Pricing Accuracy of Put-Option Valuation Models: Directional Bias Due to Risk Free Interest Rates

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The classical Black-Scholes formula reveals systematic biases in valuation of option prices (Geske and Roll 1984 and reference therein). Heo et al. (2015) also found the existence of similar biases in fractional quadratic option pricing models. These observed pricing biases depend on moneyness, the time to maturity, and volatility of underlying assets. Recently, we have noticed that pricing bias is also seemingly influenced by interest rates. This study compares pricing accuracy across several put option models and investigates pricing biases caused by risk-free LIBOR using daily data of Yahoo put options traded in CBOE from February 2005 to February 2015.

INTRODUCTION

The primary objective of this research is to examine the accuracy of quadratic option pricing models. Preliminary results, with a limited Yahoo dataset for the period January 2014-February 2015, showed with very few exceptions that all the tested models overestimate option prices regardless of moneyness and maturity (See Table 1). It contradicts the results of numerous previous studies, which are well documented in Geske and Roll (1984 and reference therein, MacBeth and Merville (1979), Rubinstein (1985)). Also, more recently, Heo et al. (2010, 2015) demonstrate that quadratic option pricing models (i.e., FMBAW and FMQuad) based on Black-Scholes' partial differential equation also contain pricing biases. These observed pricing biases depend on moneyness, the time to maturity, and variance. With a few exceptions, all models underestimate option prices with regards to moneyness and time to maturity. It is important to determine the causes for these contradictory results in pricing biases (i.e., systematic under- or over-estimation in option valuations) and provide plausible explanations.

The underlying assumption is that American quadratic option pricing models are the sum of European option price and the early exercise premium, especially in put option pricing. If European models overestimate option price, then any American model will result in even greater over-estimation, which hampers the accuracy of American option pricing models.

Previous studies conducted before the 2008 global financial crisis did not experience the subsequent historically low interest rates that prevailed in the following decade. Consequently, those studies did not include the potential influence of low interest rates on option pricing. We explore the relationship between option pricing models and interest rates and examine the existence of pricing bias that might result.

In this study, we examine the accuracy of American put valuation models base on fractional Brownian motion using Yahoo equity option data – traded on the Chicago Board of Option Exchange (CBOE) from February 1, 2005 to February 25, 2015. Accuracy is measured with mean absolute percentage error with respect to actual option price (MAPE), mean percent error with respect to option price (MPE) and root mean squared error (RMSE) by interest rates.

The remainder of this paper is structured as follows: Section II describes various valuation models of American put options. Section III discusses data and research methodology. Estimation results are reported in Section IV. Section V concludes the study with suggestions for future research.

APPROXIMATION METHODS IN PRICING OF AMERICAN OPTIONS

In this section, we describe three fractional option pricing models – European fractional Black-Sholes model (FBS) (Daye 2003) and two American quadratic approximation models, FMBAW and FMQuad (Heo et al. 2015).

Using the time variable t, $0 \leq t \leq T$, where $t = 0$ corresponds to the issue date of the option and $t = T$ corresponds to its expiration date, we let

$S = S(t)$: stock price at time t
X: strike price of option
r: current risk-free interest rate
σ: stock price volatility
$\tau = (T - t)$: time to expiration
δ: current dividend yield (for dividend paying stocks)

European put option price (**FBS**), $P_E(S,t)$, is given by

$$P_E(S,t) = Xe^{-r\tau}N(-d_2) - Se^{-\delta\tau}N(-d_1),$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + r\tau + \frac{\sigma^2}{2}(T^{2H} - t^{2H})}{\sigma\sqrt{T^{2H} - t^{2H}}}, \qquad d_2 = d_1 - \sigma\sqrt{T^{2H} - t^{2H}}, \quad N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-u^2/2}du.$$

We use two American quadratic put option pricing formulas (FMBAW and FMQuad) presented in Heo et al. (2015). These two models FMBAW and FMQuad formulas are resemble to MBAW [Barone-Adesi, and Whaley (1987)] and MQuad [Ju and Zhong (1999)], respectively. Thus, we compare the accuracy of these models as well as FBS and classical BS models.

FMBAW Model

$$P(S,t)=\begin{cases}P_E(S,t)+KA(K)(S/S^*)^{\lambda(K)} & if\ S>S^*\\ X-S & if\ S\leq S^*,\end{cases}$$

$$K=1-e^{-r\tau},\ A(K)=-\left(\frac{S^*}{K\lambda}\right)\left[1-e^{-\delta\tau}N\left(-d_1(S^*)\right)\right],$$

$$\lambda(K)=\frac{(1-\beta)-\sqrt{(1-\beta)^2+4\alpha/K}}{2},\ \ \alpha=r/[\sigma^2H\ (T-\tau)^{2H-1}],\ \text{and}\ \ \beta=(r-\delta)/[\sigma^2H\ (T-\tau)^{2H-1}].$$

FMQuad Model

$$P(S,t)=\begin{cases}P_E(S,t)+\dfrac{KA(K)(S/S^*)^{\lambda(K)}}{1-\chi} & if\ \ S>S^*\\ X-S & if\ \ S\leq S^*,\end{cases}$$

$$\chi=b\left[\log\left(\frac{S}{S^*}\right)\right]^2+c\log\left(\frac{S}{S^*}\right),\ \ b=\frac{(1-K)\lambda'(K)\alpha}{2(2\lambda+\beta-1)},\qquad \lambda'(K)=\frac{\alpha}{K^2\sqrt{(1-\beta)^2+\frac{4\alpha}{K}}}$$

$$c=-\frac{(1-K)\alpha}{(2\lambda+\beta-1)}\left[\frac{1}{KA(K)}\frac{\partial P_E(S^*,\tau)}{\partial K}+\frac{1}{K}+\frac{\lambda'(K)}{2\lambda+\beta-1}\right],$$

$$\frac{\partial P_E(S^*,\ \tau)}{\partial K}=-XN(-d_2)+\frac{S^*\delta}{r}e^{(r-\delta)\tau}N(-d_1)+\frac{X}{r}\frac{\partial N(-d_2)}{\partial\tau}-\frac{s}{r}e^{(r-\delta)\tau}\frac{\partial N(-d_1)}{\partial\tau}.$$

In both pricing formulas, the critical stock price S^* is recovered from the equation

$$X-S^*=P_E(S^*,t)-\left[1-e^{-\delta\tau}N\left(-d_1(S^*)\right)\right]S^*/\lambda(K).$$

FMBAW and FMQuad formulas are similar to MBAW and MQuad, respectively. However, the coefficients α and β involve the term $H\cdot(T-\tau)^{2H-1}$ so that the option prices highly depend on the Hurst parameter. Traditional BS, MBAW, and MQuad models are basically special cases of fractional counterparts such as FBS, FMBAW and FMQuad where $H=1/2$.

DATA AND RESEARCH METHODOLOGY

This study uses daily close prices traded on the Chicago Board of Option Exchange (CBOE) for Yahoo put options between February 1, 2005 and February 25 2015. These put options differ in exercise price and expiration date so that there are total of 7,322 different put options. This data is obtained from IVolaitity.com and uses LIBOR as risk-free interest rates.

For a variety of reasons enumerated below, it is necessary to screen the data. American put option prices must satisfy the no-arbitrage boundary conditions: $P_{actual}\leq 0.99(X-S)$. Any observation failing these conditions was deleted. Observations with put option prices less than \$0.50 were also deleted to eliminate outliers and prohibitively high transaction costs. Thinly traded put options were also deleted. Some input variables such H value and implied volatility were recovered from the previous day's data, thus the first observation of each put option was lost. The last filter required deleting options whose H values are not in the range of 0 to 1. Consequently, the final dataset consists of 139,758 usable observations.

To examine the accuracy of each model, we employ three measures - mean absolute percent error (MAPE) for accuracy, mean percentage error (MPE) for bias, and root mean squared error (*RMSE*) for variation given by the following equations respectively,

1) $MAPE = \frac{1}{N}\sum\frac{|P_{model}-P_{actual}|}{P_{actual}} \times 100(\%)$,

2) $RMSE = \sqrt{\frac{1}{N}\sum(P_{model} - P_{actual})^2}$ (\$),

3) $MPE = \frac{1}{N}\sum\frac{P_{model}-P_{actual}}{P_{actual}} \times 100(\%)$.

This study utilizes the implied volatility measures that are recovered from the binomial tree model with 100 steps, and we adopt the implied Hurst values (*H*) recovered from FBS model using the previous day's data. The Hurst parameter is the key in studying fractional models and previous results by Razdan (2002), Heo et. al. (2015), and Meng and Wang (2010) also verify that the Hurst parameter significantly influences the pricing accuracy.

RESULTS USING IMPLIED VOLATILITY

In Table 1, the preliminary results with a limited data from January 2014 to February 2015 are puzzling. Fractional models (FBS, FMBAW, and FMQuad) perform slightly but not significantly better than conventional pricing models (BS, MBAW, and MQuad). It is also noteworthy that American put pricing overestimation was more pronounced than its European counterpart simply because of premium component added to European option value to capture early exercisability. It is not consistent with evidence from previous option literature and thereby motivates this research.

TABLE 1
MAPE, RMSE, AND MPE (JANUARY 2014 – FEBRUARY 2015)

N=44311	BS	MBAW	MQuad	FBS	FMBAW	FMQuad
MAPE	3.4476	3.4389	3.4692	3.3197	3.3252	3.3672
MPE	0.3217	0.3963	0.4036	0.0500	0.1243	0.1174
RMSE	0.1368	0.1341	0.1457	0.1331	0.1335	0.1496

N = Number of observations

Table 2 reports estimation accuracy of the extended option data from February 2005 to February 2015. It shows that MBAW and MQuad models yield smaller errors than the fractional counterpart. This result is again not consistent with previous research by Heo.et. al. (2010, 2015). American option pricing models perform worse because of the added early exercise premiums. Referring to Table 3, we attempt to partially explain this anomaly by examining the effect of interest rates in the comparison of these models.

TABLE 2
MAPE, RMSE, AND MPE (FEBRUARY 2005 – FEBRUARY 2015)

N=139758	BS	MBAW	MQuad	FBS	FMBAW	FMQuad
MAPE	3.0488	2.2792	2.2879	2.3752	2.6542	2.6319
MPE	- 0.8741	0.2452	0.2011	- 0.1838	0.7336	0.6744
RMSE	0.3121	0.1021	0.1081	0.1362	0.1834	0.1831

N = Number of observations

Table 3 reports MPE of the different option pricing models across various interest rates. As the interest rate *decreases*, the MPEs of European option models (BS and FBS), gradually *increase* moving from underestimation to overestimation. This pattern of pricing bias contributes larger pricing errors to American quadratic option models, such as BS vs MBAW and MQuad and FBS vs FMBAW and FMQuad. Also interestingly, fractional option pricing models (see Heo. et al (2010, 2015)) are generally more effective than non-fractional models. This study, however, doesn't find any consistent pattern of results. Table 3 indicates that when the interest rates are small American pricing model's accuracy critically hinges upon pricing biases of European style models, either BS or FBS. The FBS model overestimates option prices. Because the BS model underestimates the option prices so that when the early exercise premium is added, MBAW and MQuad provide smaller pricing errors. The FBS model estimates within 1% of pricing error throughout regardless of interest rate and hence provides smaller pricing error than the corresponding BS model, which is consistent with the study of Meng and Wang (2010).

TABLE 3
MPE (FEBRUARY 2005 – FEBRUARY 2015)

Rates	N	BS	MBAW	MQuad	FBS	FMBAW	FMQuad
< 0.2%	25535	0.5908	0.5966	0.5839	0.1594	0.1656	0.1411
0.3	23490	0.1646	0.1895	0.2394	-0.1421	-0.1159	-0.0568
0.4	13986	0.1361	0.2004	0.2483	-0.0139	0.0518	0.1026
0.5	7076	0.0569	0.1641	0.2119	-0.0666	0.0431	0.1060
0.6	9183	-0.2184	0.0818	0.0806	-0.0995	0.2002	0.1684
0.7	6170	0.2021	0.5601	0.5671	0.1040	0.4573	0.4818
0.8	4237	-0.4678	0.0016	0.0441	-0.0975	0.3529	0.3901
0.9	2817	-0.2926	0.1604	0.1997	0.0608	0.4969	0.5231
1	1130	-0.2689	0.2373	0.2665	0.0746	0.5356	0.5556
2	7968	-0.4726	0.3546	0.3370	0.1980	0.9209	0.8836
3	7278	-0.9308	0.2010	0.1712	-0.2345	0.7516	0.7182
4	8266	-2.2446	0.1171	0.0108	-0.5486	1.3756	1.2294
5	9140	-4.4519	-0.0591	-0.3843	-0.9268	2.7151	2.3142
≥ 5	13482	-5.1749	0.0718	-0.2372	-0.9042	3.1772	2.8145

N = Number of observations

When considering the mean absolute percent error (MAPE), Table 4 presents that the FBS model yields smaller error than the BS model but it is reversed for fractional American options models. It is not surprising because MPE of FBS estimation biases is within +/-1% across various interest rates so that American option models are less accurate unless the early exercise premium is very small. We conclude that the FBS model is a reasonable choice for option pricing models regardless of option styles, consistent with some previous studies (Heo et al. (2010, 2015)). MAPE is sensitive to interest rates in the sense that when the rate is less than 0.2%, all models do not perform well. Also when the interest rate is greater than 3%, BS estimation is not as accurate as FBS, but MBAW and MQuad perform better than their counterparts, FMBAW and FMQuad. This can be explained by MPE of those models. As we can see in Table 3, using MPE, FMBAW and FMQuad significantly overestimates to a greater extent than their counterparts. Accuracy decreases with the inclusion of early exercise premiums.

From Table 5, we find that the RMSE of BS model decreases as the interest rate decreases to the 1% level. Overall, the FBS model outperforms the other models but the FMBAW and FMQuad models are not as stable as the corresponding MBAW and MQuad models are when interest rates are greater than 1%. Again, this is consistent with MPE and MAPE accuracy estimations. However, when the interest rates are between 0.3% and 1%, we cannot conclude one model is better in pricing accuracy than the others because all models perform similarly.

TABLE 4
MAPE (FEBRUARY 2005 – FEBRUARY 2015)

Rates	N	BS	MBAW	MQuad	FBS	FMBAW	FMQuad
< 0.2%	25535	4.0135	4.0136	4.0792	3.8969	3.8964	3.9665
0.3	23490	1.7882	1.7869	1.8346	1.7507	1.7471	1.8223
0.4	13986	1.9108	1.9057	1.9156	1.8894	1.8830	1.8940
0.5	7076	1.7259	1.7106	1.7206	1.7194	1.7069	1.7248
0.6	9183	2.3673	2.3129	2.3141	2.2082	2.2207	2.2679
0.7	6170	2.6503	2.5810	2.5774	2.2541	2.2733	2.3151
0.8	4237	1.9015	1.7502	1.7487	1.7743	1.7905	1.8134
0.9	2817	2.0769	1.9416	1.9401	1.9014	1.9474	2.0168
1	1130	1.7794	1.5991	1.5960	1.5286	1.6255	1.6489
2	7968	2.0550	1.7458	1.7366	1.7340	1.9624	1.9587
3	7278	2.9255	2.3538	2.3493	2.5398	2.6027	2.6085
4	8266	3.4663	1.9113	1.8767	2.3224	2.6813	2.5869
5	9140	5.0212	1.7389	1.6684	2.3070	3.7517	3.4062
≥ 5	13482	5.6706	1.5371	1.4839	2.3259	3.8304	3.5308

N = Number of observations

TABLE 5
RMSE (FEBRUARY 2005 – FEBRUARY 2015)

Rates	N	BS	MBAW	MQuad	FBS	FMBAW	FMQuad
< 0.2%	25535	0.1251	0.1251	0.1497	0.1243	0.1242	0.1521
0.3	23490	0.0838	0.0837	0.0864	0.0857	0.0851	0.1017
0.4	13986	0.0949	0.0943	0.0946	0.0960	0.0950	0.0954
0.5	7076	0.0862	0.0839	0.0856	0.0895	0.0870	0.0906
0.6	9183	0.1360	0.1278	0.1273	0.1275	0.1282	0.1316
0.7	6170	0.1631	0.1518	0.1515	0.1484	0.1489	0.1816
0.8	4237	0.1165	0.0996	0.0990	0.1118	0.1034	0.1046
0.9	2817	0.1017	0.0865	0.0861	0.0911	0.0867	0.0923
1	1130	0.0922	0.0668	0.0663	0.0677	0.0737	0.0765
2	7968	0.1537	0.0799	0.0797	0.0858	0.1116	0.1132
3	7278	0.2346	0.1150	0.1144	0.1679	0.1794	0.1810
4	8266	0.4020	0.1098	0.1086	0.1742	0.2226	0.2118
5	9140	0.6024	0.0814	0.0797	0.2142	0.3276	0.3031
≥ 5	13482	0.7305	0.0619	0.0610	0.1998	0.3728	0.3524

N = Number of observations

CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

This study investigates pricing accuracy and bias of various American put option models – classical Black-Scholes model (BS) and two quadratic approximation models (MBAW and MQuad). For the period of February 2005-February 2015, using daily close prices of 7,322 Yahoo put options, three accuracy measures are compared across three traditional models and three matching fractional counterparts.

Empirical evidence documents that different directional pricing biases are present at around a 1% risk–free rate. When interest rate is 1% or less, pricing bias is insignificantly small. Otherwise, the bias is substantially bigger as shown in Table 3. Regardless of different interest rate levels, fractional American put pricing models perform worse than FBS model because FBS estimation biases is so small. As a result, American option models generate option prices that deviate farther from accurate prices unless the early exercise premium is very small. Although it is not consistent with evidence from previous literature, traditional non-fractional quadratic approximation models (MBAW and MQuad) perform more accurately with less pricing biases than fractional counterparts (FMBAW and FMQuad) where risk-free rate is higher than a 1%.

This study cannot provide clear explanations why theoretically sound fractional pricing models do not perform better than non-fractional matching models. However, we list the following plausible explanations along with suggestions for future research.

Yahoo is a single firm from an atypical industry sector. Therefore, put option prices of Yahoo may not follow a usual return generating process. Also, the observation period is includes a significant shock in global financial markets – the 2008 subprime loan crisis triggered turmoil in financial markets.

Because accuracy and unbiasedness of fractional option pricing models are critically affected by the estimates of Hurst parameters, our selected estimation process of the H parameter perhaps may result in less accurate and more biased option valuations.

We plan to conduct research with different observation periods and put options from various industry sectors. The future research will employ alternative estimation processes of the H parameter to improve model accuracy and reduce pricing biases.

REFERENCES

Barone-Adesi, G. (2005). The Saga of American Put*, Journal of Banking and Finance*, 29, 2909-2918.

Barone-Adesi, G., and Whaley, R. (1987). Efficient Analytic Approximation of American Option Values, *Journal of Finance*, 42, 301-320.

Biagini, F., Hu, Y., Øksendal, B., and Zhang, T. (2008). Stochastic Calculus for Fractional Brownian Motion and Applications, Springer.

Black, F., and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-654.

Daye, Z. (2003). Introduction to Fractional Brownian Motion in Finance, Manuscript.

Elliott, R. J., and Van der Hoek, J. (2003). A General Fractional White Noise Theory and Applications to Finance, *Mathematical Finance*, 13, no. 2, 301-330.

Geske, R., and Roll, R. (1984). On valuing American Call Options with the Black-Scholes European Formula, *The Journal of Finance*, Vol. 39, No. 2, 443-455.

Heo, S., Sarol, Y., and Kang, J. (2010). Valuation of American Equity Options with Quadratic Approximation Models Adopting Fractional Brownian Motion, *Global Business and Finance Review* (Fall), 127-140.

Heo, S., Rhim, J., Cashel-Cordo, P. and Kang, J. (2015), An Estimation of American Put Option Price using Quadratic Approximations of the Fractional Black-Scholes (FBS) Partial Differential Equation, *Journal of Accounting and Finance*, Vol 15(8), 2015, 48-61.

Ju, N., and Zhong, R. (1999). An Approximate Formula for Pricing American Options, *Journal of Derivatives* (Winter), Vol. 7, No. 2, 31-40.

Macbeth, J. and Merville, L. (1979). An Empirical Examination of the Black-Scholes Call Option Pricing Model, *Journal of Finance*, Vol. 34, no. 5, 1173-1186.

MacMillan, W. (1986). Analytic Approximation for American Put Options, *Advances in Options and Futures Research*, 1, 119-139.

Meng, L., and Wang, M. (2010). Comparison of Black-Scholes Formula with Fractional Black-Scholes Formula in the Foreign Exchange Option Market with Changing Volatility, *Asia-Pacific Finance Markets*, 17, 99-111.

Razdan, A. (2002). Scaling in the Bombay Stock Exchange Index*, Pramana-Journal of Physics*, Vol. 58, No. 3, 537-544.