Enhancing the Dividend Discount Model to Account for Accelerated Share Price Growth

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The dividend discount model (DDM) for calculating the intrinsic value of stock assumes constant growth for both dividends and share price. While the nonconstant growth method permits estimation of the stock price during abnormal growth periods, there is nothing that accounts for these periods within the original model, obscuring the connection between the methods. I introduce a new method, tied to the firm's life cycle and dividend policy, which accounts for these periods of extraordinary growth within the original DDM. An example is given comparing the intrinsic value calculated with the enhanced DDM to that using the nonconstant growth method.

INTRODUCTION

One of the oldest and most trusted models in financial theory is the Dividend Discount Model (DDM), also known as the Gordon Model (Gordon 1962). The DDM is used to calculate the intrinsic value of a firm's common equity based on three simple inputs: the next dividend the firm will pay (D_1), the rate by which dividends are expected to grow moving forward (g), and the return investors require for buying the firm's common stock (r). The model as taught in the business school curriculum is:

$$\hat{P}_0 = \frac{D_1}{r - g} \quad \text{where g < r} \tag{1}$$

This is the simplified form of the model. The model in its original form shows that the intrinsic value is derived as the present value of the firm's expected future dividends:

$$\hat{P}_0 = \sum_{t=1}^{\infty} D_1 \frac{(1+g)^{t-1}}{(1+r)^t} \quad \text{where g < r} \tag{2}$$

The proof for how equation (2) is simplified to equation (1) is given in Appendix A.

The model makes two basic assumptions: that both the required return on the firm's common stock and the dividend growth rate are constant moving forward. The required return on common equity (r) is a function of the market's perception of the relative riskiness of the firm's cash flows, influenced by the riskiness of the firm's capital investments and the general level of risk aversion in the market. Thus barring a significant event in the firm or in the general economy, r changes slowly if at all.

The dividend growth rate (g), while influenced by those same risks, is also driven by the firm's dividend policy. Dividend increases tend to be "sticky", as rescinding them sends a negative signal to the market that is ultimately realized via a decrease in the firm's stock price. Thus the dividend growth rate tends not to fall, even when earnings decline. At the same time, g only tends to increase when the firm's executives believe the increase is permanent.

Yet there are times when the firm's share price grows faster than the firm's dividends, without any accompanying change in the firm's perceived risk profile. The DDM does not address these situations directly. Rather, theory advises us to follow the basic principle that the value of an asset is equal to the present value of the cash follows that asset is expected to produce over its economic life. Thus the "nonconstant growth" method for estimating a stock's intrinsic value was developed:

1. Estimate the dividends during the period of nonconstant growth;
2. Use the DDM to value the stock at the end of the nonconstant growth period (the beginning of normal growth);
3. Discount the nonconstant growth dividends and the value of the DDM to obtain their present value;
4. Sum the present values of the nonconstant dividends and the DDM to obtain the estimate of the stock's intrinsic value.

No attempt has been made to date to reconcile the original DDM with the nonconstant growth stock valuation method. In this paper I use dividend theory and the firm's life cycle to derive two new terms that, when added to the DDM, account for the increase in the firm's share price not recognized in the original model.

ENHANCING THE DDM

When a firm (or industry) is relatively young, the economy offers many investment opportunities that provide returns higher than the firm's cost of capital, thus creating substantial growth. During this phase, it is common for the firm's stock price to rise faster than the firm's earnings, reflected in a higher P/E ratio. Consequently during this phase the DDM is not effective for valuing the firm's common equity, as growth is not constant. When the firm reaches the maturity phase in its life cycle, increasing competition forces it to fight for fewer worthwhile investment opportunities, and growth slows to a more sustainable level. At that point the firm's P/E ratio adjusts downward, as the market recognizes the diminished opportunities for growth. That is the beginning of constant growth for the firm, and the point at which the DDM is effective in valuing the firm's shares.

The change in the firm's growth opportunities also impacts the firm's dividend policy. While the firm has its pick of positive NPV investments, it will retain a greater percentage of its earnings than usual to fund these projects, since retained earnings are the least expensive form of common equity. When the investment opportunities become sparse, the firm is obliged to distribute more of its earnings to investors, rather than invest in negative NPV projects. The firm will therefore retain less of its earnings at this point, and should return the cash to the investors in the form of a special dividend or a share repurchase. In this way the nonconstant growth in the firm's share price during the growth phase of its life cycle is tied to the firm's retention rate.

Utilizing the Retention Rate

The DDM is easily modified to include the retention rate since the retention rate is implied in two of its terms: the dividend (D_1) and the dividend growth rate (g). These terms are rewritten to explicitly include the retention rate. To simplify this process, a new term, p (for plowback) is used to represent the retention rate. The term p is used in two forms and in two different ways, thus enriching the concept of the retention rate.

The retention rate is commonly stated as the complement of the dividend payout rate ($1 - b$). Conversely, the dividend payout rate can be stated in terms of the retention rate. If p is the percentage of earnings retained and used for investment opportunities, then (1 – p) is the percentage of earnings paid out as a dividend.

The variable p is more specifically defined as the percentage of earnings retained and used for investment opportunities during the maturity phase of the firm's life cycle. A second term, p_g, is used to represent the increase in the retention rate experienced during the growth phase of the firm's life cycle. This temporary increase in the firm's retention rate is inserted into the original DDM in two places.

The dividend (D_1) is rewritten to include the retention rate by multiplying the earnings per share (EPS) by one minus the retention rate. To include both the growth phase and the maturity phase of the firm's life cycle in the dividend, it is rewritten as $D_1 = EPS_1 \cdot (1 - p - p_g)$. When the firm reaches the maturity phase in their life cycle the term p_g goes to zero and falls out of the model.

The second term $\left(\frac{(1+g)^{t-1}}{(1+r)^t}\right)$ represents the constant growth in the dividend and the discounting of the future dividends to the present. The dividend growth rate (g) is commonly calculated as $ROE \cdot (1 - b)$, but here is calculated as $g = ROE \cdot p$, and is further rewritten as $g = ROE \cdot (p + p_g)$. Again, when the firm enters the maturity phase of their life cycle, the term p_g goes to zero and falls out of the model.
Inserting the temporary increase in retained earnings into the model enhances the original DDM thusly:

$$\hat{P}0 = \sum_{t=1}^{\infty} EPS_1 (1 - p - p_g) \frac{\left[1 + ROE \cdot \left(p + p_g\right)\right]^{t-1}}{\left(1+r\right)^t} \tag{3}$$

In the first right-hand term, the dividend is reduced (and retained earnings are increased) by p_g. This modifies the expected future value of the dividends to account for the increase in the retention rate. This modification does not necessarily reflect a decrease in the actual dividend paid, but rather in the percentage of earnings that are used for paying the dividend. Assuming the projects the firm invests in are successful and growth is extraordinary, the firm would not need to pay out as much of their earnings (as a percentage) in order to increase the dividend. For example, assume a firm has \$100 in earnings and pays 50% of its earnings as a dividend. This results in a dividend of \$50. If the following year the firm has \$150 in earnings, it could reduce its payout rate to 40% resulting in a dividend of \$60, which is a 20% increase over the prior year.

In the numerator of the second right-hand term, the dividend growth rate is increased by $ROE \cdot p_g$. In the simplified form of the enhanced DDM (equation 4 below) this reduces the denominator (if $g_n > 0$ then $r - g - g_n < r - g$), thus enhancing the present value of the expected future dividends. As Appendix B shows, equation (3) simplifies to:

$$\hat{P}_0 = \frac{D_1 - EPS_1 \cdot p_g}{r - g - \left(ROE \cdot p_g\right)} = \frac{D_1 - DR}{r - g - g_n} \tag{4}$$

In its simplified form, the numerator of the original DDM is decreased by the increase in the retention rate ($DR = EPS_1 \cdot p_g$), while the denominator is decreased by a growth premium ($g_n = ROE \cdot p_g$) that is also driven by the increase in retained earnings. These two changes push the intrinsic value of the share price up to account for the growth experienced over and above the normal dividend growth reflected in the original DDM.

Interpreting the New Terms

Firms create value by investing in projects that return more than their cost of capital. During the growth phase of their life cycle, firms have an exceptional number of positive NPV projects available to them. To take advantage of those growth opportunities, these firms plow back more earnings into the firm than they normally would, as retained earnings are the cheapest form of equity capital. During this period the firm experiences extraordinary growth, and the increase in the firm's retention rate is a significant driver of that growth. The market will recognize this extraordinary growth by increasing the firm's share price beyond what it would be if normal growth were expected. Once the number of positive NPV projects declines, the firm will return the excess retained earnings to the shareholders and reduce the retention rate. At this point the firm returns to normal growth, and the market will recognize that as well.

Both of the new terms included in equation (4) are driven by the change in the firm's retention rate. The dividend reduction in the numerator ($DR = EPS_1 * p_g$) reflects the fact that a smaller percentage of the firm's earnings are paid out as a dividend while the firm is experiencing strong growth. Assuming the firm has established a sustainable dividend, reducing the payout rate during times of extraordinary growth should not result in a reduction of the cash dividend paid, and could likely include an increase.

The growth premium in the denominator ($g_n = ROE * p_g$) acts as an increase in the dividend growth rate (g) that serves to increase the future value of the stream of dividends. This term decreases the value of the denominator in the simplified model, thus increasing the intrinsic price of the stock.

Note that when the firm's growth rate is once again constant and the retention rate is reduced to its normal level, these two terms go to zero and fall out of the model, leaving the original DDM.

A SIMPLE EXAMPLE

I offer a simple example to show the mechanics of the calculations and how they are applied to a particular firm. Note that the inputs of the original DDM are entered as they would be under normal growth conditions.

Firm A just paid a dividend (D_0) of \$1.20. Their normal dividend growth rate (g) is 5%, but due to the recent release of a new product, dividends are expected to grow by 20% per year for the next three years, after which dividends are expected to grow at their normal rate of 5% per year. The required return on the firm's common equity (r) is 10%. The firm's normal retention rate (p) is 60%, but during periods of supernormal growth the firm retains 80% of their earnings (i.e., p_g = 20%). Last year Firm A had earnings per share (EPS_0) of \$3.00, which are expected to grow at the dividend growth rate of 5% during periods of constant growth.

Without the release of the new product, Firm A's intrinsic share price, calculated with the original DDM, would be $\hat{P}_0 = \frac{\$1.26}{0.1 - 0.05}$ = \$25.20. This would be the intrinsic value of the firm's stock assuming constant normal growth in the future. Using the nonconstant growth method for valuing the stock, the intrinsic value of the firm's common equity after the release of the new product is calculated as $\hat{P}_0 = \frac{1.20(1.2)}{1.1} + \frac{1.20(1.2)^2}{(1.1)^2} + \frac{1.20(1.2)^3}{(1.1)^3} + \left[\frac{1.20(1.2)^3(1.05)}{(.1 - .05)(1.1)^3}\right]$ = \$37.01. This increase in the intrinsic value of the stock is driven by the increase in the dividend growth rate during the first three years, and the increase in the dividend growth rate is in turn driven by the high number of positive NPV projects in which the firm invests. The additional investment projects are funded partially by the increase in the retention rate.

Two steps are required to apply the new terms to the enhanced DDM. First, the dividend reduction (DR) is calculated as $EPS_1 * p_g$. EPS_1 is calculated as $EPS_0 \cdot (1 + g)$ = \$3.00 · 1.05 = \$3.15. Therefore the dividend reduction term is DR = \$3.15 · 0.2 = \$0.63.

Second, the increase in the dividend growth rate (g_n) must be calculated indirectly. The intrinsic value calculated by the nonconstant growth method (\$37.01) implies a growth premium in the firm's dividend growth rate (g_n) of approximately 3.3% (see Appendix B for the method for calculating g_n). Note that this growth premium reflects an ROE for the firm of 16.5% (since $g_n = ROE \cdot p_g$, $ROE = g_n \div p_g = 0.032978 \div$

0.2 = 0.1649), which is higher than the required return on common equity. This is consistent with the firm experiencing extraordinary growth in earnings and dividends.

Given the analysis above, and using the enhanced DDM, $\hat{P}_0 = \dfrac{\$1.26 - \$0.63}{.1 - .05 - .032978} = \$37.01$.

SUMMARY

The Dividend Discount Model (DDM) calculates the intrinsic value of a stock based on the present value of the expected future dividends, but makes the assumption that the growth rates for both dividends and the stock price are constant moving forward. It is common for firms still in the growth phase of their life cycle to experience share growth higher than dividend growth, based on the availability of numerous positive NPV projects and the increase in retained earnings used to fund the projects. In this paper I enhance the DDM to account for extraordinary growth in the share price by including the increase in retained earnings common for growing firms. This enhancement results in two new terms in the enhanced DDM related to the firm's higher retention rate. Together these two terms account for the extraordinary increase in share price growth.

REFERENCES

Gordon, M. J. (1962). *The Investment, Financing and Valuation of the Corporation*, Homewood, IL: Irwin.

Farrell, J. L. (1985). The Dividend Discount Model: A Primer. *Financial Analysts Journal*, 41, (6) 16-19 + 22-25

APPENDIX A: PROOF FOR THE ORIGINAL DIVIDEND DISCOUNT MODEL

The dividend discount model states that the intrinsic value of a stock is calculated as the present value of the stream of expected future dividends:

$$\hat{P}_0 = \sum_{t=1}^{\infty} D_1 \frac{(1+g)^{t-1}}{(1+r)^t} \text{ where g < r.}$$

On the right side of the equation, divide D_1 by (1 + g) and multiply the right-hand term by (1 + g) to have a consistent exponent for the right-hand term (the effect is to multiply the right side by $\frac{1+g}{1+g}$).

$$\hat{P}_0 = \sum_{t=1}^{\infty} \frac{D_1}{1+g} \left(\frac{1+g}{1+r} \right)^t \text{ where g < r.}$$

The term $\left(\frac{1+g}{1+r} \right)^t$ is an infinite geometric series if g < r. This series can be simplified by using a geometric expansion.

Let $\beta = \left(\frac{1+g}{1+r} \right)$.

$$\sum_{t=1}^{\infty}\beta^t = \frac{\beta}{1-\beta}, \text{ therefore } \sum_{t=1}^{\infty}\left(\frac{1+g}{1+r}\right)^t = \frac{\frac{(1+g)}{(1+r)}}{1-\frac{(1+g)}{(1+r)}}, \text{ and consequently } \hat{P}_0 = \frac{D_1}{1+g}\left[\frac{\left(\frac{1+g}{1+r}\right)}{1-\left(\frac{1+g}{1+r}\right)}\right].$$

Multiply the right side of the equation by $\frac{1+r}{1+r}$ to simplify the right-hand term.

$$\hat{P}_0 = \frac{D_1}{1+g}\left[\frac{\left(\frac{1+g}{1+r}\right)}{1-\left(\frac{1+g}{1+r}\right)}\right]\left(\frac{1+r}{1+r}\right) = \frac{D_1}{1+g}\left[\frac{1+g}{(1+r)-(1+g)}\right]$$

Simplify the denominator in right-hand term of the right side of the equation.

$$\hat{P}_0 = \frac{D_1}{1+g}\left(\frac{1+g}{r-g}\right) = \frac{D_1}{r-g} \quad \text{where } g < r.$$

APPENDIX B: PROOF FOR THE ENHANCED DIVIDEND DISCOUNT MODEL

Let EPS_1 = expected annual earnings per share one year in the future at the firm's normal growth rate [$EPS_1 = EPS_0 \cdot (1 + g)$];
Let p = the normal retention rate (applicable during the maturity phase of the life cycle);
Let $(1 – p)$ = the normal dividend payout rate (applicable during the maturity phase of the life cycle);
Let p_g = the temporary increase in the retention rate experienced during the growth phase;
Let r = the required return on common equity;
Let ROE = the firm's Return on Equity ratio;
Let g = the normal (constant) dividend growth rate;
Let g_n = the increase in the dividend growth rate during the nonconstant growth phase of the firm's life cycle, attributable to the increased retention rate ($g_n = ROE * p_g$);
Let DR = the reduction in the dividend attributable to the increased retention rate ($DR = EPS_1 * p_g$).

$$\hat{P}0 = \sum_{t=1}^{\infty} EPS_1(1-p-p_g)\frac{\left[1+ROE\cdot\left(p+p_g\right)\right]^{t-1}}{(1+r)^t}$$

$$= \sum_{t=1}^{\infty}\frac{EPS_1(1-p-p_g)}{\left[1+\left(ROE\cdot p\right)+\left(ROE\cdot p_g\right)\right]}\left[\frac{1+\left(ROE\cdot p\right)+\left(ROE\cdot p_g\right)}{1+r}\right]^t$$

The second term on the right is an infinite geometric series.

$$\text{Let } \beta = \left[\frac{1+\left(ROE\cdot p\right)+\left(ROE\cdot p_g\right)}{1+r}\right]; \quad \sum_{t=1}^{\infty}\beta^t = \left[\frac{\frac{1+\left(ROE\cdot p\right)+\left(ROE\cdot p_g\right)}{1+r}}{1-\frac{1+\left(ROE\cdot p\right)+\left(ROE\cdot p_g\right)}{1+r}}\right].$$

Plugging this back into the equation,

$$\hat{P}0 = \frac{EPS_1\left(1 - p - p_g\right)}{\left[1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)\right]}\left[\frac{\frac{1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)}{1 + r}}{1 - \frac{1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)}{1 + r}}\right]$$

$$= \frac{EPS_1\left(1 - p - p_g\right)}{\left[1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)\right]}\left[\frac{\frac{1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)}{1 + r}}{1 - \frac{1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)}{1 + r}}\right]\left(\frac{1 + r}{1 + r}\right)$$

$$= \frac{EPS_1\left(1 - p - p_g\right)}{\left[1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)\right]}\left[\frac{1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)}{1 + r - \left[1 + \left(ROE \cdot p\right) + \left(ROE \cdot p_g\right)\right]}\right]$$

$$= \frac{EPS_1\left(1 - p - p_g\right)}{\left[r - \left(ROE \cdot p\right) - \left(ROE \cdot p_g\right)\right]}$$

$$= \frac{EPS_1\left(1 - p\right) - EPS_1 \cdot p_g}{r - g - \left(ROE \cdot p_g\right)}$$

$$= \frac{D_1 - DR}{r - g - g_n}$$ where $g \geq 0$, $g_n \geq 0$, and $g + g_n < r$.

Solving this enhanced model for g_n reveals $g_n = r - g - \frac{D_1 - DR}{P_0}$.