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for GSEI, clearly outperforming EGARCH with Gaussian and GED and GJR model. For NSEI, no clear results can be obtained, where GJR and EGARCH for each distribution density provides close results. However, EGARCH with GED density slightly outperforms the remainder for NSEI.

TABLE 7
FORECAST PERFORMANCE-COMPARATIVE DISTRIBUTION DENSITY

GHANA **NIGERIA**

Gaussian

Model	GJR	EGARCH	GJR	EGARCH
MAE	0.349926	0.345179	0.855411	0.855580
RMSE	0.842153	0.842036	1.203467	1.203482
MAPE	58.69737	55.80815	101.0401	101.8333
TIC	0.933933	0.943324	0.979136	0.977445

Student-T

Model	GJR	EGARCH	GJR	EGARCH
MAE	0.321145	0.321144	0.855893	0.856050
RMSE	0.843130	0.843130	1.203520	1.203543
MAPE	46.65202	46.65181	103.2910	104.0127
TIC	0.999912	0.999914	0.974427	0.972970

GED

Model	GJR	EGARCH	GJR	EGARCH
MAE	0.321642	0.321310	0.854732	0.855292
RMSE	0.843172	0.843144	1.203443	1.203458
MAPE	46.74093	46.68153	97.85564	100.4832
TIC	0.998999	0.999608	0.974427	0.980345

TABLE 8
RANKING PERFORMANCE FORECAST

GHANA **NIGERIA**

Gaussian

Model	GJR	EGARCH	GJR	EGARCH
MAE	2	1	1	2
RMSE	2	1	1	2
MAPE	2	1	1	2
TIC	1	2	2	1

Student-T

Model	GJR	EGARCH	GJR	EGARCH
MAE	2	1	1	2
RMSE	1	1	1	2
MAPE	2	1	1	2
TIC	1	2	2	1

GED

Model	GJR	EGARCH	GJR	EGARCH
MAE	2	1	1	2
RMSE	2	1	1	2
MAPE	2	1	1	2
TIC	1	2	1	2

TABLE 9
SUMMARY OF BEST PERFORMING MODEL

	Ghana	**Nigeria**
Gaussian	EGARCH	GJR
Student-t	EGARCH	GJR
GED	EGARCH	GJR

Table 8 ranks the asymmetric GARCH models when evaluated against each other with the three different distribution densities for the error term. The evidence in Tables 7 and 8 indicates that no single model completely dominates the other for either series. Nonetheless, Table 9 shows that EGARCH provides the best out-of-sample forecast for the Ghanaian stock market, while the GJR gives a better estimation for the Nigerian stock market. Furthermore, forecasting with non-normal distribution densities yields a significant reduction of the forecast error relative to the normal distribution. The predictive power of $\hat{\sigma}_t^2$ is justified due to the fact that the asymmetric GARCH models may follow a fatter-tailed distribution. Moreover, it appears that the asymmetric models with fatter-tailed distribution have a tendency to produce superior forecast and the predictor $\hat{\sigma}_t^2$ appears to possess some predictive power when a fatter-tailed distribution density is assumed for the asymmetric GARCH residuals.

CONCLUSION

The aim of this paper was to model and forecast the performance of two prominent asymmetric volatility models with the introduction of different distribution densities for Ghana and Nigeria stock markets. The study was motivated by recognising the importance of accurate volatility measurement and forecast in a wide range of financial applications including portfolio selection, value at risk, asset pricing, hedging strategies and option pricing, and the non-existence of empirical evidence available to date for Ghana and Nigeria stock markets. Our paper contributes to the literature in the following ways. First, we used data sets from emerging African stock markets, where such study has not been conducted previously. Secondly, we applied asymmetric GARCH models (GJR and EGARCH) that capture not only the time series features of skewness, kurtosis and volatility clustering but also the leverage effect. Thirdly we compared these two GARCH-type models with the introduction of three different distribution densities (i.e. Gaussian versus non-normal) for modelling and forecasting the GSEI and NSEI returns volatility.

Our statistical results indicate that news impact is asymmetric in both stock markets for all densities with statistically significant coefficients at standard levels. Furthermore, our evidence shows that both the GJR and EGARCH models capture the leverage effect in GSEI stock returns, which indicates that negative shocks imply a higher next period conditional variance than positive shocks of the same magnitude. Moreover, similar to Wan et al. (2014) evidence in China, our results in Nigeria show that the market exhibits a reverse volatility asymmetry. This means that the Nigerian market controverts the widely accepted observation of volatility asymmetry, where negative returns induce a higher return volatility than positive returns. The evidence shows that in Nigeria the return volatilities react more intensely to positive returns than to negative returns. This reverse asymmetry could be attributed to the higher trading volume associated with momentum stocks (i.e. price rising stocks) as Nigerian investors are known to rush for such stocks more than their negative counterparts and this gives rise to higher volatility. So in effect a positive return-volatility correlation is observed in Nigeria, while in Ghana we observed a negative return-volatility relationship.

Comparing models with each density shows that the fatter tails (Student-t and GED) distributions clearly outperform the Gaussian for both GJR and EGARCH models. The comparison between models with each distribution density indicates that, given the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for GSEI. However, in Nigeria, no clear results can be obtained, as both GJR and EGARCH for each distribution density provides close results. Moreover, our results can show noticeable improvement when non-normal distribution density is used to model the conditional variance. According to the different measures used to evaluate the volatility forecast performance, the EGARCH provides the best out-of-sample forecast for the Ghanaian stock market, while the GJR gives a better estimation for the Nigerian stock market. Furthermore, forecasting with non-normal distribution densities yields a significant reduction of the forecast error relative to the normal distribution.

The presence of leverage effect suggests that investors in these markets are to be rewarded for taking up additional leverage risks. This implies that by allocating portfolios, fund managers and/or investors should go beyond the mean-variance analysis in regards to these markets and look into information about volatility, information asymmetry, correlation, skewness and kurtosis (see for example Bekaert *et al*, 1996). Cost of equity capital is expected to be high in these markets due to compensation for additional leverage risk which places additional burden on indigenous Ghanaian and Nigerian firms seeking to raise finance from domestic capital markets.

Finally, there are areas where further studies might be useful. We recommend that future research be directed at modelling and forecasting realised volatility from intra-day trading data (Poon, 2003 & 2005). Furthermore, future research may also consider exploring a variety of models including other conditional variance models such as APARCH and long memory models such as FIEGARCH, FIAPARCH and CGARCH in order to allow a superior insight into the dynamics of these market series. Lastly the study should be replicated in other African countries in order to shed more light on the economic and structural variables that drive volatility in African stock markets.

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