Short and Long-Term Dynamic Dependencies of Main Latin American Stock Indexes and Commodity Prices: A Wavelet Approach

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Modern portfolio theory seeks to maximize returns and minimize risk through the selection of non-correlated investment instruments. Investment instruments in the portfolio are, often, non-linear and non-stationary making forecasting of returns difficult. The use of decomposition models has been found to improve the accuracy of predictive models. This paper extends modern portfolio theory by applying wavelet analysis to the Latin American stock markets in various time horizons as investment vehicles in a portfolio along with major commodity markets. The main findings reveal that different commodities are needed in the portfolio depending on the time horizon and market.

INTRODUCTION

Modern portfolio theory, or MPT, is an investment theory that studies how to maximize returns and minimize risk through the appropriate choice of investment portfolio components. MPT was first introduced by Markowitz in 1952 and has since become a basic principle in the study of finance. Diversification is central to MPT, since the greater the level of diversification obtained, the less risk and the greater relative return the portfolio realizes. Diversification is inversely related to the degree of association of the returns of the assets incorporated into the portfolio; the more correlated the movements of the underlying assets in the portfolio the lower the level of diversification. Conversely, the lower the correlation, the greater the realized diversification. Hence, identification of the dependencies between various financial assets is important to the overall risk assessment of the portfolio. It should be noted that the benefits to obtaining diversification are limited by the systematic risk of the market as a whole.

Investment in international markets has long been a staple in the overall diversification of a portfolio. The belief has held that international markets are largely independent of each other such that as one experiences a decline the others will not. Grubel (1968), Levy and Sarnat (1970) and Lessard (1973) studying international markets found support for the preceding belief in that their studies demonstrated a low correlation between stock markets of industrial countries. Further, they found that this low correlation makes international diversification profitable. Subsequently, the number of international diversification studies have increased with the rise of globalization and the increase in international investments. Since the late 60's and early 70's, there has been in increase in globalization, the co-movement of economies, and the correlations between stock markets thereby making it critical to test the diversification benefits in this present context. Recently, evidence of this greater correlation between stock markets was detected by Bekaert et al. (2009) for European markets. Baele and Inghelbrecht (2009) found that globalization and integration have led to a gradual convergence of country to industry especially in Europe, but not so for North America or East Asia. Moreover, geographical diversification continues to be higher than industry diversification. Christoffersen et al. (2012) found that correlations have increased between stock markets both for developed as well as developing markets while correlations between the developed markets are much higher than those between developing markets.

While it is common to include international stocks in the modern portfolio, investors are also likely to include other assets such as commodities. In particular, some studies reveal the potential benefits of adding commodities to an investment portfolio such as Erb and Harvey (2006), You and Daigler (2010), You and Daigler (2013) Gorton and Rouwenhorst (2006) point out that commodities could potentially diversify the risk component of the portfolio and that this type of asset is very correlated with the inflation rate and its changes. It is for this reason that it is not uncommon to find cases of commodities investment, such as Akey (2005), in which money is invested in commodity linked indexes through hedge and pension funds. According to Adams et al. (2008) and Anson et al. (2011), commodities can be used as an effective hedge against both expected and unexpected inflation; their prices adjust according to economic cycles, with their prices typically being designated in US dollars. Ultimately, commodity prices cannot be estimated with traditional valuation models, but instead depend on the economic conditions as well as supply and demand. In fact, investment in commodities has increased along with the total amount of commodity assets under management according to Carpenter (2011).

Some investors are incorporating alternative asset classes to diversify and hedge portfolios. In this context, the commodity market is a good alternative, mainly the major energy products and precious metals (Tang & Xiong, 2012; Hammoudeh et al., 2013). Once thought to be a good inclusion in the portfolio to diversify risk, Cao et al. (2010) and Daskalaki and Skiadopoulos (2011) have provided evidence that the correlation between financial assets and commodities has increased, decreasing diversification benefits. However, Gorton and Rouwenhorst (2006) found that commodities are an important asset to take into account for international investment portfolio diversification.

The growth of Latin American markets in recent decades led to an increase in their presence within the world markets and inclusion in portfolios. The main exports from these emerging markets are commodities (e.g. oil, metals, timber). Therefore, it is relevant for investors to understand the joint behavior of these markets as well as the behavior of commodities. However, this analysis is usually done without taking the influence of temporal frequency into account (i.e. the short-term and long-term effect or relationship). Li et al. (2003) affirm that diversification in emerging markets is an important topic due to the benefits that it can provide for international investment portfolios. This fact has also been presented in the past by Harvey (1995) and Bekaert and Urias (1996). Miralles-Marcelo et al. (2015) found that among the main topics that have gained attention in recent financial literature is establishing whether the benefits of international diversification are still substantial in the current context of high correlations among markets.

On the basis of this background, this study seeks to determine the relationship between the main Latin American stock indexes and commodities at different time scales, specifically considering dynamic correlations based on wavelet coherence. It is expected that the economies that export certain commodities do not diversify risk well with respect to their stock markets and, moreover, that the countries themselves possess co-movement through time. This approach distinguishes between different types of investors with different investment horizons. Hence, it is crucial for them to be capable of distinguishing diversification in various time horizons.

The study of the relationship between Latin American and American stock markets and the commodity markets is interesting for four reasons. First, Latin American stock markets are considered to be assets in an international portfolio investment, being classified as emerging markets. Second, the combination of emerging markets and commodities in a diversified investment portfolio is not well studied. Third, global investors can benefit from investing in American and Latin American stock markets and commodity markets. Fourth, portfolio managers can more efficiently manage international portfolios. The novelty of this paper with regards to the existing literature on the topic is associated with the dynamic correlation analysis for different horizons among emerging markets, in particular, Latin American stock markets and commodity markets using wavelet. This analysis has not been previously performed and its results are key to better portfolio management both at the individual and institutional level.

This remainder of this document is structured in the following way: first, in the following section we provide a brief summary of the literature where several studies are presented that have used wavelet analysis in economics and finance, risk diversification, and commodities. Then, the methodology used is presented with a brief explanation of the wavelet transform. Finally, the empirical results are analyzed in order to later establish the conclusions of the study.

LITERATURE REVIEW

Nearly a decade ago, Gallegati (2008) recognized the potential application of wavelet analysis to the fields of “geophysics, engineering, physics, acoustics and, more recently, in economics and finance” (p. 3068). Wavelet Analysis is based on the decomposition of a time series or signal in various functions localized both in time and frequency domain; these new functions are more elementary and include different information about the original series. The main advantage of wavelet analysis is its ability to decompose time series, and data in general, into their time scale components, allowing us to obtain results at different time horizons. In particular, decomposing time series of asset price returns, we can analyze correlations through time for different time horizons, allowing us to make conclusions on how this correlation has varied in the short, medium, and long term between assets over time. One of the first studies to use wavelet methodology applied to economics is Ramsey and Lampart (1998), who focused on studying macroeconomic variables, in particular interest rate, income, consumption, and monetary aggregates, in different time scales in order to better explain series behavior. Subsequently, Chew (2001) studied the criteria to become a member of the European Monetary Union (EMU), concluding that these may be capable of changing the relationships of money and income in potential member countries, indicating that time scale analysis provides better knowledge on money and income. In the case of stock markets, Shik & Lee (2004) study behavior between the U.S. and Korean markets using wavelet. They find strong evidence of correlation in different horizons and volatility spillover effects from the U.S. stock market to the Korean stock market.

In financial models, one of the applications of wavelet analysis has been applied to the CAPM model. Xiong, Zhang, Zhang, and Li (2005) propose a new method to estimate systematic risk (□) of the China stock market using wavelet. They were able to determine the behavior of □ at different time scales, showing that the CAPM predictions are more relevant in short-term horizons than in long-term horizons. Following the same line of study, Fernández (2005) formulates a time-scale decomposition of an international version of the CAPM, representing both market risks and exchange-rate risks.

In the case of commodities, the effect on portfolio diversification has been studied by various authors, Lummer and Siegel (1993), Kaplan and Lummer (1998), Conover et al. (2010), Garreth and Taylor (2001), Erb and Harvey (2006). More recently, Graham, Kiviaho, and Nikkinen (2013) examined short and long-term dependencies of returns for S&P 500 and S&P GSCI U.S. commodity index using wavelet transformation methodology. They found that there is a weak co-movement between both indexes; for this reason, from the perspective of diversification, both short and long-term gains can be obtained, but with different relative risk. Following the same logic, Tan, Galagedera, and Maharaj (2012) studied the relationship between oil prices and various exchange rates in dollars through multiresolution wavelet analysis finding that the prices were not dependent during the period prior to the crisis; however, evidence of contagion and negative dependence is found after the beginning of the crisis. Lai, He, Xie, and Chen (2006), focusing on the non-ferrous metal market (nickel, zinc, aluminum), applied wavelet methodology to estimate VaR. Gallegati (2008) studied the relationship between stock market returns and economic activity by using wavelet for the Dow Jones indexes and the industrial production index in the United States, finding that these indexes possess long-term co-movement. Subsequently, Gallegati (2012) proposed an approach based on wavelet to test financial market contagion due to the 2007 subprime crisis in the United States, demonstrating that stock markets have been affected by the subprime crisis. Nonetheless, given the wavelet decomposition, it can be concluded that there are scales where this contagion was not significant. Belousova and Dofleitner (2012) analyzed the benefits of diversifying investment portfolios with commodities for European investors, finding that the contribution depends on the commodity. Industrial metals, agriculture, and livestock mainly contribute to reducing the risk, while energy and precious metals contribute to decreasing the risk and increasing the expected return. In relation to oil, Jia, An, Fang, Sun, and Huang (2015), applied a wavelet decomposition analysis, concluding that oil is important for portfolio diversification strategies but that investors should take into account according to the investment horizon.

Graham and Nikkinen (2011) utilized wavelet analysis to examine the co-movement of international stock markets in the short and long term from a European perspective, finding that the co-circulation of Finland and regions with emerging markets is limited to long-term fluctuations and that co-movement is evidenced between Finland and the developed regions of Europe, the Pacific, and North America in all frequencies, with higher levels of co-movement at the highest frequencies. Later, Dajcman, Festic, and Kavkler (2012) examined co-movement dynamics between developed countries and their respective stock markets, focusing on the United Kingdom, Germany, France, and Austria. They find that co-movements between stock market returns vary depending on the scale. Li, Chang, Miller, Balcilar, and Gupta (2015) focused their study on the relationship between the U.S. housing and stock markets, finding strong evidence of co-movement as well as that causality varies in the frequencies and evolves over time.

Garrett and Taylor (2001) investigated how the incorporation of commodities into a portfolio influences returns. They explore whether periods of significant investment in commodities are correlated with excess co-movement in commodity prices, concluding that optimal portfolios include commodities in a large proportion. However, this slightly decreases as the investor's risk aversion increases. There are studies that only focus on precious metals, such as Hillier, Draper, and Faff (2006), which investigate the potential of gold, silver, and platinum as investment assets, showing that these three metals have low correlations with stock index returns, suggesting that they may provide diversification within broad investment portfolios. Hammoudeh, Nguyen, Reboredo, and Wen (2014) examined recent trends in dependence structure between the fast-growing commodity markets and Chinese stock markets, providing evidence of low and positive correlations between these markets. This suggests that commodity futures are a desirable asset for portfolio diversification. Arouri, Khuong, and Pukthuanthong (2014) investigated the benefits of diversification and optimal portfolio allocation across different U.S. asset classes, coming to the conclusion that although there is a growing trend in market integration, the five main financial markets (equities, bonds, currencies, commodities, and real estate) seem to be weakly integrated. Along the same line, Orlov and Äijö (2015) studied the linkage between eight currencies and LIBOR rates to find the best diversification opportunities, using wavelet decomposition in five time horizons, showing that portfolio composition on the basis of wavelet correlation of returns varies depending on the time scale, where the results are more pronounced in the period prior to the crisis.

While this is not an exhaustive review of the literature as it applies to modern portfolio theory and wavelet analysis, it does illustrate the point that globalization has resulted in increased co-movements of markets; commodities have the potential to improve portfolio performance and reduce risk; and, that wavelet analysis has the potential to improve portfolio allocation relative to investment horizon. In this context, it is appropriate to consider the application of wavelet analysis to the Latin American stock market indexes and commodity markets at different time horizons. To build a Latin American and commodity portfolio, it is necessary to perform a study about this relationship. This study extends our understanding of portfolio theory since these relationships have yet to be studied. In the following section, we detail our approach and discuss the data.

METHODOLOGY AND DATA

Wavelet

The basis of the discrete wavelet transform (DWT) is a Fourier transformation analysis and its derivatives. DWT corrects the problem of resolution which Fourier has associated with the loss of information when determining time and frequency position at the same time, as Heisenberg's uncertainty principle poses (Doroslovacki, 1994). Wavelet analysis seeks to provide an adequate time and frequency resolution when the series has high and low frequencies (for further information, see Mallat, 1989; and, Daubechies and Laboratories 1993). The main difference between the wavelet transform and the windowed Fourier transform is that the wavelet transform is calculated for each spectral component, and this changes the width of the window, which the Fourier transform does not calculate.

The main benefits of wavelet transform are that it can be used for non-stationary series, it delivers a good approximation of the original series, and it locates specific phenomena in time (Crowley, 2005). Wavelet transform functions through the mother wavelet, which breaks the series down into different frequency components making up a family of functions that are translations and dilations of a mother function ψ (t), where the first families were created by Haar (1910). Dilation and translation are described mathematically in Equation (1).

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right) \tag{1}$$

where τ is the translation and s is the dilation. In this study, discrete wavelet transform is used. For this case, the dyadic scale is used, with the discrete wavelet transform according to Equation (2).

$$C(j,k) = \sum_{n\,\epsilon\,\mathbb{Z}} f(n)\psi_{j,k}(n) \tag{2}$$

where $s = 2^j$; $\tau = k2^j$; $j\ \epsilon\ \mathbb{N}$; $k\ \epsilon\ \mathbb{Z}$; and, $\psi_{j,k}(n) = 2^{-\frac{j}{2}}\psi\left(2^{-j}n - k\right)$

Given the multiresolution analysis, it is possible to reconstruct the original time series. For this purpose, filters with different cut-off frequencies were used in order to analyze the series at different scales. The series was passed through high-pass filters to analyze high-frequency components and low-pass filters to analyze low-frequency components. These operations change the resolution of the series, and the scale is changed by operations of interpolation and subsampling. Mathematically, multiresolution analysis is the obtaining of successive approximations of a series, $\cdots A_J, A_{J-1}, A_{J-2}, \cdots$ such that each approximation is a better representation of the original series; therefore, A_{J-1} represents a better approximation of the series than A_J. The difference between the different successive approximations of a series is detailed according to Equation (3).

$$D_J \equiv A_{J-1} - A_J \tag{3}$$

In this study, Daubechies least asymmetric mother wavelet of length LA (8) is used, because it is a better approximation to the ideal band pass filters by allowing less information loss (Härdle, Kerkyacharian, Picard, and Tsybakov, 2012). The convolution of discrete wavelet transform is defined according to Equation 4.

$$C_n(s), = \frac{1}{\sqrt{s}}\sum_{t=1}^{N} f(l)\,\psi_0^*\left(\frac{(l-n)\delta t}{s}\right) \tag{4}$$

where $*$ denotes complex conjugate, $n = 0,1,\cdots,N$ and δt represents equal steps in time. The results of the coefficients are used to describe certain characteristics of the series. If the absolute value of the coefficients is taken, the results can be interpreted as the power or amplitude of the series: $|C_n(s)|$. Variance can be obtained through $|C_n(s)|^2$ (that is associated with wavelet energy). Co-movement calculation for two time series is described according to Equation 5.

$$C_n^{XY}(s) = C_n^X(s) * C_n^Y(s) \tag{5}$$

where $C_n^X(s)$ and $C_n^Y(s)$ are transformed from discrete wavelets of x_n and y_n. $C_n^{XY}(s)$ represents the total covariance between the two time series at each scale and frequency. Finally, obtaining a practical measure of dependencies (R^2) of two time series is defined according to Equation 6.

$$R_n^2(s) = \frac{\left|S\left(s^{-1}C_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|C_n^X(s)\right|^2\right)\cdot S\left(s^{-1}\left|C_n^Y(s)\right|^2\right)} \tag{6}$$

where S is a smoothing operator. With this, the results have been normalized to values between 0 and 1, and it can be used as the classic correlation coefficient.

Data

The information that is used are the main stock indexes of Latin American countries and the price of various commodities; these prices are from March 24th, 2004 to January 8th, 2014. Regarding the stock markets, they are the main Latin American markets: Brazil, Mexico, Chile, Argentina, Colombia, and Peru. The largest stock market is the Brazilian, which is characterized by the Sao Paulo stock market index (IBOV). The second in size is the Mexican stock market, which is represented by the Indice de Precios y Cotizaciones (IPC). The remaining markets are the Chilean, Argentine, Colombian, and Peruvian markets, characterized by the Santiago stock exchange Indice de Precios Selectivo (IPSA), the Buenos Aires stock market index (Merval), the Bogota stock market index (IGBC), and the Lima stock market index (IGBVL). Moreover, the commodities analyzed are: gold, silver, copper, aluminum, oil (West Texas Intermediate, WTI), corn, and cocoa. The evolution of the Latin American commodity indexes can be observed in Figure 1. It can be observed that the Peruvian stock market experiences the most growth in the period before the crisis (mar-04:sep-07), but it also experiences the most significant drop. The Merval was the market that experiences the least growth before the crisis. Furthermore, the Brazilian and Chilean markets exhibit the least growth during the entire analyzed period.

In the case of commodities, it can be seen that the behavior is not the same for the case of gold and silver as for the other commodities. Their prices were relatively stable and experienced high growth between 2008 and 2013, due to the fact that they are used as safety assets. Aluminum was the most stable commodity; however, it dropped in value during the analyzed period. It is important to note that it is the only asset used that was expressed in Chinese Yuan.

The original dataset included daily prices which were reduced to just the weekly set; the Wednesday price was chosen so as to remove weekend effects. Then, the prices were transformed to return (r_t) according to Equation 7.

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{7}$$

FIGURE 1
EVOLUTION OF LATIN AMERICAN STOCK INDEXES

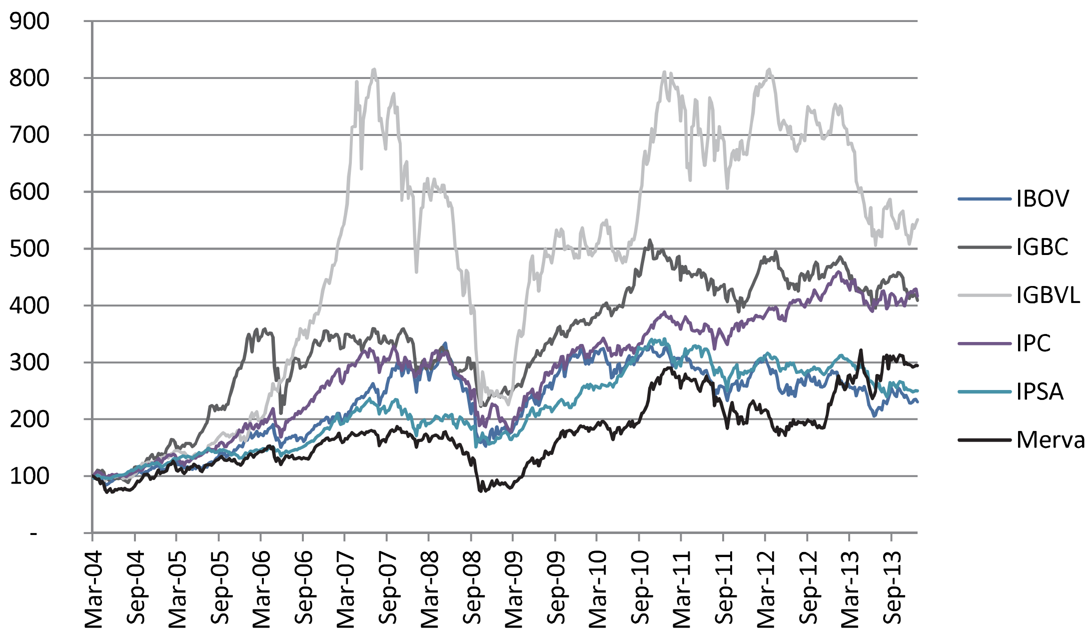


Figure created on the basis of 100 for each index for March 24, 2004.

FIGURE 2
TIME SERIES OF COMMODITY PRICES

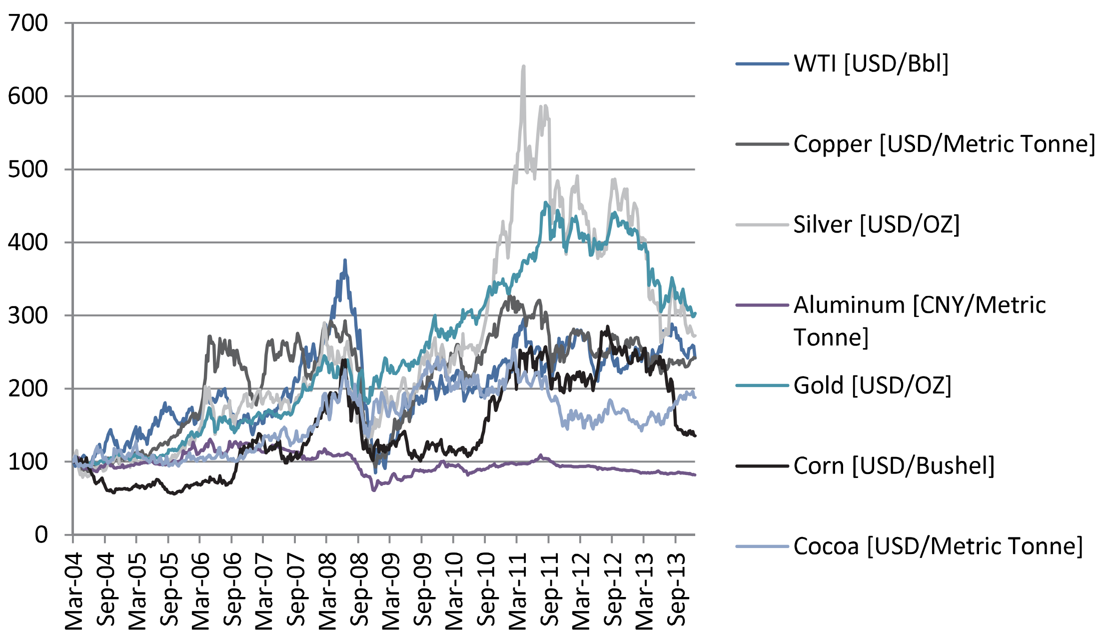


Figure created on the basis of 100 for each price for March 24, 2004.

As can be seen in Table 1, the descriptive statistics of the study variables are represented by logarithmic returns and present a large dispersion given that the standard deviation is significantly higher than the mean. Cocoa has a very similar kurtosis to that of normal distribution; thus, it can be inferred that a large amount of data is concentrated around the mean, and that the IPSA has the highest kurtosis. Therefore, it is likely that the tails carry a lot of weight in the distribution. WTI possesses the greatest interval, while cocoa has the lowest; this makes sense with respect to their kurtosis and asymmetry coefficient.

TABLE 1
DESCRIPTIVE STATISTICS

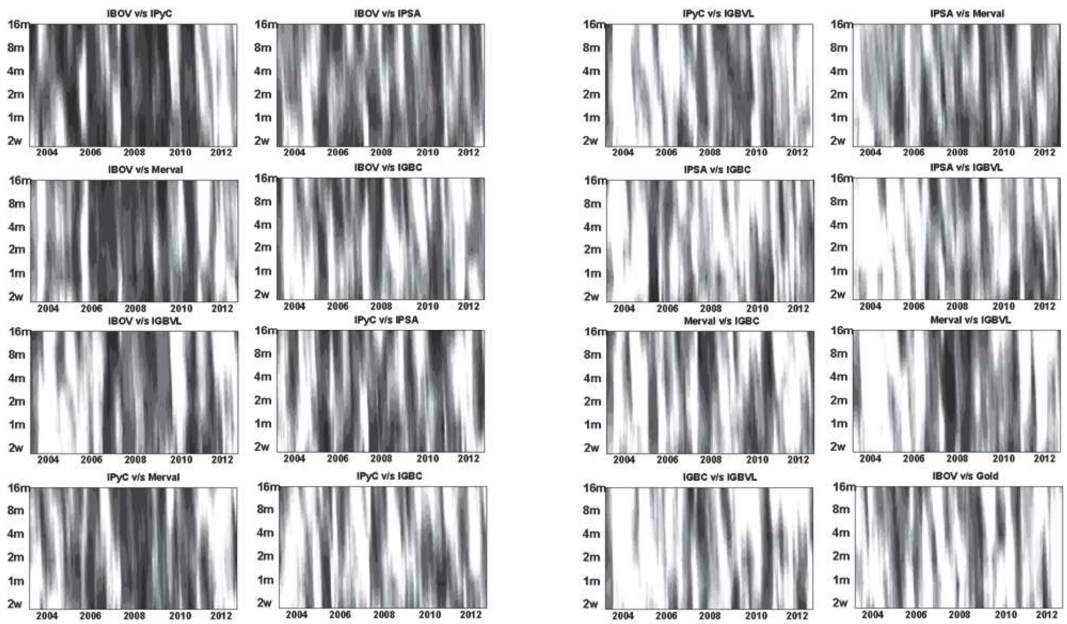
	Mean	Max	Min	Kurtosis	Skewness	Median	Std	Var
IBOV	0.0016	0.1058	-0.2549	7.9458	-0.9558	0.0046	0.0357	21.8266
IGBC	0.0028	0.1370	-0.2245	10.6239	-1.1761	0.0049	0.0321	11.6592
IGBVL	0.0033	0.2287	-0.1859	7.3308	-0.3339	0.0043	0.0416	12.4815
IPC	0.0028	0.0979	-0.1944	8.2376	-0.9249	0.0043	0.0287	10.3042
IPSA	0.0018	0.1016	-0.2153	13.2789	-1.4050	0.0056	0.0258	14.3885
MERVAL	0.0021	0.2124	-0.2314	6.4228	-0.5576	0.0055	0.0428	20.3342
WTI	0.0017	0.3211	-0.2138	7.3591	0.1508	0.0037	0.0493	28.5965
Copper	0.0017	0.1385	-0.1824	4.9428	-0.3670	0.0045	0.0414	24.0087
Silver	0.0020	0.1644	-0.2830	5.6872	-0.8072	0.0059	0.0507	25.9038
Aluminum	-0.0004	0.0963	-0.1123	8.3904	-0.1634	-0.0006	0.0211	-53.7799
Gold	0.0022	0.1002	-0.1284	5.8640	-0.7262	0.0042	0.0279	12.8740
Corn	0.0006	0.1632	-0.1878	4.2711	-0.2394	0.0054	0.0483	81.6595
Cocoa	0.0012	0.1374	-0.1361	3.4952	-0.0771	0.0014	0.0402	32.6560

ANALYSIS OF RESULTS

All analysis was run using Matlab R2015B. The results are presented graphically for each of the pairs analyzed, designated as stock market v/s commodity; each graph illustrates the correlation of the two assets studied. In the x-axis, the time axis details the years of the period analyzed (2004-2012), while in the y-axis, the various time horizons are represented: short-term (2 weeks-1 month), medium-term (2 months-4 months), and long-term (8 months-16 months). Correlation is reflected according to color; for a higher correlation, the color is darker. Therefore, it can be deduced that there is a very low correlation in the white areas (below 0.2), implying that for these years and in the time horizon that defines the area, diversification existed; low correlation is higher diversification. After the graphs, a Correlation Coloring table is shown in the figures that follow.

The first analysis performed, illustrated in Figure 3, focused on determining the possibility of diversification between the Latin American stock market indexes. To do this, the graphs of stock index pairs are analyzed. In the case of the BOVESPA, its relationship with the Mexican and Argentine index is characterized by a high correlation in the period 2006-2010 (Graphics: IBOV v/s IPyC and IBOV v/s Merval); in the last two years of study, this correlation declines. With respect to the IPSA (Graph IBOV v/s IPSA), while the correlation is of lower intensity than the 2006-2010 period with the Mexican and Argentine index, it shows consistency in the correlation during the entire period of study. The relationship of the BOVESPA with the Peruvian index (Graph IBOV v/s IGBVL) was very low until 2007 and has been irregular post-2010. In the case of the relationship with the IGBC (Graph IBOV v/s IGBC), its relationship is erratic. However, in 2012, the plot suggests a moderate correlation at all time horizons.

FIGURE 3
WAVELET ANALYSIS: MARKETS



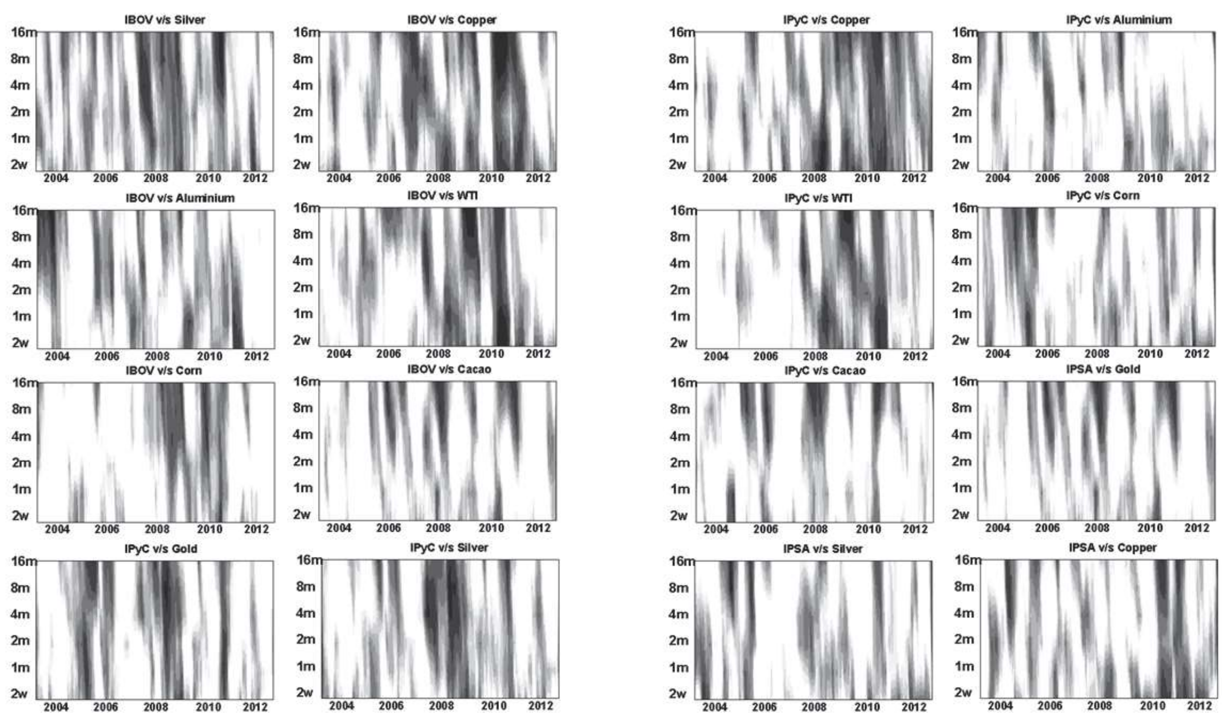
The Mexican index presents a high correlation in general with the IPSA (Graph IPyC v/s IPSA) during the entire period analyzed and for all terms. Regarding its behavior with the MERVAL (Graph IPyC v/s Merval), it presented a high medium and long-term correlation until 2010, while high short-term correlation occurred between 2005 and 2011. There is no correlation for 2012; thus, it is possible to conclude that the MERVAL serves to diversify the IPC. The IPC shows a low correlation over time with the Colombian and Peruvian stock indexes (Graphics: IPyC v/s IGBC and IPyC v/s IGBVL). Therefore, it can be concluded that a portfolio that is representative of the Mexican stock market can be diversified with Argentine, Colombian, and Peruvian portfolios.

Regarding the IPSA, its correlation with the Argentine market (Graph IPSA v/s Merval) has been increasing since 2005, while its correlation has been relatively low with the Colombian index (Graph IPSA v/s IGBC); however, in 2012, it increased in all horizons. The relationship of the IPSA with the IGBVL (Graph IPSA v/s IGBVL) had some episodes of high correlation between 2007 and 2010, but then this relatively high correlation became concentrated in the short-term. These last two relationships are interesting as the economies of Colombia, Peru, and Chile are characterized by having a strong commodity export sector. Nonetheless, this fails to permeate the correlation of their stock indexes. In fact, the graph comparing the IGBC and IGBVL (Graph IGBC v/s IGBVL) shows a relatively low correlation that is intermittent over time.

The MERVAL and the Colombian stock index (Graph Merval v/s IGBC) generally have had a low correlation, with the exception of 2008 and 2010. Meanwhile, the MERVAL with the IGBVL (Graph Merval v/s IGBVL) show a high correlation between 2007-2010, which later drops. We can conclude that the MERVAL can be diversified with Colombian and Peruvian instruments.

The second analysis, illustrated in Figure 4, focuses on the correlation between each of the Latin American markets and commodities. The graphs represent each market for all commodities under consideration: gold; silver; copper; aluminum; WTI; corn; and, cocoa). Analyzing the BOVESPA with gold (Graph IBOV v/s Gold), it can de determined that there was a high correlation at all horizons in 2005 and 2008-2009. From there, there was a long-term correlation until 2011 and then a short-term correlation during the same year. For 2012, there is no strong correlation in any of the horizons. Furthermore, it can be observed that in 2012, the portfolio does not have a strong correlation with the price of aluminum and corn; thus, it may be concluded that these two assets are useful for diversification of the BOVESPA. Moreover, the correlation between the IBOV and cocoa, copper, and silver (Graphs IBOV v/s Cacao, IBOV v/s Copper, and IBOV v/s Silver), has had a high correlation in all investment horizons analyzed. Finally, oil and corn show high correlation with the IBOV (Graphs IBOV v/s WTI and IBOV v/s Corn), between 2008-2012, while after this year the correlation centers around the short term; meanwhile, in the case of aluminum (Graph IBOV v/s Aluminum) it can be observed that there is no correlation since 2011. Thus, it could be used as an asset for long-term diversification.

FIGURE 4
WAVELET ANALYSIS: MARKETS & COMMODITIES

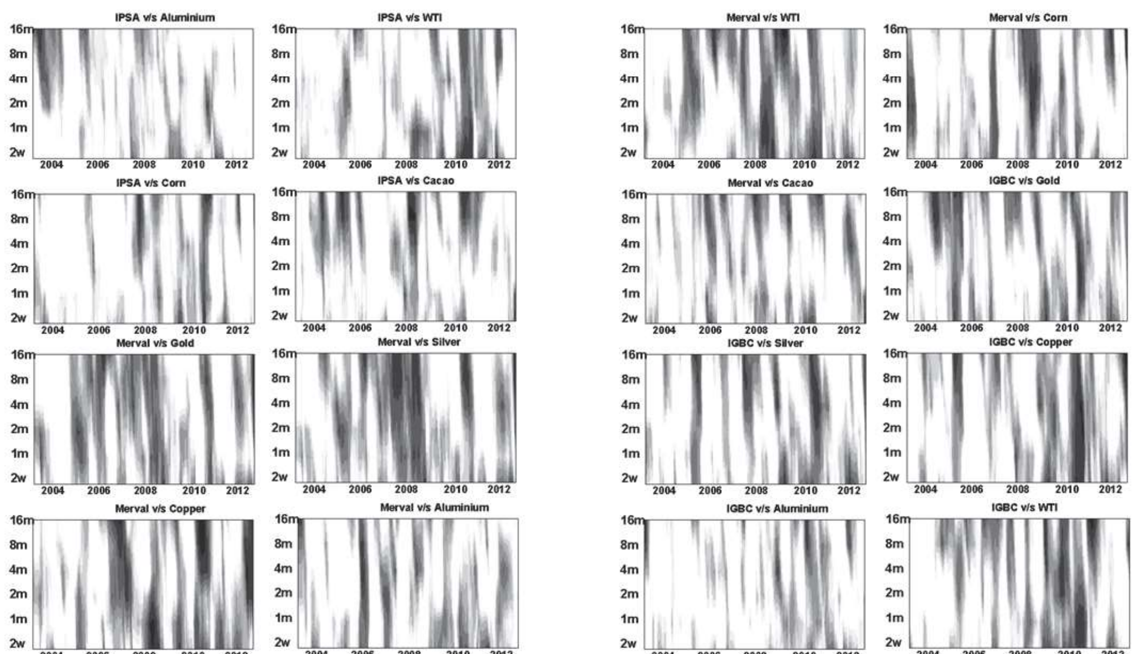


In the case of the Mexican stock index, it can be observed that gold (Graph IPyC v/s Gold) showed a high correlation in all terms in 2006, 2008, and 2010. However, its correlation was very low in 2011 and 2012. In the case of silver (Graph IPyC v/s Silver), there is a period of high correlation in 2008-2009, but then the correlation is low and irregular. With respect to copper (Graph IPyC v/s Copper), there is a high correlation since 2007 for all horizons, while there is no long-term correlation since 2009 for aluminum (Graph IPyC v/s Aluminum). Oil had a period of high correlation with the Mexican index between 2008 and 2011 in all horizons (Graph IPyC v/s WTI); this correlation later drops severely. Corn (Graph IPyC v/s Corn) presents a low and irregular correlation for the different terms since 2006, showing a greater short-term correlation in

2012. Cocoa (Graph IPyC v/s Cocoa) has a high long-term correlation in 2006, 2008, and 2010, but its correlation is generally low or null.

In the case of the IPSA, its correlation with gold (Graph IPSA v/s Gold) is irregular, demonstrating periods of higher long-term correlation in 2006, 2008, and 2011, with a relatively low correlation since 2011. The correlation of the IPSA with silver (Graph IPSA v/s Silver) has a greater association and persistence in the short term since 2008, while a high long-term correlation is only observed in 2010. Copper, Chile's main export commodity, shows an irregular relationship with periods of high correlation, but the correlation is greater and regular for all horizons since 2011 (Graph IPSA v/s Copper). Co-movement between the IPSA and aluminum (Graph IPSA v/s Aluminum, Figure 5) shows a high long-term correlation in 2004-2005, after which the correlation has been relatively low. Thus, aluminum is a commodity to diversify a portfolio with Chilean stocks. The case of oil (Graph IPSA v/s WTI) presents a negligible correlation with the IPSA until 2008, but subsequently the correlation is high and in general for all horizons. Corn has a high irregular correlation only in the 2008-2010 period (Graph IPSA v/s Corn). For higher time horizons, the correlation is almost null. Finally, correlation with cocoa (Graph IPSA v/s Cocoa) presents episodes of high long-term correlation between 2004 and 2010, but the correlation is negligible in 2011-2012.

FIGURE 5
WAVELET ANALYSIS: MARKETS & COMMODITIES (CONTINUED)

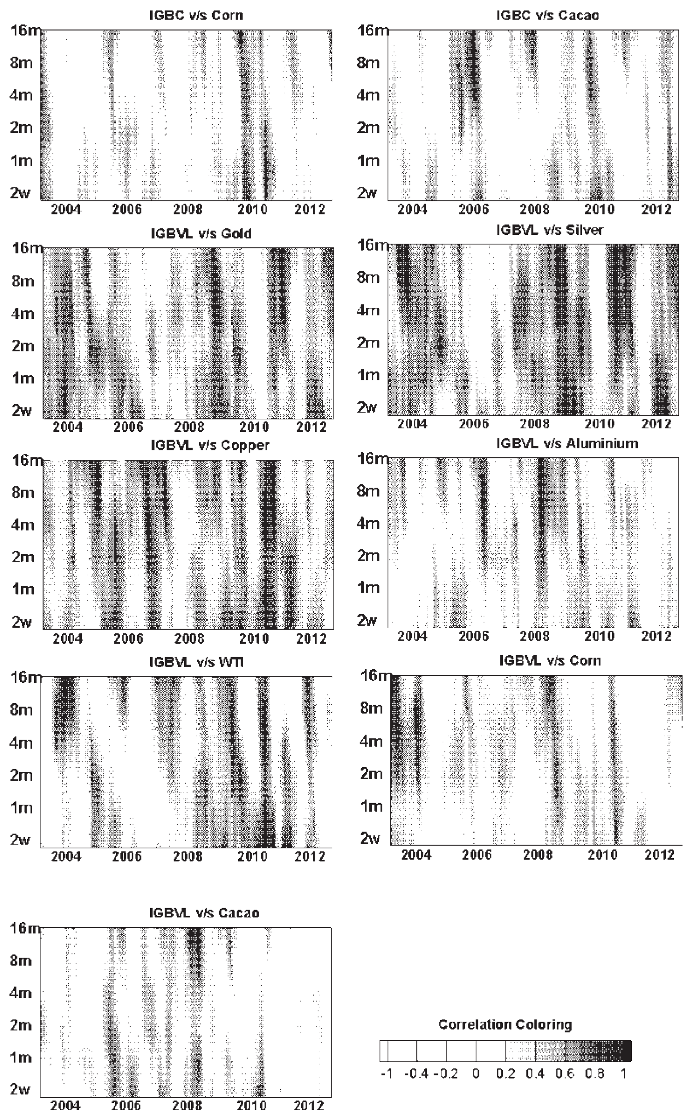


The MERVAL shows high correlation with gold (Graph Merval v/s Gold) for all horizons between 2005 and 2009 and then later reappears in 2012. Silver, copper, and WTI (Graphs Merval v/s Silver, Merval v/s Copper and Merval v/s WTI) have similar behavior to that of gold with the Argentine index, indicating that gold, silver, copper, and oil do not act as commodities to diversify the MERVAL. Aluminum and cocoa present an irregular medium level of correlation throughout the period of analysis for all horizons (Graphs Merval v/s Aluminum and Merval v/s Cocoa). Corn in particular exhibits a low short-term correlation in the last two years, making it the only potential commodity to diversify (Graph Merval v/s Corn).

The Colombian index has a similar correlation with the three metals analyzed (Graphs IGBC v/s Gold, IGBC v/s Silver and IGBC v/s Copper), a concentration of high correlation in 2006, 2008, 2010, and 2012, meaning that there is cyclical correlation behavior. WTI has a period of high correlation between 2008 and 2011 in all horizons, to later have a short-term correlation at the beginning of 2012, and finally, a long-term correlation (Graph IGBC v/s WTI). Correlations with aluminum and corn have similar behavior: they are relatively low with higher correlation in all horizons in 2010 and later decrease again (Graphs IGBC v/s Aluminum and IGBC v/s Corn). Finally, the correlation with cocoa (Graph IGBC v/s Cocoa) has been relatively low and non-constant; however, a moderate correlation can be noticed in all horizons at the end of the study period.

The Peruvian stock index presents a strong and regular correlation during the study period and in all horizons with the commodities gold, silver, and copper, the metals studied, showing a heavy dependence on the price of these commodities for the value of Peruvian stocks as illustrated in Figure 6 (Graphs IGBVL v/s Gold, IGBVL v/s Silver and IGBVL v/s Copper). For the case of WTI, (Graph IGBVL v/s WTI), there is a period of high correlation between 2008 and mid-2012, primarily in the short term. Correlation with aluminum is relatively low, and there is no long-term correlation in 2012, (Graph IGBVL v/s Aluminum). Finally, correlations with cocoa and corn have similar behavior: they are relatively low and erratic, tending to disappear in the last two years, (Graphs IGVL v/s Cocoa and IGBVL v/s Corn).

FIGURE 6
WAVELET ANALYSIS: MARKETS & COMMODITIES (CONTINUED)



CONCLUSIONS

Modern Portfolio Theory is predicated on the basis that risk-adverse investors can build portfolios that reduce risk and maximize expected returns. Commodities and international markets have been an important part of the diversified portfolios that investors build but this approach is in question given the increased interdependencies introduced by the process of globalization. Different financial techniques have been used to better forecast the volatility in markets in order to reduce risk (e.g. artificial neural networks, tail dependence, GARCH models). In this paper, we utilized wavelet analysis which allows one to decompose a time-series data into orthogonal components with different frequencies. The decomposition of components allows for the identification of correlations between investment instruments in different time horizons.

The focus of this paper was the measurement of diversification of various Latin American and commodities portfolios. In general, there is a change in behavior for some pairs of indexes and commodities between 2007 and 2008. This fact can clearly be attributed to an effect of the subprime financial crisis. Moreover, this change in behavior is towards a greater correlation between the pairs. With regard to the Colombian index, there is high correlation with the metals in 2006, 2008, 2010, and 2012; thus, there is probably a cyclical nature. It is important to highlight the notorious correlation of Mexican, Argentine, and Brazilian markets with WTI in different time horizons. This may be due to rising oil prices, after which the price was below 35 dollars in December 2008, thanks to the recovery of the economies emerging from recession, the incessant increase in demand by emerging economies, the events of the Arab Spring, and the diplomatic crisis due to Iran's nuclear program from 2011-2013. In the case of copper-exporting countries, there is a high correlation between their stock markets, especially from 2009 to 2013, where the price of copper increased significantly.

With respect to diversification possibilities for these stock markets regarding commodities, cocoa and corn provide the greatest potential since their co-movements with the majority of the stock markets of the countries in study are small. Aluminum is a metal that has high diversification possibilities in the Colombian market in practically all time horizons. For the case of the Mexican market, the highest time horizons are recommended for the diversification of this metal. In contrast, lower horizons are recommended for Chile. The Brazilian market has a high correlation with the stock markets of Chile, Mexico, Argentina, and Colombia; therefore, diversification is not recommended.

We believe that this is the first study to analyze the diversification possibilities of Latin American stock markets and commodities. It is verified that there are commodities with low correlations with these stock markets, allowing the diversification of an international investment portfolio. While our results are robust for the Latin America market, there is still opportunity to extend the work. For instance, one extension might be to look at the portfolio from a *global treasury* perspective wherein the portfolio has investments from multiple markets rather than merely a regional market. Within this perspective, one might consider both emerging and mature markets simultaneously as a portfolio.

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