Analyzing the Contagion Effect of Foreclosures as a Branching Process: A Close Look at the Years that Follow the Great Recession

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We lean on the phenomenon called the "foreclosure contagion effect," generate proxy measurements of contagious foreclosures, construct mathematical branching processes that depict the spread of these consequential defaults, and analyze the gradual progression of these foreclosures across the 366 metropolitan areas in the U.S. throughout the 15 quarters that follow the great recession. We find that the foreclosure epidemic was far from its conclusion by the end of 2013, although the U.S. housing crisis is broadly defined as lasting from 2007 to 2009. In this period, prime-loan (subprime-loan) associated contagious foreclosures had further worsened in about 52% (27%) of the metropolitan areas in the U.S.

INTRODUCTION

The great recession (associated with the U.S. financial crisis of 2007–2008 and the subprime mortgage crisis of 2007–2009) demonstrated that the real estate market plays a significant role in a nation's broader economy. The collapse of the U.S. housing market during these years has confirmed the influence of this economic sector on other modules of the economy and illuminated the systematic risk involved in it. This modern real estate meltdown had two distinct attributes: a significant reduction in housing prices and a widespread foreclosure epidemic. These two developments are tightly connected through a perpetual self-feeding mechanism often called the "foreclosure contagion effect." In this study we assess this well documented continual time-series feedback system, construct universal branching processes to depict the prospective spread of consequential foreclosures, and analyze the gradual progression of these contagious foreclosures across the 366 metropolitan areas in the U.S. throughout the four years that follow the great recession.

We assume that each foreclosed house has the potential to trigger other foreclosures in its neighborhood. By acquiring proxy measurements of contagious foreclosures and deploying branching process methodologies, we model the probable diffusions of these consequential foreclosures and their ergodic properties. In particular, we postulate the necessary derivations for the number of contagious foreclosures in a closed metropolitan area and within predefined time intervals. We also depict the prospective distributions of the total number of transmissible foreclosed properties under varying economic settings. We designate the likelihood for a complete elimination of foreclosed residences from a given urban region. In addition, we provide essential Bayesian estimates and confidence intervals for the model's parameters, and further derive the likely duration of a foreclosure epidemic given various velocities of progression.

In the subsequent empirical section we first collect genuine data of foreclosure rates across the 366 metropolitan areas in the U.S., as classified by the U.S. Office of Management and Budget (OMB), from the beginning of 2010 until the end of the third quarter of 2013. Our accessible database is divided into three subsamples: the inclusive mortgage market, the prime loans market, and the subprime loans market. This allows us to separately explore the trends of contagious foreclosures in these three groups. We then generate representative figures for the respective foreclosure rates that are associated with the foreclosure contagion effect and deploy the model's mathematical derivations over these proxy measurements.

Our empirical findings repeatedly indicate that the foreclosure epidemic of the great recession not only did not end, as many perceived, in 2009, but was still far from its conclusion even towards the end of 2013, with significant gradual increments in contagious foreclosure rates in many of the test areas in the years that followed this crisis. To authenticate this observation, we analyze a large set of distributional properties of these new consequential foreclosures.

This investigation is crucial to our understanding of how the recent housing downturn cycle continues to affect not only the real estate market but also the wider economy. In particular, our empirical findings convey valuable insights into themes that involve future city plans, new constructions, rentals, mortgage rates, lending banks and financial institutions, employment opportunities, government and local assistance and emergency programs, personal finance, imminent investments in schooling, roads, and other municipal services to communities.

Numerous academic studies, newspaper articles, biographies, and anthologies have been written thus far on the structural and financial failures that led to the toxic fallout throughout the recent U.S. foreclosure epidemic. Countless experts have stretched their testimonies in front of the U.S. Congress and other regulatory bodies to explain the root causes and the unprecedented expansion of this modern housing crisis.[1] Nonetheless, to the best of our knowledge, the literature has been relatively silent so far about the aftermath of the great recession.[2]

Our contributions in this study, therefore, reside along three dimensions. First, we present a complete set of mathematical procedures that together can monitor, almost in real-time, the ongoing evolution of contagious foreclosures in specified metropolitan areas. These combined policy tools are universal; they do not rely on any particular assumptions (besides accepting the "foreclosure contagion effect"), and essentially they can be activated by regulators under a broad class of economic circumstances. Second, we deploy the resulting statistical and distributional techniques over a comprehensive sample of foreclosure rates and reveal the lasting deterioration of the U.S. housing market, long after the period by which the great recession is commonly defined. And third, we also embed into our proposed framework a balanced policy discussion on the possible regulatory actions and their likely consequences.

The study proceeds as follows. In Section 2 we provide a literature review that substantiates the prevalence of the phenomenon called the "foreclosure contagion effect."[3] In Section 3 we present multiple channels of the mathematical theory of branching processes and explain how the modules of this framework can be applied to trace the expansion of virulent foreclosures. In Section 4 we conduct an empirical investigation where we deploy the various components of the theory over proxy records of contagious foreclosure rates across the 366 metropolitan areas within the U.S. from the first quarter of 2010 until the third quarter of 2013. In Section 5 we summarize our findings, stage a concise policy debate, and conclude.

RELEVANT LITERATURE

Numerous studies document the "foreclosure contagion effect" throughout the great recession of 2007–2009. This phenomenon entails both the reduction in housing prices because of neighboring foreclosures and the ignition of further foreclosures due to a devaluation of nearby properties. Immergluck and Smith (2006a, 2006b) were among the first scholars to recognize that foreclosures of conventional single-family houses (defined as one- to four-bedroom units) mount a significant pressure on nearby property values. In fact, their studies preceded the great recession and were based on data on foreclosures in the city of Chicago in 1997 and 1998 with property sales recorded in 2000. Schuetz, Been, and Ellen (2008) further explored the foreclosure contagion effect in the city of New York and emphasized the need for government intervention to remedy the consequential reduction in local tax bases. The latter authors found that the magnitude of the sale price discount increases with the number of foreclosed properties, yet not in a linear response curve.

Other studies provide arguments complementary to the foreclosure contagion effect. Demyanyk and Henert (2009) deploy a proportional odds model and show that the subsequent low house price appreciation for 2006 and 2007 loans was the key determinant for the modern foreclosure epidemic. Gerardi, Shapiro, and Willen (2009) utilize both purchase and sale records of residential mortgage and foreclosure transactions in Massachusetts from 1989 to 2008 and estimate the effects of changes in housing prices on residential foreclosures. The authors conclude that the recent foreclosure crisis took place not so much because of the relaxation of underwriting standards but rather in light of the severe decline in property values, which began at the end of 2005.

Another branch of the literature looks at the effects of physical damages to distressed properties, which decrease the value of nearby houses and feed the foreclosure contagion effect. Harding, Rosenblatt, and Yao (2009) describe the gross neglect, abandonment, and vandalism often associated with foreclosed properties. In addition, the authors pronounce the "excess supply" economic aspect of foreclosures, which naturally lowers property values. The authors report that these destructive elements have two dimensions, time and distance; they can stretch along 0.9 kilometer (roughly 10 blocks) and within 5 years from a liquidation event.

Lin, Rosenblatt, and Yao (2009) use data from Chicago, IL, explain that foreclosures often result in vandalism, disinvestment, and other negative spillover effects in respective neighborhoods, develop a theoretical model to analyze these spillover effects, and find that these harmful aspects depress neighborhood property values by as much as 8.7% per event. Rogers and Winter (2009) focus on the St. Louis real estate submarket and rationalize the increase in neighborhood crime rates as a result of foreclosures that leave properties vacant for extended periods of time, which also reduces adjacent house values and destabilizes neighborhoods. Interestingly, these authors detect that while the impact of foreclosures is both statistically and economically significant across communities, the marginal impact of foreclosures declines as the number of foreclosures increases.

Campbell, Giglio, and Pathak (2011) explain that foreclosed houses in Massachusetts are likely to sell at low prices, both because they may be physically damaged during the foreclosure process and because their respective original lenders aim to sell them quickly. These additional price reductions naturally affect the values of nearby houses as well. Rauterkus, Miller, Thrall, and Sklarz (2012) utilize distressed and non-distressed sales in Chicago, IL, to inspect Real Estate Owned (REO, a class of properties owned by the lenders) discounted sales. The authors observe that neighborhoods with relatively high foreclosure rates have a smaller gap between distressed and non-distressed sale prices, when compared to areas with low foreclosure rates.

Fisher, Lambie-Hanson, and Willen (2013) further study the "supply effect" and the "physical externality effect" of foreclosures on neighboring condominium properties in Boston, MA. The authors find evidence that investment externalities drive foreclosures' impacts on nearby condominium prices. In particular, adjacent foreclosures reduce the sale price of Boston condominiums by more than 6% compared to other properties in the same census tract that sell without any foreclosures nearby.

Gangel, Seiler, and Collins (2013a) examine the magnitude of the foreclosure contagion effect through a simulated agent-based modeling approach and discover that the time a foreclosed property is left to flounder on the market is the most detrimental factor to a market's stability. To further assist policymakers in identifying necessary-to-intervene submarkets, Gangel, Seiler, and Collins (2013b) use a Latin Hypercube Sampling technique and mathematically specify the foreclosure contagion threshold, i.e. the precise boundary that separates surviving real estate markets from those that crash. Towe and Lawley (2013) further use micro-data and identify highly localized foreclosures contagion effects in Maryland.

Another strand of research explores several human behavioral aspects of real estate equity deficiencies and foreclosure sales. Shefrin and Statman (1985) introduce the "disposition effect," describing homeowners reluctant to lose equity. Genesove and Mayer (1997, 2001) and Engelhardt (2003) find that real estate sellers often have a strong element of loss aversion. More recently, Ong, Neo, and Tu (2008) focus on the influence of price expectations, volatility and equity losses on foreclosure transactions. Using data from Singapore, the latter authors empirically show that differences in seller response to market expectations and equity losses exist across foreclosure and non-foreclosure sales, while past price movement is the most important determinant.[4]

The well documented phenomenon called the "foreclosure contagion effect" forms the basis of the following notional model. There are other studies that delve into this subject matter. Many of these articles scrutinize various legal aspects of the impact of foreclosures on the values of nearby properties as well as legal issues of the opposite causality effect. We shall refer to some in our later policy discussion.

THE MODEL OF BRANCHING PROCESS

The Underlying Theory

In this exploration we presume that new foreclosures may arise because of either a foreclosure contagion effect within the same metropolitan area or in light of macroeconomic factors such as changes in the local labor market. As observed during the recent subprime mortgage crisis in the U.S., a bank foreclosure process immediately adds more pressure on home values in the neighboring region. As a result of depressed housing prices, further foreclosures in that area could strike. This situation arises when existing mortgage loans carry higher debts than their corresponding home values hence when properties are "underwater" or in "negative equity." These circumstances, as recently witnessed, cause homeowners to simply walk away from their properties, even when borrowers are fully capable of paying their existing mortgage loans. In these "strategic default" cases, mortgagers lose any incentives to service their respective debts; they tend to abandon their houses as bank foreclosed properties, while these new foreclosures mount even more contagious pressure on home values in the respective metropolitan areas.

In many instances, however, foreclosures are instigated by sudden losses of income among mortgagers. A significant wave of layoffs not only prompts new bank foreclosures, but also causes higher unemployment rate in the respective region, which then creates more economic pressure on that metropolitan area in the forms of lower purchasing power by consumers and further deterioration in the regional economic output, thus more job losses and consequently additional distressed mortgagers and foreclosures.

Among others, Calomiris, Longhofer, and Miles (2013) use a dynamic panel vector autoregressive model (PVAR) and authenticate how foreclosures and home prices interact within a larger macroeconomic framework that includes U.S. state-level variables such as employment, construction permits, and housing sales for the period 1981–2009.[5] Rogers and Winkler (2013) reveal that in the majority of the U.S. metropolitan areas, domestic labor markets declined prior to the respective housing markets, somewhat contrary to the national pattern.[6] Rana and Shea (2015) use a local projection method and further validate the U.S. state-level relationship among unemployment, foreclosures, and housing prices during the great recession.

Altogether, in light of the foreclosure contagion effect, in the following investigation we shall assume that a bank foreclosure i generates $x_i(\tau) \in \{0, 1, 2, 3, \dots\}$ of new foreclosed properties during the successive time unit τ. We can further assume that $x_i(\tau)$ are independent and identically distributed (i.i.d.) random variables. While α denotes the initially recorded number of foreclosures within a given region, $\beta(\tau)$ accumulates the total number of new foreclosures created within time interval τ, and $\lambda = E[x]$ represents the average number of contagious foreclosed properties that arise within the same metropolitan area throughout the measured time units (in our later empirical section, a standard time unit is set to be a quarter of a year). These notations assist us in depicting the systemic risk of bank foreclosures as a set of spanning trees.

We can therefore model the potential spread of transmissible foreclosures in a specified metropolitan area with a homogeneous Bienaymé-Galton-Watson (BGW) branching process. In this setting, λ is a random number that controls both the length (the expected duration) and the depth (the scaled number of contagious foreclosed homes) of a foreclosure epidemic in a confined housing market. When $\lambda > 1$, a domestic foreclosure outbreak in any metropolitan area has a real potential to become a widespread national epidemic. Conversely, when $\lambda \leq 1$, the ergodic properties of the snowball effects described above dictate that the BGW branching process is interrupted with probability 1, regardless of any regulatory intervention.[7] Nevertheless, in practice, it is advisable to maintain λ well below unity. Depending on whether $\lambda < 1, \lambda = 1$, or $\lambda > 1$, in the relevant mathematics literature a branching process is called subcritical, critical, or supercritical, respectively.

To formalize this BGW branching process, we express the spread of foreclosures as:

$$\begin{cases} \beta(0) = \alpha \\ \beta(1) = x_1(0) + x_2(0) + \cdots + x_\alpha(0) \\ \beta(2) = x_1(1) + x_2(1) + \cdots + x_{\beta(1)}(1) \\ \cdot \\ \cdot \\ \cdot \\ \beta(j) = x_1(j-1) + x_2(j-1) + \cdots + x_{\beta(j-1)}(j-1) = \sum_{i=1}^{\beta(j-1)} x_i(j-1) \end{cases} \quad . \tag{1}$$

Since λ denotes the average number of new contagious foreclosed properties that arise within the same metropolitan area throughout the measured time units, we can observe that the foreclosure branching process starts from α cases. In the next time unit there will be, on average, $\alpha\lambda$ new foreclosures. In the subsequent time unit there will be $\alpha\lambda^2$ cases, which are then followed by $\alpha\lambda^3$ foreclosures, and so forth. All together a foreclosure epidemic is expected to yield the power series $\alpha + \alpha\lambda + \alpha\lambda^2 + \alpha\lambda^3 + \cdots$. Thus, for the purpose of model tractability we can now assume that x has a generalized power series distribution. This broad family of discrete distributions includes the Binomial, Poisson, Negative Binomial (in particular, Geometric), and Logarithmic series distributions, as well as the truncated forms of these disseminations, as special cases. Under these circumstances we obtain

$$P(x_i(\tau) = k) = \frac{a_k \theta^k}{A(\theta)}, \tag{2}$$

where $a_k \geq 0$, $A(\theta) = \sum_k a_k \theta^k$, $\theta > 0$ is a canonical (natural generator) parameter, and k is a non-negative integer. The mean of x is defined as

$$\lambda = E[x] = \frac{\theta A'(\theta)}{A(\theta)} = \frac{\theta dA(\theta)}{A(\theta) d\theta}. \tag{3}$$

In our context, however, the Binomial distribution could be an over-simplification of reality. This dissemination does not faithfully represent the potential spread of contagious foreclosures in light of its reliance on binary situations of either transmissions or no-transmissions of necessary conditions for bank foreclosures across nearby houses.[8] We therefore direct our attention to the Poisson and the Geometric distributions as representative disseminations of x, as described hereafter.[9]

In the case that x follows a Poisson distribution, $a_k = \frac{1}{k!}$, $A(\theta) = e^\theta$, $P(x_i(\tau) = k) = \frac{e^{-\theta}\theta^k}{k!}$, where $k \in \{0, 1, 2, 3, \ldots\}$, and $\lambda = \theta$. When x follows a Geometric distribution, $a_k = 1$, $A(\theta) = \frac{1}{1-\theta}$, $P(x_i(\tau) = k) = \theta^k(1-\theta)$, where $k \in \{0, 1, 2, 3, \ldots\}$, and $\lambda = \frac{\theta}{1-\theta}$.

The distribution of the possible magnitude of a foreclosure epidemic is of high interest to regulators and policymakers. We now denote γ as the final (total) recorded number of foreclosures within a specified area (along all feasible time units), thus by definition $\gamma = \sum_{\tau=0}^{\infty} \beta(\tau)$. Consequently, the distribution of γ is the r-th convolution of x hence

$$P(\gamma = r) = \frac{\alpha}{r} P(x_1 + x_2 + x_3 + \cdots x_k = r - \alpha), \tag{4}$$

where $r \in \{\alpha, \alpha+1, \alpha+2, \ldots\}$, and $x_1, x_2, x_3, \ldots, x_k$ are i.i.d random variables.

When x follows a Poisson distribution, i.e. when $P(x = k) = \frac{e^{-\lambda}\lambda^k}{k!}$, we can use the sum of i.i.d Poisson random variables with parameter λr and get:

$$P(x_1 + x_2 + x_3 + \cdots x_r = k) = \frac{e^{-\lambda r}(\lambda r)^k}{k!}. \tag{5}$$

Therefore, in this discrete domain, the Probability Mass Function (PMF) of the total recorded number of foreclosures γ becomes:

$$P(\gamma = r) = \frac{\alpha}{r} P(x_1 + x_2 + x_3 + \cdots x_k = r - \alpha) = \frac{\alpha}{r} \frac{e^{-\lambda r}(\lambda r)^{r-\alpha}}{(r-\alpha)!}, \tag{6}$$

where $r \in \{\alpha, \alpha+1, \alpha+2, \ldots\}$, i.e. γ has a Borel-Tanner distribution, as extended by Haight and Breuer (1960). When computation challenges arise in light of an excessive number of contagious foreclosed homes beyond the initial level, i.e. when $r - \alpha$ is quite large and both the power term in the numerator and the factorial term in the denominator of equation (6) become rather difficult to calculate, we can approximate this PMF by using Stirling's formula and obtain:

$$P(\gamma = r) \cong \frac{\alpha}{\sqrt{2\pi}} \lambda^{-\alpha} r^{-1.5} e^{-r[\lambda - \ln(\lambda) - 1]}, \ 1 \ll r - \alpha. \tag{7}$$

Approximation (7) is mostly useful to assess the right tail of the distribution of γ.

When x, however, follows a Geometric distribution, i.e. when $P(x = k) = \frac{\lambda^k}{(1+\lambda)^{k+1}}$, the sum of i.i.d Geometric random variables becomes a Negative Binomial distribution thus:[10]

$$P(x_1 + x_2 + x_3 + \cdots x_r = k) = \binom{r+k-1}{k} \frac{\lambda^k}{(1+\lambda)^k} \frac{1}{(1+\lambda)^r}. \tag{8}$$

In this case, the PMF of the final recorded number of foreclosures γ is:

$$P(\gamma = r) = \frac{\alpha}{r} P(x_1 + x_2 + x_3 + \cdots x_k = r - \alpha) = \frac{\alpha}{r} \binom{r + r - \alpha - 1}{r - \alpha} \frac{\lambda^{r-\alpha}}{(1+\lambda)^{r-\alpha}} \frac{1}{(1+\lambda)^r}$$

$$= \frac{\alpha}{r} \binom{2r - \alpha - 1}{r - \alpha} \frac{\lambda^{r-\alpha}}{(1+\lambda)^{2r-\alpha}} = \frac{\alpha}{r} \frac{(2r-\alpha-1)!}{(r-\alpha)!(r-1)!} \frac{\lambda^{r-\alpha}}{(1+\lambda)^{2r-\alpha}} = \frac{\alpha}{2r-\alpha} \binom{2r-\alpha}{r-\alpha} \frac{\lambda^{r-\alpha}}{(1+\lambda)^{2r-\alpha}}. \tag{9}$$

where $r \in \{\alpha, \alpha+1, \alpha+2, \ldots\}$, i.e. the total observed number of foreclosures γ has a distribution of Haight (1961), while the additional number of foreclosed homes beyond the initial level $(\gamma - \alpha)$, i.e. the marginal impact of a domestic housing crisis, has a Lagrangian Generalized Negative Binomial distribution, as first named by Jain and Consul (1971).

In fact, we can designate the likelihood for a complete elimination of foreclosed properties from a given metropolitan area when the branching process starts from a single foreclosure as $\xi = P(\beta(\tau) = 0, \text{for some } \tau \geq 1 \mid \beta(0) = 1)$. Then for $\lambda < 1$, $\xi = 1$, and for $\lambda > 1$, $\xi < 1$. When the branching process, however, starts from α infectious foreclosed properties (this is a more "natural" economic setting since there are almost always subprime mortgage delinquencies and foreclosure candidates, even in the most stable rural areas), then $P(\beta(\tau) = 0, \text{for some } \tau \geq 1 \mid \beta(0) = \alpha) = \xi^\alpha$.

In light of our prior introduction of the generalized power series distributions, since ξ is indeed a function of λ, we can also write it as $\xi(\lambda)$ and realize that a BGW branching process that starts from a single transmissible foreclosure will be completely eliminated with $\beta(\tau) = 0$ for some $\tau \geq 1$ with a probability equals to the smallest root of $A(\xi\theta) = \xi A(\theta)$. For instance, for the Geometric distribution $\xi(\lambda) = Min\left\{\frac{1}{\lambda}, 1\right\}$, thus for $\lambda > 1$, $\xi(\lambda) < 1$.

We should further note here that any metropolitan area under investigation must be large enough to sustain the snowball effects of contagious foreclosures. If a local housing submarket is too small, the contagion effect will not have sufficient time to develop before the supply of potential foreclosures runs out, regardless of any regulatory intervention. Moreover, these miniature sporadic markets convey little economic importance to regulators and policymakers. We therefore disregard isolated and narrow rural areas and consider in our later empirical section only sizable metropolitan regions within the U.S. housing market.

Bayesian Estimation of λ

The mathematical literature provides several methods to infer λ in a BGW branching process (in our context, λ denotes the expected number of contagious foreclosed properties within the same metropolitan area in the subsequent time units). If we let $\tilde{\gamma}_\tau$ signify the total number of foreclosures up to and including time unit τ, accumulated from the initially recorded α foreclosures, then a Maximum Likelihood Estimator (MLE) of λ given observation of the branching process up to some pre-determined time unit $\tau^* \geq 1$ yields:

$$\hat{\lambda} = \frac{\tilde{\gamma}_{\tau^*} - \alpha}{\tilde{\gamma}_{\tau^*-1}}. \tag{10}$$

Yanev (1975) shows that this MLE is consistent and asymptotically unbiased in the limit where α is very large. Since most metropolitan areas in the U.S. normally exhibit a large number of foreclosures, regardless of any housing crisis, this MLE can serve us well. Somewhat anticipated, when the arrival of new contagious foreclosures follows a Poisson distribution, for example, and when the historically recorded $\lambda = 1$, i.e. when the branching process thus far is classified as critical, then the first two moments of $\hat{\lambda}$ are:

$$E\left[\hat{\lambda}\right] = \frac{\alpha}{\alpha+1}, \quad \sigma^2_{\hat{\lambda}} = \frac{2\alpha}{(\alpha+1)^2(\alpha+2)}, \tag{11}$$

which suggests that for a large enough initial number of foreclosures α, the BGW foreclosure branching process should remain active for an extended period of time as $E\left[\hat{\lambda}\right] \to 1$ and $\sigma^2_{\hat{\lambda}} \to 0$. On the other hand, for a relatively modest α, the foreclosure branching process would gradually degenerate since $E\left[\hat{\lambda}\right] < 1$, although having a relatively high variance. In the latter case the forward-looking projection of the parameter λ is sufficiently below unity, which prevents a "house of cards" phenomenon in light of mode zero among the arrivals of new foreclosures in the subsequent time units hence $Mode\{x_i(\tau)\} = 0$.

A Confidence Interval for λ

We can also deploy an alternative method to estimate λ by using an upper 95% profile confidence interval, for instance, while further utilizing Monte Carlo simulations as described hereafter. This approach allows policymakers to examine the posterior probability that $\lambda > 1$; depending on this posterior likelihood, a regulatory intervention could be considered. In fact, to use this alternative estimation method we are only required to observe the initial number of foreclosures α and the total (accumulated along the time) number of foreclosed properties γ in a given region. Within this framework, the likelihood function for γ given λ takes the form:

$$\mathcal{L}(\gamma \mid \lambda) = P(\Gamma = \gamma; \alpha, \lambda), \tag{12}$$

which corresponds to either the Borel-Tanner PMF in equation (6), in the case where the appearance of new foreclosures follows a Poisson distribution, or the Haight PMF in equation (9), in the case where the arrival of new foreclosures tracks a Geometric distribution.

We may denote $\pi(\lambda)$ as the *prior distribution* of λ and simulate it through two representative disseminations: First, a Uniform distribution within the closed interval $[0.92, 1.12]$, i.e. $\sim U[0.92, 1.12]$, and second, a Normal distribution having a mean of 1.02 and standard deviation of 0.1, i.e. $\sim N(\mu = 1.02, \sigma = 0.1)$. Both of these prior distributions have means and medians equal to 1.02, as persistently recorded in our later empirical section across all subsamples.[11] The Uniform prior distribution maintains all simulated data within a relatively narrow range as identified by most actual observations. The Normal distribution, however, is expected to generate less compact dissemination of simulated random numbers than the Uniform distribution, thus to allow for sporadic outliers beyond the closed interval $[0.92, 1.12]$. These disseminations can be generated through Monte Carlo computerized simulations by most statistical packages.

We can now use the Bayes' formula and construct the *posterior distribution* of λ given γ up to a normalizing constant $\omega := \left[\int_0^\infty \mathcal{L}(\gamma \mid \lambda)\pi(\lambda)d\lambda\right]^{-1}$ (while it is not necessary to actually compute it as demonstrated in our later empirical section) as:

$$f(\lambda \mid \gamma) = \frac{\mathcal{L}(\gamma \mid \lambda)\pi(\lambda)}{\int_0^\infty \mathcal{L}(\gamma \mid \lambda)\pi(\lambda)d\lambda} = \omega\mathcal{L}(\gamma \mid \lambda)\pi(\lambda). \tag{13}$$

By minimizing the squared of errors, the mean of the *posterior distribution* yields a robust point estimate of λ as:

$$\hat{\lambda} = E[\lambda \mid \gamma] = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \dots + \lambda_N}{N}, \tag{14}$$

after simulating large enough N estimates $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots + \lambda_N \sim f(\lambda \mid \gamma)$. At this stage of the analysis we can select a posterior confidence interval $[\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$ for $\hat{\lambda}$ according to identified regulatory needs with a specific $100(1-\vartheta)\%$ upper profile. In this general setting we require that $P(\lambda \in [\tilde{\lambda}_{low}, \tilde{\lambda}_{high}] \mid \gamma) = 1 - \vartheta$ for a fixed $\vartheta \in (0, 1)$, and if $\lambda_1 \in [\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$ but $\lambda_2 \notin [\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$, then $f(\lambda_1 \mid \gamma) > f(\lambda_2 \mid \gamma)$.

The Duration of a Foreclosure Epidemic

A duration analysis of a foreclosure epidemic assesses the distribution of the number δ of time units (quarters of a year in our context) until a complete elimination of contagious foreclosed properties from a given metropolitan area.[12] When the BGW branching process starts from a single foreclosure and when $\lambda \leq 1$, i.e. when the branching process is classified as either subcritical or critical, Farrington and Grant (1999) show that the duration distribution function $f_k = P(\delta \leq k)$ satisfies the following recursive relation:

$$f_0 = \varphi(0), \;\; f_{k+1} = \varphi(f_k), \;\; k = 0, 1, 2, 3, \dots, \tag{15}$$

where $\varphi(x) := \frac{A(x\theta)}{A(\theta)}$ is the Probability Generating Function (PGF) of the new foreclosures in the successive time units. Nevertheless, when the BGW branching process starts from α foreclosed homes, this distribution function evolves to $f_{k,\alpha} = (f_k)^\alpha = P(\delta \leq k \mid \alpha)$. In the case where the arrival of new foreclosures follows a Poisson distribution with mean $\lambda \leq 1$ the duration distribution function is:

$$f_k(\sim Poisson, \lambda \leq 1) = e^{-\lambda}E_k\left(e^{-\lambda e^{-\lambda}}\right), \tag{16}$$

where $\mathcal{C} := e^{-\lambda e^{-\lambda}}$ and $E_k(\mathcal{C})$ satisfies the recursive relation

$$E_0(\mathcal{C}) = 1,\;\; E_{k+1}(\mathcal{C}) = \mathcal{C}^{E_k(\mathcal{C})} = \mathcal{C}^{\mathcal{C}^{\cdots^{\mathcal{C}}}}. \tag{17}$$

When the appearance of new contagious foreclosures in a given metropolitan area follows a Geometric distribution with mean $\lambda < 1$ the duration distribution function is:

$$f_k(\sim Geometric, \lambda < 1) = \frac{1-\lambda^{k+1}}{1-\lambda^{k+2}},\;\; k = 0, 1, 2, 3, \ldots, \tag{18}$$

and when $\lambda = 1$ the duration distribution function is attained by using L'Hôpital's rule as:

$$f_k(\sim Geometric, \lambda = 1) = \lim_{\lambda \to 1} \frac{1-\lambda^{k+1}}{1-\lambda^{k+2}} = \lim_{\lambda \to 1} \frac{\partial(1-\lambda^{k+1})/\partial\lambda}{\partial(1-\lambda^{k+2})/\partial\lambda} = \lim_{\lambda \to 1} \frac{-(k+1)\lambda^k}{-(k+2)\lambda^{k+1}} = \frac{k+1}{k+2}. \tag{19}$$

In this case, the generating probabilities are:

$$P_k(\sim Geometric) = \frac{\lambda^k(1-\lambda)^2}{(1-\lambda^{k+1})(1-\lambda^{k+2})}. \tag{20}$$

In the next section we shall utilize the above derivations and examine both the length (the expected duration) and the depth (the scaled number of infectious foreclosed homes) during the aftermath of the foreclosure epidemic that took place in the U.S. from 2007 to 2009.

EMPIRICAL INVESTIGATION

Data Collection

According to the U.S. National Bureau of Economic Research (NBER) the U.S. great recession began in December 2007 and ended in June 2009. This period was closely synchronized with the U.S. subprime mortgage crisis, which is commonly defined as the period of 2007–2009. During these years a major foreclosure epidemic hit numerous communities within the U.S., while only sporadic areas were able to avert this housing calamity.

In this section we are interested in exploring the consequences of this foreclosure epidemic. We therefore collect data on local area unemployment rates from the Bureau of Labor Statistics (BLS). We assemble additional data on new foreclosure rates across 366 Metropolitan Statistical Areas (including Micropolitan Statistical Areas, Combined Statistical Areas, and New England City and Town Areas), as classified by and the U.S. OMB for the general use of Federal statistical activities. We shall refer to these regions as the 366 Statistical Areas (SA) hereafter.

We cluster the new foreclosure records including the specific names of the areas, unique metro identification numbers, and flags that indicate whether these areas are part of the 100 largest U.S. metros from Local Initiatives Support Corporation (LISC), a partnership between the Foreclosure-Response and the MetroTrends organizations. These new foreclosure rates were collected, assessed, and adjusted to various economic and demographic parameters with the help of the Mortgage Bankers Association (MBA) and LPS Applied Analytics, formerly known as McDash Analytics, a vendor of loan performance data from the nation's largest loan servicers.[13]

Quarterly records of local area unemployment rates and new foreclosure rates (with accuracy of one hundredth of a percentage) are available for us from March 2010 until September 2013 with observations every March, June, September, and December, i.e. at the end of each quarter.[14] All 366 SA contain 15 time-series data points, and they are further categorized into three groups: the inclusive rates of foreclosures, the rates of foreclosures among prime mortgages, and the rates of foreclosures among subprime mortgages.

To isolate the foreclosures that are associated with the contagion effect, i.e. to remove the likely influence of the local area unemployment rates, we form $366 \times 3 = 1{,}098$ chain reactions along their respective 15 time-series data points of transmissible foreclosure rates by running the following regressions (all standard assumptions of cross-section-over-time regressions apply):

$$Y_t = \alpha + \beta Y_{t-1} + \gamma X_{t-1} + \delta(Y_{t-1}X_{t-1}) + \varepsilon_t, \tag{21}$$

where Y_t denotes the foreclosure rate at time t, X_{t-1} represents the local area unemployment rate at the previous time t-1, and $(Y_{t-1}X_{t-1})$ is a lagged interaction term between the unemployment and the foreclosure rates. Essentially, the term βY_{t-1} serves here as a proxy for the contagion effect, because it measures how prior foreclosures affect later foreclosures in the same SA.[15]

We verify that these regression models are statistically significant (they have sizeable F values, R^2, adjustable R^2, and highly robust coefficients). We also refute multicollinearity by examining their variance inflation factors (all well under 10). These 1,098 chain reactions now contain proxy measurements of contagious foreclosures in their corresponding SA. We provide summary statistics on the three subsamples of these contagious foreclosure rates in **Table 1.**

TABLE 1
SUMMARY STATISTICS FOR THE THREE FINAL SUBSAMPLES (INCLUSIVE, PRIME, AND SUBPRIME LOAN MORTGAGES)

Contagious Foreclosure Rates in the Inclusive Mortgages Subsample (Over 366 Statistical Areas in the U.S.)															
	Q1/10	*Q2/10*	*Q3/10*	*Q4/10*	*Q1/11*	*Q2/11*	*Q3/11*	*Q4/11*	*Q1/12*	*Q2/12*	*Q3/12*	*Q4/12*	*Q1/13*	*Q2/13*	*Q3/13*
Minimum	1.00%	1.09%	0.98%	1.00%	1.03%	0.99%	1.19%	1.27%	1.16%	1.25%	1.14%	0.94%	0.76%	0.77%	0.63%
Maximum	17.83%	19.50%	18.85%	18.79%	19.34%	18.94%	19.71%	20.98%	19.74%	20.43%	17.57%	16.72%	15.51%	16.15%	12.64%
Mean	4.36%	4.76%	4.64%	4.96%	5.26%	5.15%	5.31%	5.68%	5.44%	5.80%	5.12%	4.93%	4.51%	4.41%	3.56%
Median	3.76%	4.11%	4.02%	4.32%	4.64%	4.54%	4.66%	5.03%	4.80%	5.22%	4.57%	4.40%	3.93%	3.82%	3.10%
Mode	2.94%	3.22%	2.73%	4.34%	3.60%	5.37%	3.93%	6.23%	4.11%	5.73%	3.47%	2.91%	4.48%	5.46%	2.86%
St. Dev.	2.49%	2.73%	2.63%	2.71%	2.84%	2.75%	2.88%	3.08%	2.93%	3.08%	2.65%	2.62%	2.46%	2.40%	1.86%
Contagious Foreclosure Rates in the Prime Mortgages Subsample (Over 366 Statistical Areas in the U.S.)															
	Q1/10	*Q2/10*	*Q3/10*	*Q4/10*	*Q1/11*	*Q2/11*	*Q3/11*	*Q4/11*	*Q1/12*	*Q2/12*	*Q3/12*	*Q4/12*	*Q1/13*	*Q2/13*	*Q3/13*
Minimum	0.61%	0.64%	0.56%	0.60%	0.60%	0.56%	0.61%	0.66%	0.59%	0.63%	0.66%	0.54%	0.43%	0.38%	0.36%
Maximum	13.26%	13.93%	14.66%	15.16%	14.53%	14.82%	14.56%	16.02%	15.58%	15.14%	13.61%	12.25%	10.66%	10.07%	7.67%
Mean	3.01%	3.16%	3.30%	3.63%	3.61%	3.61%	3.51%	3.85%	3.84%	3.90%	3.61%	3.31%	2.97%	2.90%	2.29%
Median	2.52%	2.65%	2.76%	3.10%	3.07%	3.12%	3.02%	3.33%	3.32%	3.42%	3.18%	2.91%	2.60%	2.48%	1.99%
Mode	1.78%	1.87%	2.54%	2.48%	2.28%	3.23%	2.88%	2.29%	3.28%	2.96%	3.05%	3.43%	2.27%	2.38%	1.49%
St. Dev.	1.92%	2.02%	2.12%	2.20%	2.14%	2.15%	2.12%	2.33%	2.26%	2.26%	2.02%	1.88%	1.70%	1.65%	1.23%
Contagious Foreclosure Rates in the Subprime Mortgages Subsample (Over 366 Statistical Areas in the U.S.)															
	Q1/10	*Q2/10*	*Q3/10*	*Q4/10*	*Q1/11*	*Q2/11*	*Q3/11*	*Q4/11*	*Q1/12*	*Q2/12*	*Q3/12*	*Q4/12*	*Q1/13*	*Q2/13*	*Q3/13*
Minimum	4.73%	4.71%	6.40%	6.58%	8.00%	6.36%	6.64%	8.24%	5.89%	7.28%	6.81%	5.38%	5.49%	4.81%	5.01%
Maximum	40.59%	40.45%	39.00%	41.02%	50.08%	36.20%	41.59%	48.02%	39.56%	46.29%	37.25%	44.47%	43.14%	42.40%	37.13%
Mean	17.39%	17.33%	16.91%	17.78%	22.08%	17.67%	19.89%	23.28%	18.80%	22.37%	19.49%	20.36%	18.28%	17.46%	15.94%
Median	16.64%	16.58%	16.05%	16.92%	20.90%	16.50%	18.64%	22.10%	17.64%	21.29%	18.65%	19.58%	17.63%	16.65%	15.20%
Mode	14.29%	14.24%	13.89%	17.43%	19.72%	20.52%	14.21%	23.40%	17.25%	26.08%	18.63%	14.13%	25.41%	13.81%	12.88%
St. Dev.	5.76%	5.74%	5.48%	5.58%	6.68%	5.33%	6.05%	7.06%	5.94%	6.78%	5.73%	6.37%	6.15%	6.05%	5.46%

The table provides descriptive statistics on the contagious foreclosure rates across the 366 U.S. statistical areas under investigation (as defined by the U.S. Office of Management and Budget) within the three subsamples (inclusive market, prime mortgages market, and subprime mortgages market) and along the 15 time-series data points (from the first quarter of 2010 until the third quarter of 2013) available to us.

Methodology and General Analysis

To enhance intuition and to allow computations of factorials in the model's derivations, the computed rates of contagious foreclosures across the 366 SA in the U.S. are uniformly multiplied by a scaling factor of 100. This simple transformation introduces scaled numbers of contagious foreclosed properties. This scaling procedure, however, does not affect the Bayesian estimation of λ, which is insensitive to different scaling factors.[16] These scaled figures serve us well in the following analyses since some SA naturally incorporate many more houses than others, and we cannot contrast the aftermath of a foreclosure epidemic across disproportionate regions through absolute quantities.

Since our three subsamples (for the inclusive, prime, and subprime mortgage markets) contain 14 data points of contagious foreclosure rates for each of the 366 SA, we can now assess the progress of $\hat{\lambda}$ using the Bayesian estimation in equation (10) along the 13 accessible time intervals; thus, each subsample reads $366 \times 13 = 4{,}758$ estimates of $\hat{\lambda}$. Within the subsample of inclusive mortgages we detect that $\hat{\lambda}$ ranges between 0.9437 and 1.1626. Within the subsample of prime loan mortgages we find that $\hat{\lambda}$ ranges between 0.9225 and 1.2041. Within the subsample of subprime loan mortgages we discover that $\hat{\lambda}$ ranges between 0.8568 and 1.2058. These arrays help us to set in our later simulations the likely confidence intervals of $\hat{\lambda}$.

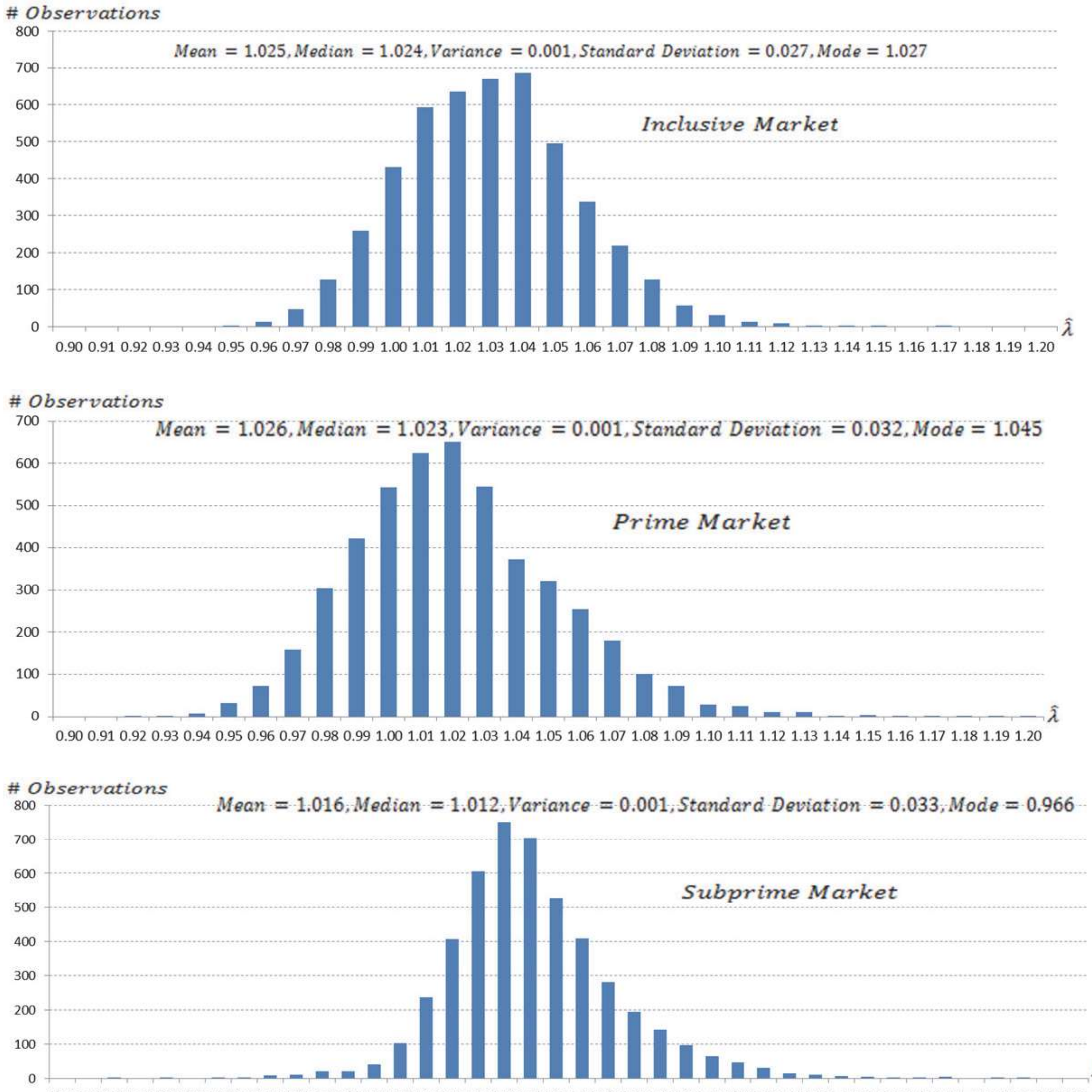
We also count how many of the 366 SA can be clearly classified as subcritical (arbitrarily defined as having their $E[\hat{\lambda}] + \sigma_{\hat{\lambda}} < 1$) or supercritical (arbitrarily defined as having their $E[\hat{\lambda}] - \sigma_{\hat{\lambda}} > 1$), while all the others are labeled as generally critical (with alternating $\hat{\lambda} \cong 1$). Within the subsample of inclusive mortgages we detect only 1 (0.2%) subcritical SA, 133 (36.4%) critical SA, and 232 (63.4%) supercritical SA. Within the subsample of prime loan mortgages we find only 2 (0.5%) subcritical SA, 173 (47.3%) critical SA, and 191 (52.2%) supercritical SA. Within the subsample of subprime loan mortgages we discover 9 (2.4%) subcritical SA, 259 (70.8%) critical SA, and 98 (26.8%) supercritical SA.

Thus far, the findings are rather striking. Although the U.S. core housing crisis is widely agreed to be confined to the years of of 2007–2009, towards the end of 2013 this foreclosure epidemic was still far from its conclusion. Across all three subsamples a clear majority of SA in the U.S. exhibited either critical or supercritical branching patterns, and only a tiny minority of the 366 SA in the U.S. conveyed distinct subcritical behavior in their scaled contagious foreclosures from the first quarter of 2010 until the end of the third quarter of 2013.

At this phase, we need to examine the distributional properties of x and infer when to deploy the Poisson distribution and when to use the Geometric distribution for modeling contagious foreclosures. In **Figure 1** we present the respective histograms of $\hat{\lambda}$ in all three subsamples along with their corresponding statistical merits. It is evident that $\hat{\lambda}$ is approximately Normally distributed with most estimates closely surrounding 1.02. The vast majority of estimates across all three subsamples appear within the closed interval $[0.90, 1.20]$. These statistical properties motivate our selections for the Uniform and Normal distributions underlying the point estimates and the confidence intervals hereafter.

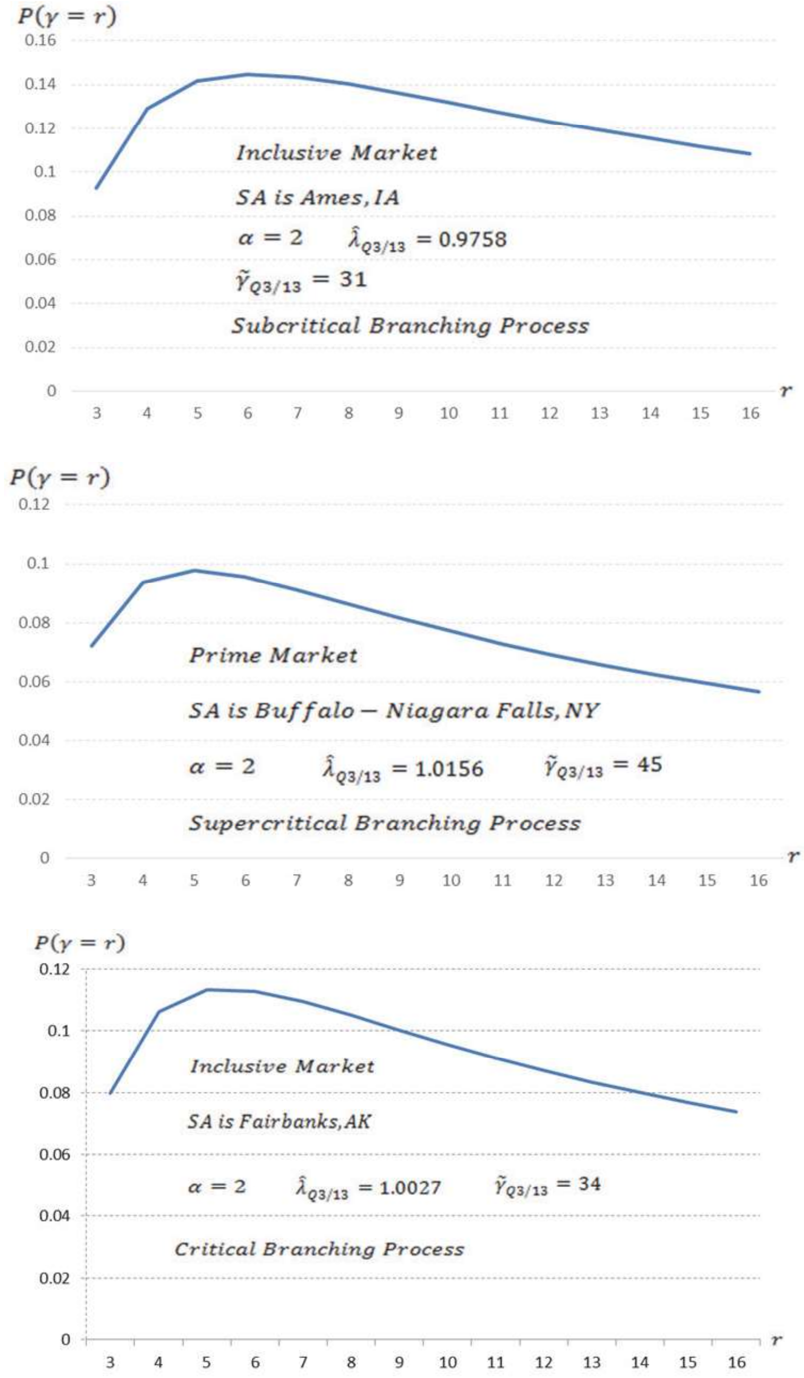
In this environment, both the Poisson and the Geometric distributions for modeling x are noticeably dominated by the parameter k, since when $\hat{\lambda} \cong 1$, $P_{Poisson}(x = k) = \frac{e^{-\lambda}\lambda^k}{k!} \cong \frac{1}{ek!}$ and $P_{Geometric}(x = k) = \frac{\lambda^k}{(1+\lambda)^{k+1}} \cong \frac{1}{2^{k+1}}$. These approximations reveal higher probabilities for $k = 0$ yet lower likelihoods for all $k \geq 1$ within the Geometric dissemination when compared to the Poisson distribution. Therefore, the Geometric distribution for modeling x is valid only in the extreme subcritical branching processes, i.e. when $\hat{\lambda} \ll 1$ and when x exhibits substantial chances to diminish. The Poisson distribution, on the other hand, would be a preferable method for modeling x in all critical and supercritical branching processes and in moderate subcritical branching processes.

FIGURE 1
DISTRIBUTIONS OF $\hat{\lambda}$ IN THE THREE SUBSAMPLES (INCLUSIVE, PRIME, SUBPRIME)



In **Figure 2** we present three examples for the computations of the left tails of the Borel-Tanner distribution as expressed in equation (6) for three different SA. These cases are selected to illustrate how the distributions for the total number of foreclosures γ may evolve under moderate subcritical, critical, and supercritical conditions, while all three SA have $\alpha = 2$ initial scaled number of foreclosed properties as recorded in the second quarter of 2010, and λ is nominated here merely based on the latest Bayesian estimate in the third quarter of 2013.

FIGURE 2
THREE ECXAMPLES FOR THE LEFT TAILS OF THE BOREL-TANNER DISTRIBUTION



Regrettably, our subsamples are truncated at the end of the third quarter of 2013. Thus, although we do not possess ultimate records of γ, we can still compare these tails of distributions with the ad hoc actual readings of the aggregate numbers of foreclosures in the respective SA. Within the inclusive market and the SA of Ames, IA, which exhibits a subcritical branching process with $\hat{\lambda}_{Q3/13} = 0.9758$, we observe $\tilde{\gamma}_{Q3/13} = 31$. Within the prime loans market and the SA of Buffalo–Niagara Falls, NY, which displays a moderate supercritical branching process with $\hat{\lambda}_{Q3/13} = 1.0156$, we perceive $\tilde{\gamma}_{Q3/13} = 45$.

Within the inclusive market and the SA of Fairbanks, AK, which reveals a critical branching process with $\hat{\lambda}_{Q3/13} = 1.0027$, we detect $\tilde{\gamma}_{Q3/13} = 34$. We can recognize that the above tails of the Borel-Tanner distributions of γ perfectly match the temporary readings of $\tilde{\gamma}_{Q3/13}$, where the subcritical process indeed attains the lowest reading of actual foreclosures, the critical process in fact presents an intermediate volume of foreclosures, and the moderate supercritical process truly reaches the highest number of contagious foreclosures by the third quarter of 2013.

In **Figure 3** we offer three examples for the calculations of the left tails of the Haight distribution as captured in equation (9). All three cases are collected from SA in Florida (one of the very few states that have experienced a significant recovery in their real estate submarkets). All three instances show robust subcritical branching processes, yet they are originated with different initial scaled number of foreclosed properties as recorded in the second quarter of 2010.

The SA of Naples–Marco Island, FL, starts with $\alpha = 40$ contagious foreclosures in the second quarter of 2010, $\hat{\lambda}_{Q3/13} = 0.9735$, and $\tilde{\gamma}_{Q3/13} = 513$. The SA of Cape Coral–Fort Myers, FL, begins with $\alpha = 18$ contagious foreclosures in the second quarter of 2010, $\hat{\lambda}_{Q3/13} = 0.9591$, and $\tilde{\gamma}_{Q3/13} = 199$. The SA of Punta Gorda, FL, commences with $\alpha = 39$ contagious foreclosures in the second quarter of 2010, $\hat{\lambda}_{Q3/13} = 0.9616$, and $\tilde{\gamma}_{Q3/13} = 448$. Evidently, the subcritical process of Cape Coral–Fort Myers, FL, originates with the lowest number of foreclosed properties, and in addition it has the fastest rate of decay (hence the lowest $\hat{\lambda}_{Q3/13}$ within this group), therefore the tail of its distribution starts to bend downward after only a few increments of r. Naturally, this SA realizes the lowest actual reading for $\tilde{\gamma}_{Q3/13}$, when compared to the other two SA within this group.

In **Figure 4** we provide three examples for the computations of the right tails of the Stirling's approximation to the Borel-Tanner distribution as stated in equation (7). As already stated, these are valid approximations when $r - \alpha$ is large enough, therefore we take all three cases from the subprime mortgages market where the initial scaled numbers of foreclosures α are relatively high, and therefore the branching processes have higher chances to progress to higher total volumes γ. As expected, all three right tails exhibit downward sloping convex curves.

In **Figure 5** we demonstrate the general behavior of the forward-looking first moments of $\hat{\lambda}$ along with the respective second moments as postulated in equation (11) with a cluster of only critical branching processes having $0.999 \leq \hat{\lambda}_{Q3/13} \leq 1.001$ selected from all three subsamples. For higher accuracy in these and the following tests we nominate the initial scaled numbers of contagious foreclosures α with two digits after the decimal points (as originally noted in our database). As observed across the three subsamples, for relatively low levels of α the projected means of $\hat{\lambda}$ are well below unity, hence the contagious foreclosure branching processes are expected to gradually subside although having relatively higher variances. Conversely, for somewhat modest and high levels of α the projected means of $\hat{\lambda}$ are converging to unity in addition to having lower variances, thus the contagious foreclosure branching processes are expected to persist for prolonged periods of time.

FIGURE 3
THREE EXAMPLES FOR THE LEFT TAILS OF THE HAIGHT DISTRIBUTION

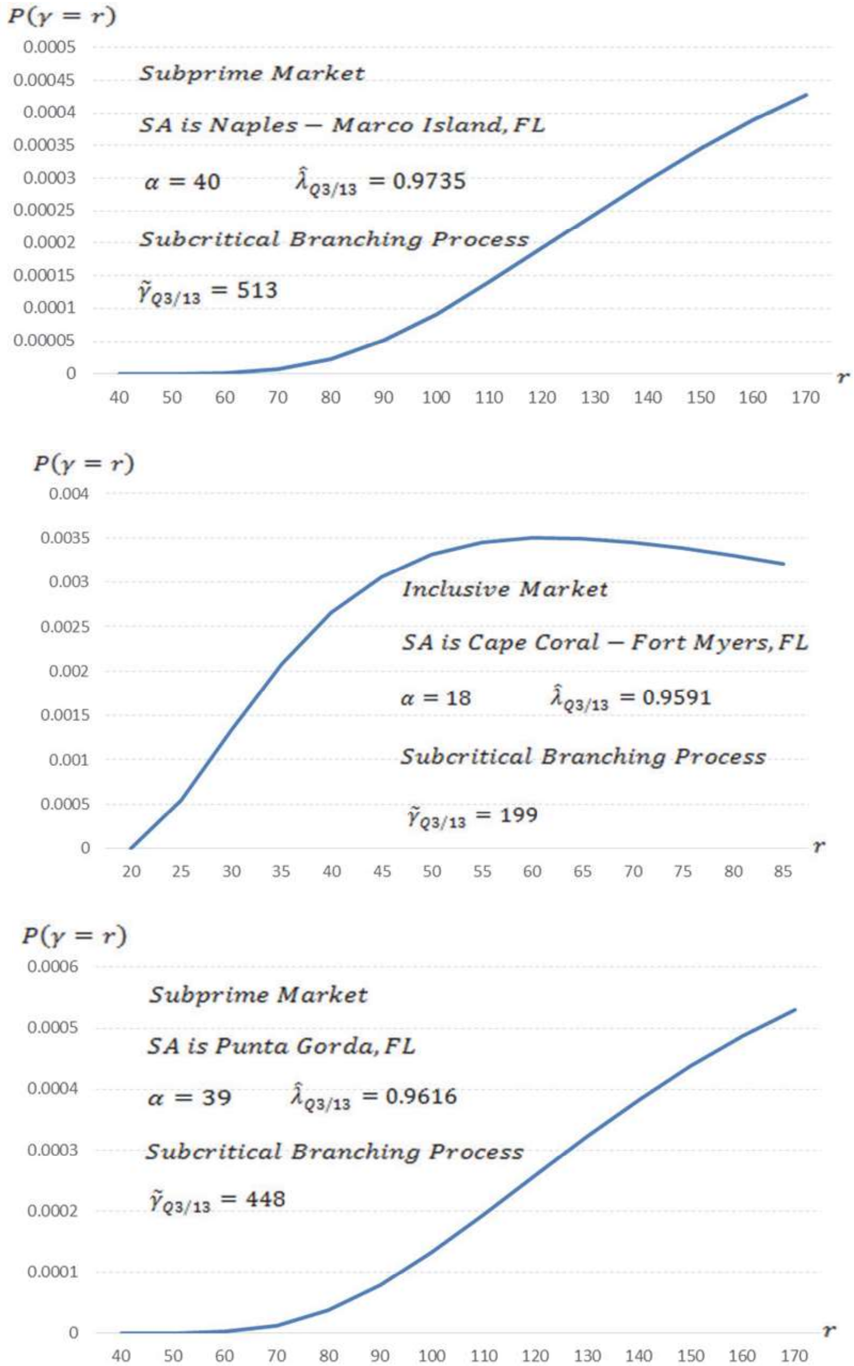


FIGURE 4
THREE EXAMPLES FOR THE RIGHT TAILS OF THE STIRLING'S APPROXIMATION

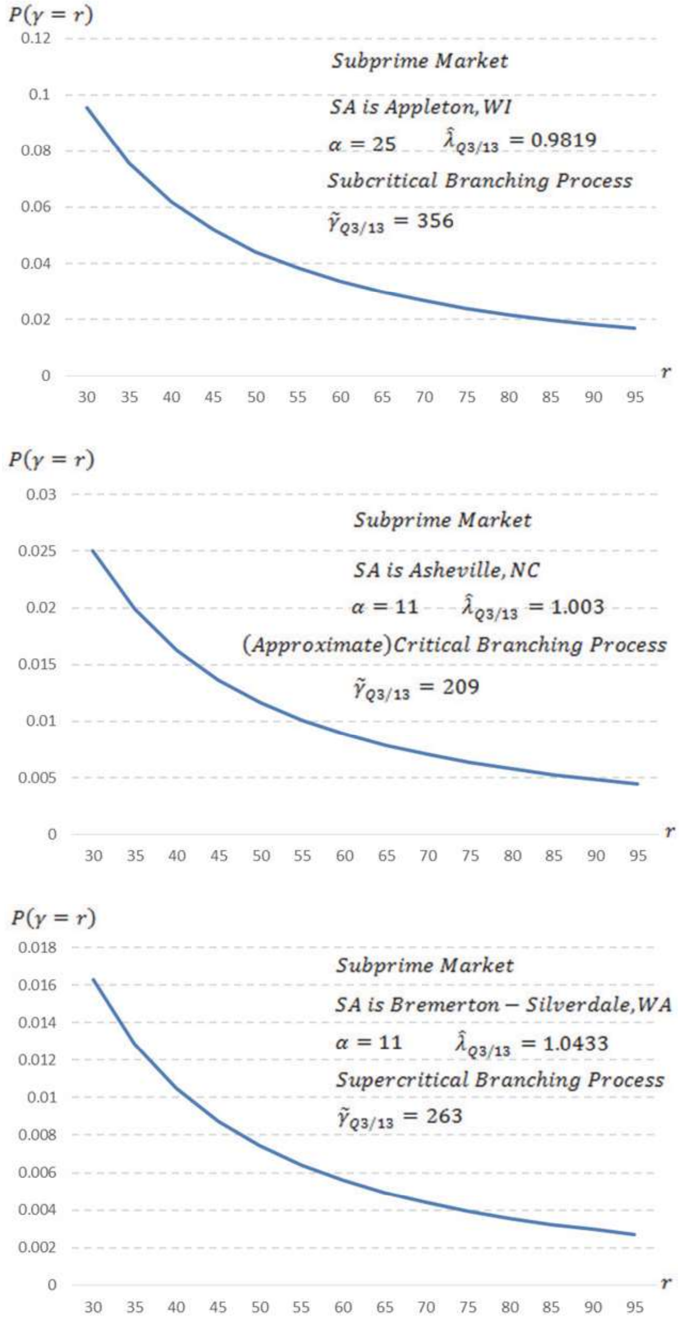
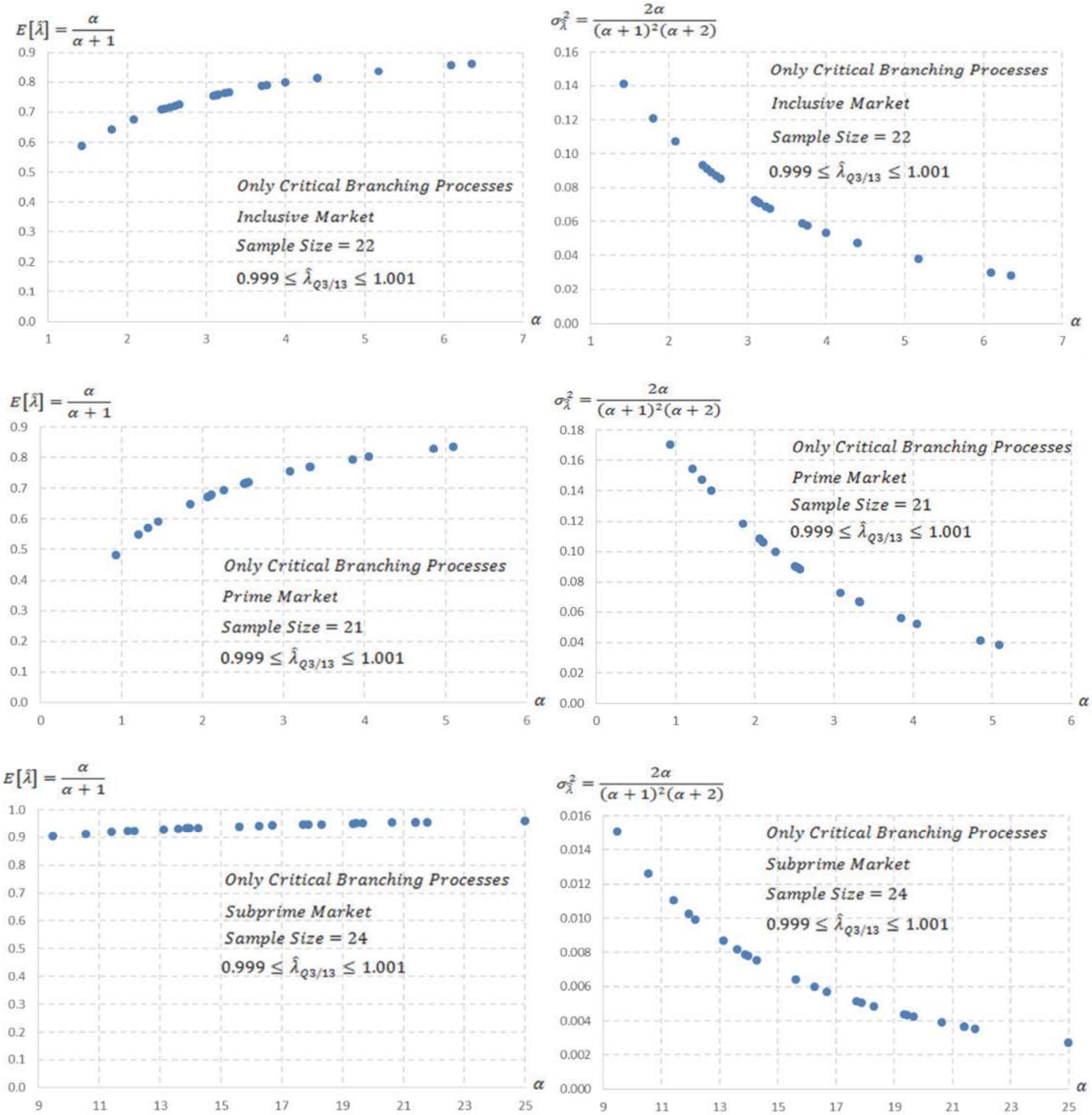


FIGURE 5
THE FIRST TWO MOMENTS OF $\hat{\lambda}$ IN EQUATION (11) FOR CRITICAL BRANCHING PROCESSES



Confidence Intervals for λ

Thus far, we have noticed that only a negligible fraction of the 366 SA in the U.S. conveyed clear subcritical behavior (having their $E[\hat{\lambda}] + \sigma_{\hat{\lambda}} < 1$) from the second quarter of 2010 until the third quarter of 2013. We have also recognized that much larger portions of these SA exhibited either obvious supercritical patterns (having their $E[\hat{\lambda}] - \sigma_{\hat{\lambda}} > 1$) or approximated critical patterns (with alternating $\hat{\lambda} \cong 1$) within the same time frame. For these latter ambiguous SA policymakers might be interested to learn more about the overall trends of their housing submarkets, whether their respective economic conditions are indeed improving or deteriorating. Similar desire may hold for the other processes as well.

The latest estimates of $\hat{\lambda}$ (at the end of the third quarter of 2013 in our sample) can provide some information on that, yet to regulators this might be insufficient. We therefore provide further statistical views on λ by constructing both point estimates and confidence intervals around them.

In **Table 2** we present three examples (arranged in three separate panels). To assess the quality of our point estimates and confidence intervals hereafter we intentionally select the same three SA from **Figure 2** that represent subcritical, critical, and supercritical branching processes, as previously acknowledged by their corresponding $\hat{\lambda}_{Q3/13}$. For each SA we nominate two prior disseminations $\pi(\lambda)$ as either the Uniform distribution or the Normal distribution (the latter is slightly more scattered), both calibrated around the empirically observed means (and medians) of 1.02. We allow the computer to simulate for each test 5,000 random numbers.[17]

TABLE 2
THREE EXAMPLES FOR POINT ESTIMATES AND CONFIDENCE INTERVALS OF λ

Panel A:	Inclusive Market, Subcritical Branching Process, SA is Ames, IA, $\alpha = 2$, $\tilde{\gamma}_{Q3/13} = 31$			
Test Number	Prior Distribution $\pi(\lambda)$ Uniform / Normal	Arrival of New Foreclosures Distribution (Equation No.)	Point Estimate $\hat{\lambda} = E[\lambda \mid \gamma]$	C.I. with $\vartheta = 0.05$ $[\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$
1	$\sim U[0.92, 1.12]$	Borel-Tanner – Equation (6)	0.9855	$[0.9806, 0.9903]$
2	$\sim U[0.92, 1.12]$	Haight – Equation (9)	0.9854	$[0.9804, 0.9906]$
3	$\sim N(\mu = 1.02, \sigma = 0.1)$	Borel-Tanner – Equation (6)	0.9865	$[0.9772, 0.9939]$
4	$\sim N(\mu = 1.02, \sigma = 0.1)$	Haight – Equation (9)	0.9854	$[0.9763, 0.9854]$
Panel B:	Prime Market, Supercritical Branching Process, SA is Buffalo – Niagara Falls, NY, $\alpha = 2$, $\tilde{\gamma}_{Q3/13} = 45$			
Test Number	Prior Distribution $\pi(\lambda)$ Uniform / Normal	Arrival of New Foreclosures Distribution (Equation No.)	Point Estimate $\hat{\lambda} = E[\lambda \mid \gamma]$	C.I. with $\vartheta = 0.05$ $[\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$
1	$\sim U[0.92, 1.12]$	Borel-Tanner – Equation (6)	1.0055	$[1.0005, 1.0107]$
2	$\sim U[0.92, 1.12]$	Haight – Equation (9)	1.0056	$[1.0010, 1.0101]$
3	$\sim N(\mu = 1.02, \sigma = 0.1)$	Borel-Tanner – Equation (6)	1.0058	$[0.9980, 1.0131]$
4	$\sim N(\mu = 1.02, \sigma = 0.1)$	Haight – Equation (9)	1.0056	$[0.9987, 1.0125]$
Panel C:	Inclusive Market, Critical Branching Process, SA is Fairbanks, AK, $\alpha = 2$, $\tilde{\gamma}_{Q3/13} = 34$			
Test Number	Prior Distribution $\pi(\lambda)$ Uniform / Normal	Arrival of New Foreclosures Distribution (Equation No.)	Point Estimate $\hat{\lambda} = E[\lambda \mid \gamma]$	C.I. with $\vartheta = 0.05$ $[\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$
1	$\sim U[0.92, 1.12]$	Borel-Tanner – Equation (6)	0.9914	$[0.9864, 0.9959]$
2	$\sim U[0.92, 1.12]$	Haight – Equation (9)	0.9914	$[0.9856, 0.9969]$
3	$\sim N(\mu = 1.02, \sigma = 0.1)$	Borel-Tanner – Equation (6)	0.9917	$[0.9812, 1.0013]$
4	$\sim N(\mu = 1.02, \sigma = 0.1)$	Haight – Equation (9)	0.9918	$[0.9819, 1.0007]$

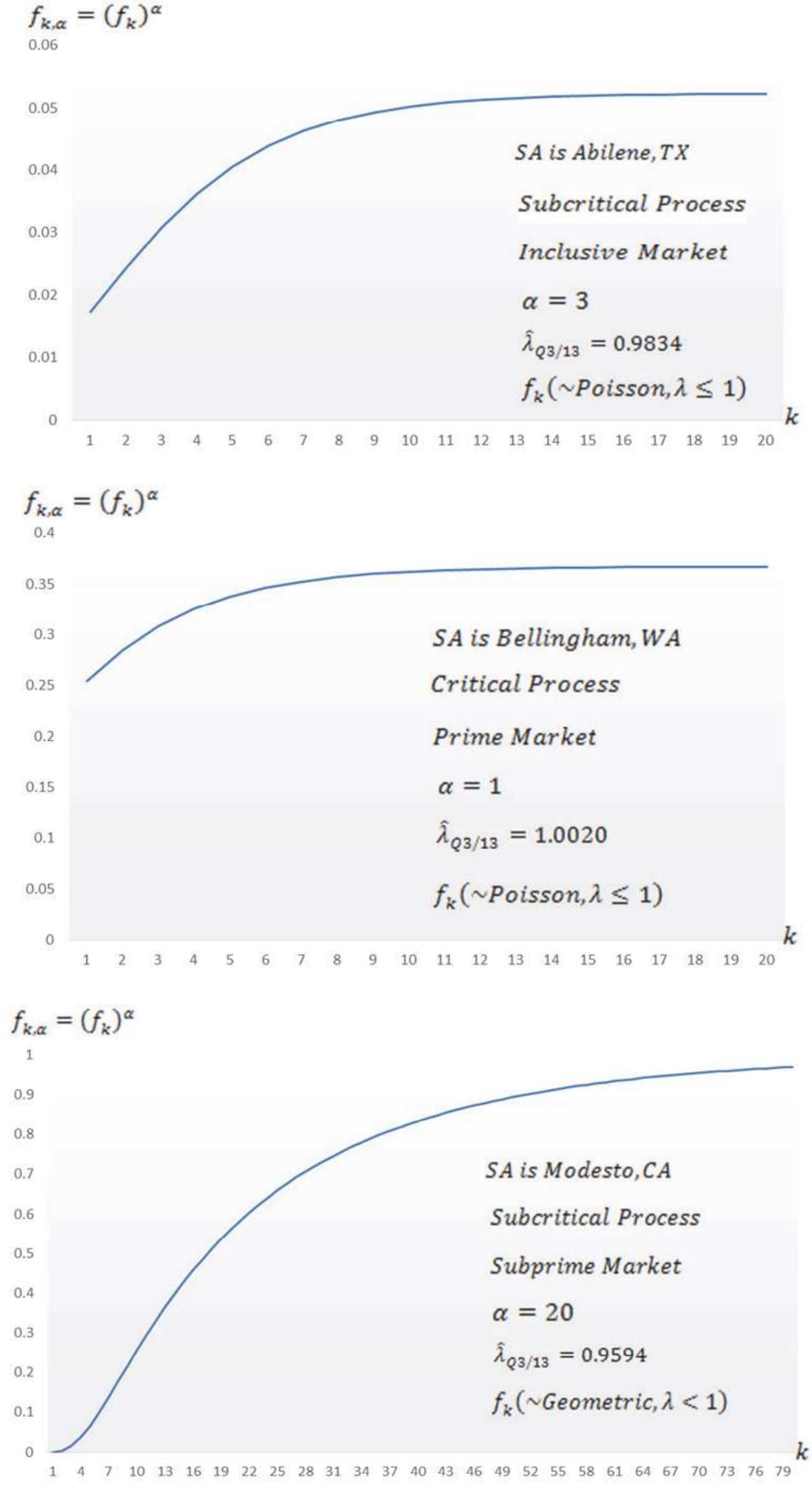
Once prior estimates of $\pi(\lambda)$ are designated, we instate them in either the Borel-Tanner distribution in equation (6) or the Haight distribution in equation (9), as possible disseminations of the arrivals of new foreclosures. Derivations (6) and (9) entail further parametrization, thus, as earlier recorded, for the SA of Ames, IA we utilize $\alpha = 2$ and $\tilde{\gamma}_{Q3/13} = r = 31$, within the SA of Buffalo–Niagara Falls, NY we employ $\alpha = 2$ and $\tilde{\gamma}_{Q3/13} = r = 45$, and for the SA of Fairbanks, AK we appoint $\alpha = 2$ and $\tilde{\gamma}_{Q3/13} = r = 34$. We now have three panels, each containing four tests, with 5,000 possible estimates of λ. We select the $\vartheta = 5\%$ highest posterior density intervals (i.e. the $100(1 - 0.05)\% = 95\%$ upper profiles in each cluster) and compute for each set of the 250 chosen (highly-probable) estimates of λ the corresponding minimum, maximum, and average, which correspond to $[\tilde{\lambda}_{low}, \tilde{\lambda}_{high}]$ and $\hat{\lambda} = E[\lambda \mid \gamma]$.

In panel A we present the results for the SA of Ames, IA. This SA has been classified earlier as subcritical in light of its $\hat{\lambda}_{Q3/13} = 0.9758$. All four point estimates of $\hat{\lambda}$ faithfully resemble this figure, while the respective confidence intervals provide further assurance that this SA is indeed a subcritical branching process. In panel B we display the results for the SA of Buffalo–Niagara Falls, NY. This SA has been labeled before as slightly supercritical due to its $\hat{\lambda}_{Q3/13} = 1.0156$. Although all four point estimates are somewhat lower than this figure, they are all sufficiently above the critical threshold. The respective confidence intervals are satisfactorily above unity as well, which corroborates the overall supercritical direction of this SA. In panel C we stage the results for the SA of Fairbanks, AK. This SA has been previously categorized as critical in light of its $\hat{\lambda}_{Q3/13} = 1.0027$. All four point estimates of $\hat{\lambda}$ closely resemble this figure (although marginally lower than 1). The respective confidence intervals provide further assurance that this SA is in fact a critical branching process. Although we deploy a large number of computer replications, the Monte Carlo simulations inherently include some variability, and therefore are subject to yielding slightly different findings with every new stochastic run.

The Duration of a Foreclosure Epidemic

We conclude our analyses by estimating the probable durations of the foreclosure epidemics across some of the 366 SA under study. We recall that the derivations of the duration distributions in equations (15)–(19) are applicable only for subcritical and critical branching processes, and that the Geometric distribution for modeling x is valid only in the more extreme subcritical branching processes.[18] Therefore, to illustrate the computations of the probable durations of foreclosure epidemics we select three prominent examples as displayed in **Figure 6**.

FIGURE 6
THREE EXAMPLES FOR THE DURATIONS OF FORECLOSURE EPIDEMICS



We use the Poisson distribution for modeling x for the subcritical branching process at the SA of Abilene, TX, and the critical branching process at the SA of Bellingham, WA. We use the Geometric distribution for modeling x for the more extreme subcritical branching process at the SA of Modesto, CA. While the branching processes within the first two SA start with relatively low initial foreclosures, $\alpha = 3$ and $\alpha = 1$, respectively, their corresponding $\hat{\lambda}_{Q3/13}$ are fairly moderate. Thus, for these two SA the corresponding distributions of foreclosure epidemic durations do not converge so quickly. Conversely, the branching process at the third SA starts with a higher $\alpha = 20$ initial number of foreclosures, yet its respective $\hat{\lambda}_{Q3/13}$ is far lower than the first two instances. In the latter case, the distribution of possible durations of a foreclosure epidemic begins with negligible likelihoods in the first year, but then rapidly converges to much higher probabilities than the corresponding possible durations in the first two examples.

SUMMARY AND POLICY DISCUSSION

In this study we have analyzed the outcome of the foreclosure epidemic during the great recession of 2007–2009. We have explored the ongoing progression of transmittable foreclosures across the 366 metropolitan areas in the U.S. from the first quarter of 2010 until the end of the third quarter of 2013. For this purpose we have segregated proxy measurements for contagious foreclosures, and then developed and examined subcritical, critical, and supercritical branching processes for the different segments of the U.S. housing market.

We have recognized that the U.S. foreclosure epidemic was far from concluding towards the end of 2013, long after the period which the housing crisis is commonly defined as covering. We detected that, throughout the inspected period, the vast majority of the metropolitan areas in the U.S. can be classified as having either critical or supercritical processes; they have experienced harsh though fluctuating economic circumstances in their respective housing submarkets, while many of these submarkets continued worsening. The economic importance of these findings is quite robust across our datasets. Within the inclusive market sample, 65 out of the 232 supercritical processes are part of the largest 100 U.S. metros. Within the prime-loans market sample, 60 out of the 191 supercritical processes are part of the largest 100 U.S. metros. Within the subprime-loans market sample, 21 out of the 98 supercritical processes are part of the largest 100 U.S. metros. These metropolitan areas have experienced lasting severe meltdown in their housing submarkets and therefore induce some regulatory intervention.

In practice, a regulatory interference could block a potential foreclosure epidemic. Its objective would be to constantly maintain the average rate of contagious foreclosures λ well below unity. There is, however, an optimal dosage of intervention when regulatory bodies and policymakers aim towards this goal. Too light interference might not be sufficient to terminate a foreclosure epidemic. Too aggressive interference would stop an infectious foreclosure epidemic, yet it could excessively harm nonprime mortgage borrowers who seek to become homeowners.[19] To better understand the tradeoff at hand and to put the following policy discussion in the right context we briefly summarize hereafter a few milestones from the relevant legal literature.

Meador (1982) and Schill (1991) detect a small yet consistent influence of state and local lending and foreclosure laws on mortgage interest rates. Clauretie (1987) focuses on the impact of state foreclosure laws on residential lending risk over the period 1980–1986 and finds that a judicial foreclosure requirement and a statutory right of redemption add significantly to mortgage risk, while anti-deficiency judgment statues preclude the amelioration of such risk. Pence (2006) studies the impact of state and local lending and foreclosure laws on the numbers and the scopes of mortgage originations and shows that mortgages are 3.5% larger when non-judicial foreclosure is permitted. Bostic et al. (2008) and Pennington-Cross and Ho (2008) both use state-border fixed effect models to measure the effect of predatory and abusive lending laws on the cost of credit. They find that these laws are associated with a modest increase in the cost of credit among fixed rate loans and a small decrease in the cost of credit among adjustable rate loans.

Mian, Sufi, and Trebbi (2011) study the non-judicial foreclosure process as an instrument for foreclosure rates to evaluate the impact of foreclosures on housing prices. They do not distinguish between prime and subprime loans and do not find a significant effect of foreclosure laws on mortgage terms in the late 1990s and early 2000s. Desai, Elliehausen, and Steinbuks (2013) analyze the effects of state bankruptcy asset exemptions and foreclosure laws on mortgage default and foreclosure rates across different segments of the mortgage market. The authors report that legal provisions are overall more influential on subprime and adjustable rate loans than on prime and fixed rate mortgage loans. More recently, Curtis (2014) explains that lender-favorable foreclosure laws and procedures are typically associated with more lending activity (both mortgage applications and originations) in the subprime market, yet they exhibit a lower impact on the prime market. On the other hand, this study reports that strong anti-predatory lending laws and slow foreclosure procedures are usually associated with less subprime activity.

As these and other studies show, state and local lending and foreclosure laws that generally benefit more lenders at the expense of borrowers overall promote mortgage lending activity, mainly within the subprime market and across adjustable rate loans, but also in the prime market and amidst fixed rate mortgage loans. On one hand, these laws help low credit-score borrowers to obtain mortgages and to become homeowners. Yet, they also tend to amplify the foreclosure rates in the mortgage inclusive market and thus increase the systemic risk in the aggregate economy. These conflicting forces create a situation that is not necessarily socially preferable (assisting one group while jeopardizing the entire population).

Conversely, mortgage lending activity tends to shrink under state and local lending and foreclosure laws that generally protect more borrowers while somewhat disadvantaging lenders, in light of a higher associated risk to the lenders. This reduced interest to provide mortgage loans typically impairs low credit-quality borrowers, precludes them from acquiring mortgage loans, and prevents them from becoming homeowners. However, this diminished mortgage market has higher credit quality, and thus exhibits lower systemic risk to the combined economy. Once again, these opposing dynamics are not necessarily socially desirable (hurting one group while protecting the entire population).

Maintaining the average rate of contagious foreclosures λ well below unity across the different metropolitan areas seems like a prudent goal of regulation. Achieving this target would depress new foreclosure rates and enhance stability in both the housing and the financial markets. Yet, vigorously preventing new foreclosures means denying mortgage applications from low credit-score applicants in the first place. Therefore, the objective to preserve $\lambda < 1$ at all times may also bring, according to some views, some level of social injustice.

The literature promotes several ideas that could mitigate the foreclosure contagion effect and would result in the least amount of market distortion, such as the use of financial instruments like home equity insurance (as discussed by Shiller and Weiss (1999)), or the “shared-responsibility mortgage” approach, which would automatically lower mortgage payments when home values decline. It is beyond the scope of this study, yet we recommend future lines of research to seek an optimal level of λ that maximizes the wealth of both lenders and borrowers, promotes stability in the real estate market, and at the same time promotes social justice. This optimum level of λ may be located in either equilibrium or disequilibrium resolutions.

ENDNOTES

1. The U.S. Department of Housing and Urban Development (2009) is one comprehensive report on these issues.
2. Very few studies have attempted to explore the repercussions of the great recession. Among them, Molloy and Shan (2011) examine what happened to borrowers and their households after their mortgages were foreclosed during the recent U.S. housing crisis.
3. To save space we do not aim to cover here the entire economic literature on bank foreclosures.

4. We largely dismiss housing prices and quantitative easing measures from our later empirical analyses since data on these matters is typically available only at the state or nation levels and not within the metropolitan areas, while the foreclosure contagion effect is greatly reduced at these broader levels.

5. The authors comment that because housing markets are local and highly segmented, one would expect to find a stronger foreclosure contagion effect using micro-level data than within the state-level aggregates. This motivates us to direct our analysis hereafter towards narrowed metropolitan areas.
6. These findings, along with our own inspections, convince us to utilize time-series lags in our empirical analyses.
7. Among others, the U.S. Conference of Mayors (2007) expresses the demand for an aggressive Federal response by local lenders.
8. In the case where the arrival of new foreclosures follows a Bernoulli distribution, the number of time units that a housing crisis may last (until a complete elimination of foreclosures) in a given metropolitan area equals the total (accumulated) number of foreclosed houses in that region.
9. In our later empirical investigation we shall clarify under what circumstances it is preferable to model x with either the Poisson distribution or the Geometric distribution.
10. We recall that $\lambda = \frac{\theta}{1-\theta}$ in terms of the generalized power series distributions and express the probability with λ.
11. Alternatively, these prior distributions can be tuned to have means and medians equal to 1, thus to become completely neutral to whether the foreclosure branching process is subcritical, critical, or supercritical. Clearly, other representative prior distributions are also feasible.
12. Although this scenario seems highly unrealistic, it can be achieved if lenders would considerably tighten mortgage underwriting standards for an extended period of time and allow only the highest credit borrowers to acquire mortgage loans. This situation, however, has profound social implications as discussed in our policy debate.
13. Li and White (2009), who also investigate the foreclosure contagion effect, also use this database of LPS Analytics, but over earlier years.
14. BLS does not explicitly quote the local area unemployment rates across the 366 SA for the month of September 2013. To best assess these figures, we calculate the average unemployment rates of August and October 2013.
15. Because data on raw foreclosure rates is available to us only from March 2010 (15 observations for each SA), throughout this trailed process we lose the first quarter of records and are left with data on contagious foreclosure rates from June 2010 (14 observations per SA). The Bayesian estimation of $\hat{\lambda}$ in equation (10) hereafter commands the subtraction of the initial measure of contagious foreclosure rates and thus leaves us with 13 measurements of $\hat{\lambda}$ within each SA. In light of this gradual reduction in the number of time-series estimates we do not extend any further the regression model in (21) to VAR system of equations. Nonetheless, these single time lags appear to be economically significant throughout our analyses.
16. Essentially, both the numerator and the denominator in equation (10) are multiplied by the same scaling factor.
17. Most statistical packages, including the Excel function $RAND(\quad)$, would generate Uniformly distributed random numbers within the default interval $[0, 1]$. To tune this range to any other interval $[a, b]$ of our choice we need to use the simple transformation $a + (b - a) \times RAND(\quad)$. The Normally distributed random numbers are appointed by using the Excel function $NORMINV(RAND(\quad), 1.02, 0.1)$.
18. Clearly, for supercritical processes, with no regulatory intervention, the expected durations of foreclosure epidemics would converge to infinity.

19. Bernanke (2013) explains (on page 43) that he prefers the term “nonprime” over “subprime” to include Alt-A and other types of mortgagers who are also not up to the traditional standards of credit underwriting.

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