A Dynamic Model for the Valuation of a Capital Investment: The Use of a Monte Carlo Simulation and the Stratification of the Modified Internal Rate of Return Within a European Put Option Strategy

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The assumption of equality of risk across investments is common in most capital budgeting situations. Within a European Put Option strategy, the valuation process for two actual real estate investment properties is undertaken. Specifically, this paper builds upon a previous work that introduced the "Stratified Modified Internal Rate of Return" (SMIRR) and further extends the financial literature by utilizing a Monte Carlo Simulation (MCS) allowing for a more 'realistic' quantitative analysis. A major point is that stochastic modeling is used to supplement the static discounted cash flow technique. The results incorporate the probability of risk and return into the valuation mix. Where discounted cash flow methods led to conflicting results, the use of the latter two techniques enabled the comparison of the two projects thereby enhancing the investment decision process.

INTRODUCTION

Real estate capital budgeting decisions follow a uniform process. It includes analyzing competitive locations based upon the unique real estate properties themselves and other available investment factors. Primary concerns center on the expected capitalization rates and the degree of risk associated with expected cash flows of each of the capital projects. Recently, Lifland (2011), looking at commercial real estate investments, argued that this endeavor created real options that impact the value of the capital budgeting investment process. Similar to a financial put option on common stock, a real option gives management the right but not the obligation to make a future sell decision. Just as with financial options, the value of the real option is contingent on future event(s) such as lease revenue receipts and the expected future re-sale or reversion value. Dixit and Pindyck (1994) argue that management will choose to exercise the option when it is perceived to be 'in-the-money'. Traditional capital budgeting analysis adheres to following NPV and IRR calculations. However, the results can create an illusion of certainty. These capital budgeting techniques do not address the probability of the project incurring a loss or meeting a target. This paper extends the literature by incorporating a Monte Carlo Simulation (MCS) in the determination of the expected capitalization rates, the corresponding degree of risk, and probabilities of projects achieving desired payoffs. This approach ran one hundred probability scenarios to produce probability distributions involving a number of random variables. This paper first looks at the related capital budgeting literature and the source of the data for the analysis. A discussion of the DCF process is followed by a detailed breakout of the SMIRR as a supplementary measure in the capital budgeting process. Next, the incorporation of the Monte Carlo methodology is reviewed. The paper finishes with the analysis of the empirical results and the concluding remarks.

RELATED LITERATURE

In a corporate capital budget analysis, the impact of risk and uncertainty on rational decision-making has been a major finance topic for discussion and research. The expected return in a capital budgeting case was found to be an increasing function of the risk-free rate of return, the market price of dollar risk, the project's variance of returns, the aggregate present value of the project and its co-variance with existing assets of the firm, and the co-variance of the project with other projects included in the capital budget (Litner, (1965)). Applying the Sharpe-Lintner-Black model of market equilibrium, the discounted cash flow models were found to depend on the periodic risk-adjusted discount rates. These rates, in turn, were adjusted for risk over the time periods as potential future cash flows were reassessed (Lucas and Prescott, 1971), (Fama, 1977), (Levy and Sarnat, 1984), and (Huang and Litzemberger, 1988). One weakness in the capital budgeting literature is that is generally accepted that investors will follow a set of rigid rules and will not alter a project at any specific stage of its useful operating life (Trigeorgis and Mason, 1987), (Trigeorgis, 1993).

Real options can exist in a capital budgeting framework allowing a strategic approach to decision making. Just as financial options derive their value from the underlying asset, the value of real options is contingent on future events (Xie, 2009). In a few hypothetical examples, the final decision on capital investments is influenced by future cash flows and discount rates but even more by the potential value that could be added from abandonment and end of period resale options (Bonini, 1977), (Berger, Ofek, Swary, 1996), and (Xie and Qi, 2008). Ignoring embedded options could result in underestimating the value of a project (Berger, Ofek, and Swary, 1995) and (Rose, 1998).

The implications are that management and investors gain a degree of flexibility through the recognition of the existence of real options and its impact on the valuation process. Traditional discounted cash flow (DCF) analysis settles project acceptance conflicts by deferring to the net present value (NPV) rule due to its focus on the cash flows of the project. Lifland (2011) introduced a new measure of risk associated with expected cash flows, the stratified modified internal rate of return (SMIRR). Within an American Put Option framework, risk averse investors can make choices counter to the conventional NPV rule prevalent in capital budgeting literature. Even though the modified internal rate of return of investments may be similar, the chance of significant risk differences between the projects can exist and the SMIRR reveals it.

DATA REVIEW

The financial facts and data for two mutually exclusive commercial real estate properties were obtained from REIS, Inc. The company provides impartial commercial real estate performance data and analysis. It specifically focuses on the metro (city) and submarket (neighborhood) for the office, apartment, retail, and industrial sectors.

Two actual commercial real estate properties are analyzed as a mutually exclusive project. There is a comparison of two downtown Chicago office building investments, Projects Riverside and LaSalle. A traditional discounted cash flow (DCF) process and valuation is adhered to. The physical characteristics for Projects Riverside and LaSalle are presented in Table 1 while the pertinent dollar per square foot data, used in the DCF model, for each property, is reported in Table 2. The net rentable area per square foot (psf) and the sale price (psf) are used to determine the initial outlay for each building. The initial outlay for Project LaSalle was approximately $108.7 million and the initial outlay for Project Riverside was approximately $144.1 million. These are historical purchase prices from 2008 based on data from REIS, Inc.

TABLE 1
PHYSICAL CHARACTERISTICS FOR PROJECTS RIVERSIDE AND LASALLE

Project Name	**Project Riverside**	**Project LaSalle**
City	Chicago	Chicago
Property Type	Multi-Tenant	Multi-Tenant
Building Area (sf)	702,439	621,428
Buildings/Floors	1/22	1/30
Year Built/Renovated	1965/1994	1984/not yet

TABLE 2
DOLLAR PER SQUARE FOOT (PSF) DATA FOR PROJECTS RIVERSIDE AND LASALLE

Property Address		Project Riverside	Project LaSalle
Net Rentable Area	psf	702,439	621,428
Sale Price	psf	$205.00	$175.00
Average Asking Rent	psf	$ 27.51	$ 26.36
Vacancy Loss Rate	%	14.40%	9.20%
Expense Stop	psf	$ 12.89	$ 10.91
Free Rent Concessions	psf	$ 0.23	$ 0.25
Credit Loss	%	1.00%	1.00%
Operating Expenses	psf	$ 14.12	$ 11.89
Capital Reserves	psf	$ 0.10	$ 0.11
Going-In-Cap-Rate	%	5.20%	7.50%

Notes for Table Two's line items:

- All per square foot (psf) figures are on an annual basis.
- Net Rentable Area (NRA) of a building included in the transaction, expressed in square feet, is an approximation based on verified public records.
- The potential rent revenue is the product of the building rentable area estimate and the average asking rent which is the market rent paid by a potential tenant.
- Sale Price (psf) is the purchase price of the property per square foot of net rentable area (NRA).
- Asking Rent for office properties is a weighted average quoted as annual gross rent per square foot.
- Vacancy losses are estimated rent losses from unoccupied space and unpaid rents.
- The Expense Stop creates an upper limit on the amount of operating expenses that the owner will be responsible for.
- Expense Reimbursement Recovery is the difference between the operating expense psf and the expense stop psf. The excess must be paid by the tenant. The recoverable operating expenses are property taxes, insurance, and maintenance.
- Free Rent Concession, to induce the lease signing, is the offer of a free rent period during which no rent is required to be paid. It is the total dollar amount or number of months free rent granted per lease terms.
- Credit Loss is the total amount of rent due that the landlord is unable to collect due to tenant default.
- Operating Expenses are the average annual costs, per square foot, of operating buildings that include property taxes, energy, janitorial service, insurance, common area maintenance, and management and leasing fees.
- Capital Reserves is an allowance that provides the periodic replacement of building components that wear out more rapidly than the building itself. They must be replaced during the economic life of the building.
- The reported estimated Going-in Capitalization Rate (Cap Rate) can be compared to the Reis Indexed Metro Office Cap Rate of 7.4%. The REIS Indexed Metro Office Cap Rate is modeled as a function of risk-free interest rates, metro rent growth expectations, current construction activity, and by running measures of volatility in rents. These measures are proxies for capital conditions, income expectations, and risk.

Critical property benchmarks for both the Chicago area and the specific properties under review are obtained from Metro Analysis and Rent and Sales Comparable reports supplied by REIS, Inc. and are presented in Table 3 below.

TABLE 3
RELEVANT DATA AND RISK FACTORS FROM METRO AREA ANALYSIS CONDUCTED BY REIS, INC.

	Chicago	Riverside	LaSalle
Annualized 5-year Vacancy Rate	17.6%	14.4%	9.2%
Annualized 5-year Rent Growth	2.1%		
Average Lease Term (years)	5.5		
Average Leasing Commissions	4.1%		
Annualized 5-year Construction/Absorption	1.9		
Inflation Rate per www.InflationData.Com	3.85%		

Notes:
Vacancy Rate is the amount of available space expressed as a percentage of total inventory.
Lease term is the average term currently being quoted for new leases, in years. This paper utilizes a 5 year lease life.
Leasing Commission is an amount paid to a real estate broker in exchange for bringing together the parties of the lease agreement. Usually it's paid in the form of a percentage of the yearly rent.
Construction/Absorption is the construction or completions during the time period divided by absorption during the same time period.

These commercial assets are acquired subject to existing leases as noted by the lease terms and leasing commissions in Table 3. All the components of Table 2 and the majority of Table 3 are accounted for in the calculation of the property's relevant future net operating income (NOI) and future reversion (RV) or sale price.

METHODOLOGY

In order to address the mutual exclusive situation between Project Riverside and Project LaSalle, a traditional DCF method is followed resulting in the determination of critical output variables such as the net present value (NPV), internal rate of return (IRR), and the modified internal rate of return (MIRR). This paper then goes beyond the DCF by introducing the stratification of the modified internal rate of return (SMIRR) and incorporates a Monte Carlo simulation (MCS) creating probability distributions that enable an investor to assess whether the risk associated with these projects is commensurate with their expected payoffs. This in-depth review of the risk-return tradeoff is done within a European Put Option framework.

The relevancy of the MIRR and hence the SMIRR, accrues from the overall strengths of the MIRR over the IRR. Kierulff, (2008) argues the IRR can give an unrealistic view of a project's potential value. A project with positive and negative cash flows delivers multiple IRRs. It also ignores the firm's cost of capital. Regarding the existence of a real option, Plath and Kennedy (1994) state that both the future operating cash flows and the timing of the estimated future resale value allow the incorporation of relative risk which enables an investor to compare projects.

DISCOUNTED CASH FLOW METHOD (DCF)

Under the (DCF), the expected future net operating income associated with the property is capitalized to determine the asset's estimated net present value (Gallinelli 2009). The DCF analysis helps to determine if a proposed project can generate strong enough risk-adjusted returns (DeLisle, 2009). Both projects, Riverside and LaSalle, supply a standard framework for multi-period real estate investment analysis. There are changing rent rolls and lease renewals and lease variables (inflation and cost of capital) that can change the level of net operating income (NOI), net terminal value or reversion value (RV) and net present valuation (NPV).

The basic DCF model to evaluate the property's net present value (NPV) is:

$$\text{NPV Office Building} = \sum NOI_t / (1+ capr)^t + RVt / (1+capr)^t - IO_0 \quad (1)$$

The NPV is equal to the present value of future cash inflows – initial investment.
Where NOI = expected net operating income (cash flows) for the office building.
RV = reversion (resale) value of the property; net terminal value.
IO = initial investment outlay.
capr = Capitalization Rate for the office building.
t = unique time period for each of the expected future cash flows.

Tables 4 and 5, below, present the projected net cash flows for Project Riverside and Project LaSalle over the time period of 2008 through 2013.

TABLE 4
PROJECTED NET CASH FLOW FROM OPERATIONS: PROJECT RIVERSIDE

Year	Factoid	2008	2009	2010	2011	2012	2013
	1	2	3	4	5	6	
Rentable Area psf	702,439						
Average Asking Rate	3.85%	$27.51	$28.57	$29.67	$30.82	$32.00	$33.23
Potential Rent Revenue	3.85%	$19,327,015	$20,071,105	$20,843,843	$21,646,331	$22,479,714	$23,345,183
Vacancy Loss	14.40%	2,783,000	2,890,239	3,001513	3,117,072	3,237,079	3,361,706
Effective Rent Revenue		$16,543,925	$17,180,866	$17,842,329	$18,529,259	$19,242,635	$19,983,477
Operating Expense psf	3.85%	$14.12	$14.66	$15.25	$15.81	$16.42	$17.06
Expense Stop psf	3.85%	12.89	13.99	13.90	14.44	14.99	15.57
Expense Reimbursement		$1.25	$1.28	$1.33	$1.38	$1.43	$1.49
Expense Reimbursement		$864,000	$897,264	$931,809	$967,683	$1,004,939	$1,043,629
Free Rent Concession	$.23	161,561	161,561	161,561	161,561	161,561	161,561
Credit Loss	1.00%	193,270	200,711	208,438	216,463	224,797	233,452
Effective Gross Revenue		$17,053,094	$17,715,858	$18,404,139	$19,118,918	$19,861,216	$20,632,093
Total Operating Expenses		$9,918,439	$10,300,299	$10,696,860	$11,108,689	$11,536,374	$11,980,524
Capital Reserves	$.10	70,244	70,244	70,244	70,244	70,244	70,244
Total Expenses		$9,988,683	$10,370,542	$10,767,104	$11,178,933	$11,606,618	$12,050,768
Net Cash Flow or (NOI)		$7,064,411	$7,345,315	$7,637,035	$7,939,985	$8,254,599	$8,581,325

*Expected inflation/growth rate is 3.85%
**Other variable % and $ from Table 2

TABLE 5
PROJECTED NET CASH FLOW FROM OPERATIONS: PROJECT LASALLE

Year	Factoid	2008	2009	2010	2011	2012	2013
	1	2	3	4	5	6	
Rentable Area psf	621,428						
Average Asking Rate	3.85%	$26.36	$27.37	$28.43	$29.52	$30.66	$31.84
Potential Rent Revenue	3.85%	$16,380,842	$17,011,505	$17,666,447	$18,346,606	$19,052,950	$19,786,489
Vacancy Loss	9.20%	1,507,037	1,565,058	1,625,313	1,687,888	1,752,871	1,820,357
Effective Rent Revenue		$14,873,805	$15,446,446	$16,041,134	$16,658,718	$17,300,079	$17,966,132
Operating Expense psf	3.85%	$11.89	$12.35	$12.82	$13.32	$13.83	$14.36
Expense Stop psf	3.85%	10.91	11.33	11.77	112.22	12.69	13.18
Expense Reimbursement		$ .98	$1.02	$1.06	$1.10	$1.14	$1.18
Expense Reimbursement		$608,999	$632,446	$656,795	$682,082	$708,342	$735,613
Free Rent Concession	$.26	161,571	161,571	161,571	161,571	161,571	161,571
Credit Loss	1.00%	163,808	170,115	176,664	183,466	190,529	197,865
Effective Gross Revenue		$15,157,424	$15,747,206	$16,359,694	$16,995,762	$17,656,320	$18,342,308
Total Operating Expenses		$7,388,779	$7,673,247	$7,968,667	$8,275,461	$8,594,066	$8,924,937
Capital Reserves	$.11	68,357	68.357	68,357	68,357	68,357	68,357
Total Expenses		$7,457,136	$7,741,604	$8,037,024	$8,343,818	$8662,423	$8,993,294
Net Cash Flow or (NOI)		$7,700,288	$8,005,602	$8,322,670	$8,651,945	$8,993,897	$9,349,014

*Expected inflation/growth rate is 3.85%
**Other variable % and $ from Table 2

Notes for Tables 4 and 5:
The estimated average annual inflation rate adjustment is 3.85%. The NOI increases each year even if leases are not renewed.
Vacancy losses are estimated rent losses from unoccupied space and unpaid rents.
Expense Stop creates an upper limit on the amount of operating expenses that the owner will be responsible for.
Expense Reimbursement Recovery is the difference between the operating expense psf and the expense stop psf. The excess must be paid by the tenant. The recoverable operating expenses are property taxes, insurance, and maintenance.
Free Rent Concession, to induce the lease signing, is the offer of a free rent period during which no rent is required to be paid. It is the total dollar amount or number of months free rent granted per lease terms.
Credit Loss is the total amount of rent due that the landlord is unable to collect due to tenant default.
Effective Gross Revenue is determined as the effective rent income plus the operating expense recoveries less the provisions for the free rent period and potential credit losses.
Operating Expenses are the average annual costs, per square foot, of operating buildings that include property taxes, energy, janitorial service, insurance, common area maintenance, and management and leasing fees.
Capital Reserves is an allowance that provides the periodic replacement of building components that wear out more rapidly than the building itself. They must be replaced during the economic life of the building.
Net operating income (NOI) is calculated as the net of the effective gross revenue and both the operating expenses and the provision for future capital outlays.
Even though the worksheet calculates the NOI, the measure is not income as described under generally accepted accounting principles (GAAP) but is cash flow. The term NOI is interchangeable with the net cash flow from operations.

If the appraised value of the project is a function of the income stream and the NOI results from the income stream that is generated from the operations of the property, the real estate investment is independent of external factors such as taxes or financing. The investor is deciding upon a property's income potential not the property itself. The before-tax NOI serves as an objective means of measuring the potential income stream from the property while the going-in capitalization rate acts as an investor's subjective estimate of how well the capital is required to perform (Gallinelli, 2004). Tax benefits are not ignored, rather, the implication is that an investor will consider the before tax cash flows, understanding that a tax benefit will be realized. (Brueggeman and Fisher, 2008).

The existing financing terms are assumed to be similar for both properties and as such, the expected returns for any particular group of investors should not be impacted by the financing of the project. It's not that interest rates or access to debt markets don't impact value, but under any economic climate, an investor will choose the equity-debt allocation based on the degree of risk that they are most comfortable with (Fisher 2008).

STRATIFICATION OF THE MODIFIED INTERNAL RATE OF RETURN (SMIRR)

Both the net present value (NPV) and the internal rate of return (IRR) are accepted measures of analyzing the attractiveness of investments. However, the weaknesses of the IRR (assumption of reinvestment at the IRR and possible multiple IRRs) can give an unrealistic view of a project's potential value (Kierulff, 2008). The use of the modified internal rate of return (MIRR) results in a more conservative return than the IRR; negative cash flows are cancelled out by positive ones, and the cash flows are compounded forward at a more realistic reinvestment rate based on the project's cost of capital. It then discounts future cash flows back to the initial outlay date at a rate that more fairly represents the investment risk of the project. The basic model to find the MIRR is presented below.

$$\text{Zero} = FVNOI_t / (1 + MIRR)^t + RV_t / (1 + MIRR)^t - IO_0 \quad (2)$$

The MIRR is the rate which equates the NPV to Zero
Future value of the sum of each NOI @ capr

$$\sum NOI_t (1+capr)^t = FVNOI \text{ at the end of the lease term} \quad (3)$$

where

RV = the reversion (sale) value at the end of the lease term
NOI = the net operating income or net cash flow for each year in the investment horizon.
capr = the capitalization rate used to determine the future value of net cash flows
FVNOI = the future value of the sum of each periodic NOI by the end of the lease term
RV = the Reversion value for the office building at the end of the lease term
MIRR = the modified internal rate of return for each office building
IO = the Initial investment outlay
t = the time period as of the end of the lease term.

This paper posits that the stratification of the MIRR provides another layer of analysis where the MIRR is weighted by two major strata: the present value of the operating net cash flows and the present value of a project's reversion value. Assuming there is generally more certainty associated with cash flows that happen earlier in the investment horizon, the calculated weights of the SMIRR reveal the relative risk associated with the return. The manager/investor can now see the sources of uncertainty in the valuation process that influences the decision to accept or reject a project. Specifically, this stratification of risk makes it a more robust technique for the mutually exclusive Riverside and LaSalle projects; especially, where there is a conflict between the decision rules of the NPV and MIRR. The process of the stratifying of the modified internal rate of return is presented below.

THE STRATIFYING OF THE MODIFIED INTERNAL RATE OF RETURN (SMIRR)

Step 1: Calculate the MIRR as described above.
Step 2: Use the MIRR to discount back the NOI cash flows and the RV cash flow
Step 3: Formulate the weight or strata of the MIRR

[a] $PVNOI_t + PVRV_t = TPVCF_t$ (4)

[b] $PVNOI_t / TPVCF_t$ = relative proportion of MIRR from the discounted total future NOI (5)

[c] $PVREV_t / TPVCF_t$ = relative proportion of MIRR from the discounted future RV (6)

where

$PVNOI_t$ = present value of future net operating income from the end of lease term

$PVRV_t$ = present value of future reversion value from the end of lease term

$TPVCF_t$ = Total present value of both operating and reversion cash flows at time period zero

Note: European Put Option – future reversion or sale of the office building can only occur at the end of the holding term.

MONTE CARLO SIMULATION

The Monte Carlo Simulation (MCS) is used to analyze models that contain uncertainty. It offers the ability to simulate a model so a variety of scenarios that might occur can be seen rather than a single best guess scenario. This is a perfect application for this Riverside/LaSalle case where the supplied REIS, Inc. data gave an inflation/growth rate of 3.85% and a cost of capital of 5.2% for Riverside and 7.5% for LaSalle. Both are treated as the critical decision variables and are modeled with probability distributions. These probability distributions require parameters such as the mean and standard deviation of a normal distribution. In the running of the simulation, there is a recalculation of one hundred iterations. After each iteration, a sample random variable is generated for each decision variable containing the probability distribution. For any stochastic model, the first requirement is the ability to generate random variables. The reproduction of a sequence of random numbers is important for reducing the variance of the distribution (McLeish, 2005). The analysis is performed in Excel and makes use of @RISK software. There are two key functions. The first is the =NORMINV(Rand(), mean, standard_dev) which returns the inverse of the normal cumulative distribution for the specified mean and standard deviation. It uses an iterative search technique. Within this function, the Rand() function returns a random number greater than or equal to zero and less than 1, evenly distributed and changed on recalculation. It takes the cumulative probability as input and provides the value of the decision variable corresponding to that cumulative probability. Those components in the DCF analysis impacted by the MCS are presented below:

Components of the DCF Analysis Impacted by the MCS

1. Average asking price of the property per square foot (psf)
2. Operating expenses per square foot (psf)
3. Expense stop per square foot (psf)
4. Present value interest factors (PVIF) leading to net present value (NPV)
5. Future value interest factors (FVIF) leading to Modified Internal Rate of Return (MIRR)

The probability distributions of the inflation/growth rate and the cost of capital for both the Riverside and LaSalle projects, respectively, are presented in Figures 1 and 2 below. The motivation for the MCS is evident as the DCF analysis is no longer dependent on single static decision variables but now incorporates a stochastic distribution of values.

FIGURE 1
PROBABILITY DISTRIBUTION OF INFLATION GROWTH RATE AND THE COST OF CAPITAL RIVERSIDE PROJECT

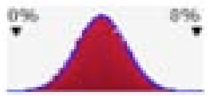
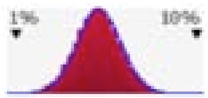
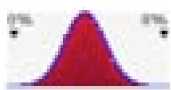
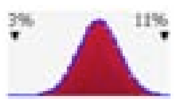
Decision Variable	Graph	Min	Mean	Max	Lower 5%	Upper 95%
Inflation Growth Rate: Riverside Project		0.74%	3.85%	7.48%	2.20%	5.49%
Cost of Capital: Riverside Project		1.74%	5.20%	9.16%	3.55%	6.84%

FIGURE 2
PROBABILITY DISTRIBUTION OF INFLATION GROWTH RATE AND THE COST OF CAPITAL LaSALLE PROJECT

Decision Variable	Graph	Min	Mean	Max	Lower 5%	Upper 95%
Inflation Growth Rate: LaSalle Project		0.69%	3.85%	7.61%	2.20%	5.49%
Cost of Capital: LaSalle Project		3.87%	7.50%	10.72 %	5.85%	9.14%

EMPIRICAL RESULTS

Under a European Put Option strategy, an investor can approach the valuation process where the property will be sold at the end of its lease term. The reversion value (RV) is determined by dividing the net operating income (NOI) (Table 4 and 5) by the given property's estimated going-in capitalization rate (Cap Rate) (Table 2). An important tenant of valuation based on direct capitalization, is that the investment properties being reviewed need to be comparable. The two office buildings appear to be similar in terms of their construction, size, age, location, and functionality. The expected future reversion value from the resale of each property is presented in Table 6 below.

TABLE 6
CASH FLOW FROM RESALE (REVERSION VALUE) FOR PROJECT RIVERSIDE AND PROJECT LASALLE UNDER THE EUROPEAN PUT OPTION STRATEGY

European Put Option:

	Project Riverside	Project LaSalle
NOI period 6 (2013)	$8,581,325	$9,349,014
Cap Rate	5.20%	7.50%
Cash Flow from Reversion	$165,025,487	$124,653,520

Note: European Put Option – Reversion value only at the end of the holding term.

Next, Table 7 presents the NPVs of each project:

TABLE 7
NET PRESENT VALUE (NPV) FOR PROJECT RIVERSIDE AND PROJECT LASALLE UNDER THE EUROPEAN PUT OPTION STRATEGY

European Put Option:

	Project Riverside	Project LaSalle
Initial Outlay	$143,999,995	$108,749,900
Cap Rate	5.20%	7.50%
Net Present Value	$16,878,443	$11,611,930

Note: European Put Option – NPV based on reversion only at the end of the holding term.

The Monte Carlo Simulation probability distributions provide a more detailed NPV profile with the results reported in Figures 3 and 4:

FIGURE 3
NPV PROFILE FOR RIVERSIDE PROJECT

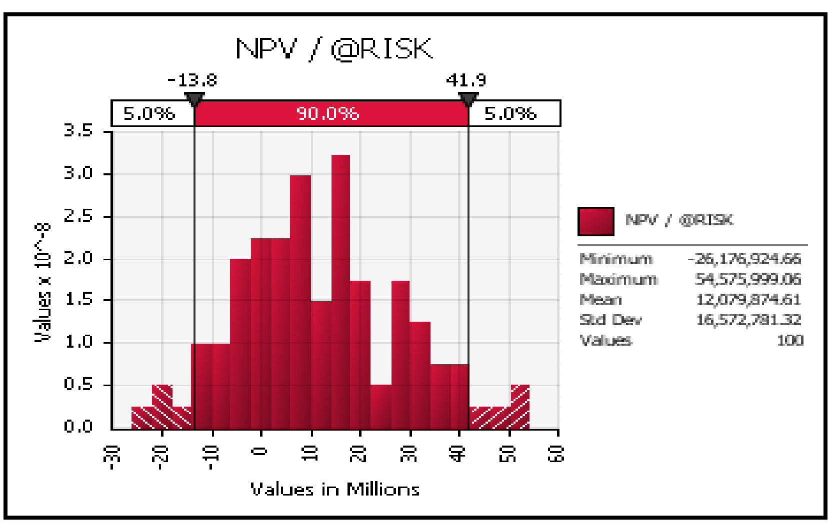
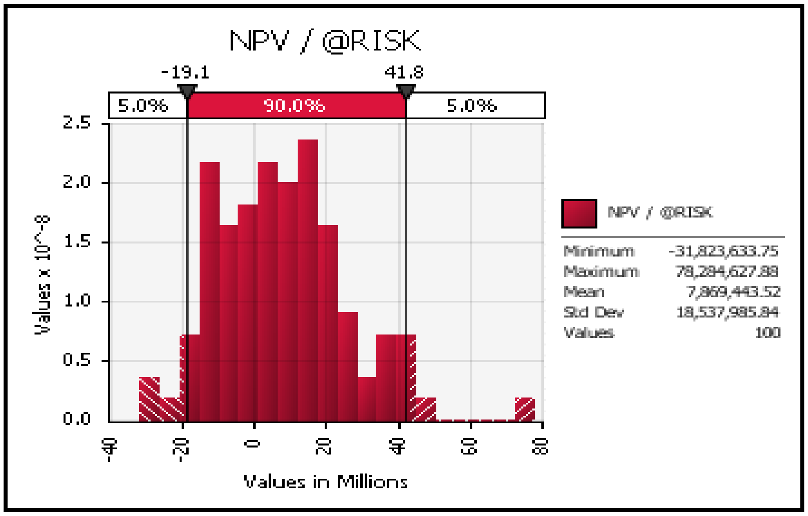


FIGURE 4
NPV PROFILE FOR LASALLE PROJECT



It's important to note that both Riverside and LaSalle have probability distributions that reveal potential negative net present values respectively. This would not be known under a traditional static analysis.

The next step in the DCF process is presented in Table 8; the determination of both the internal rate of return and the modified internal rate of return with their respective excesses over the project's capitalization rate.

TABLE 8
INTERNAL RATE OF RETURN and MODIFIED INTERNAL RATE OF RETURN FOR PROJECTS RIVERSIDE AND LASALLE UNDER THE EUROPEAN PUT OPTION STRATEGY

European Put Option:

	Project Riverside	Project LaSalle
Capitalization Rate	5.20%	7.50%
Internal Rate of Return (IRR)	7.78%	10.00%
Modified Internal Rate of Return (MIRR)	7.56%	9.70%
Excess of MIRR over Cap Rate	2.36%	2.20%
Excess of IRR over Cap Rate	2.58%	2.50%

European Put Option – IRR and MIRR based on reversion only at the end of the holding term.
The relatively larger excesses of the IRR compared to the MIRR, support the idea that the IRR overstates the valuation process. MIRR is more conservative.

An investor's expectation is that the respective modified internal rates of return will be greater than the project's going-in capitalization rate. The Riverside property reflects a MIRR of 7.56% while Property LaSalle generated a MIRR of 9.7%. Project Riverside's excess of the MIRR over its capitalization rate in comparison to Project LaSalle is only .16%. While the mutually exclusive conflict between the two capital budgeting techniques is not resolved, the MIRR does deliver a more conservative return measurement as evidenced by the IRR consistently overstating the return associated with each project.

A byproduct of the MCS analysis is a 'tornado' chart. This chart shows the impact of the two decision variables (inflation/growth and the cost of capital) on the MIRR. The longest bar appears at the top of the chart followed by the shorter bars. The interpretation of this chart is that the longer the bar the more effect the decision variable has on the MIRR. It reports the data as regression coefficients. Figures 5 and 6 report the findings for both properties; the cost of capital had the greatest impact on the MIRR and subsequently the NPV. The expected negative coefficient sign implies that an increase in the Cap Rate led to a decline in value while the inflation/growth rate had a positive relation. In the case of LaSalle, the positive impact of the inflation is somewhat greater for it than for the Riverside project. The degree of influence would not be known with a traditional analysis.

FIGURE 5
TORNADO CHART OF MIRR FOR RIVERSIDE

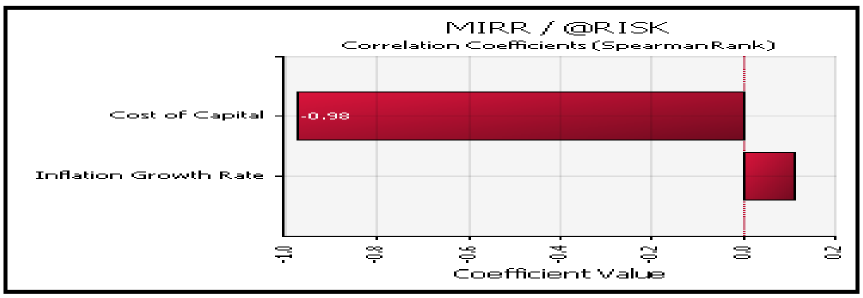
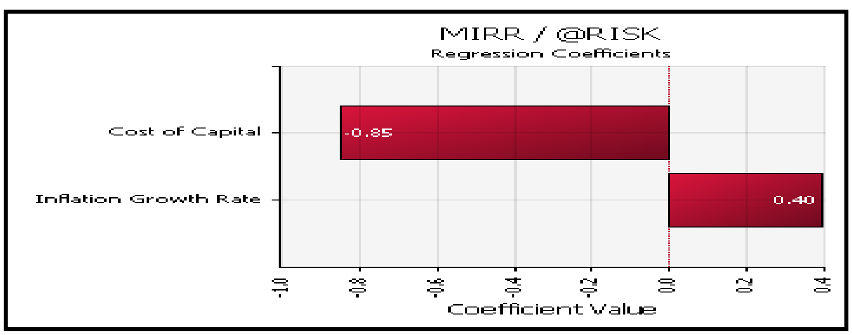


FIGURE 6
TORNADO CHART OF MIRR FOR LASALLE



For both projects, the cost of capital is the greater influence on the MIRR and subsequently the project's expected future cash flows. The regression gives the expected negative sign for the cost of capital and positive sign for inflation growth rate.

A major conjecture of this paper is that the investment strategy is better served by implementing a MIRR technique over the traditional IRR and more than that, the additional step of stratifying the MIRR (SMIRR) is a valuable tool in decision making process under conditions of uncertainty. The SMIRR results are presented below in Table 9.

TABLE 9
STRATIFYING THE MODIFIED INTERNAL RATE OF RETURN FOR PROJECTS RIVERSIDE AND LASALLE UNDER THE EUROPEAN PUT OPTION

European Put Option:	Project Riverside	Project LaSalle
Relative Proportions of the MIRR		
From operational (NOI) cash flows	21.14%	28.72%
From reversion (RV) cash flow	78.86%	71.28%

Note: European Put Option – based on reversion only at the end of the holding term.

ANALYSIS OF EMPIRICAL RESULTS

The decision regarding investing in Project Riverside and Project LaSalle is being made under a mutually exclusive investment situation. In the DCF framework, Project Riverside is preferred as its positive NPV of $16,878,773 is greater than the $11,611,930 NPV of Project LaSalle. The Monte Carlo resulted in both projects showing potential negative values that would have been hidden under a static analysis but comes to life with the creation of the probability distributions for both projects.

TABLE 10
TWO KEY PROBABILITIES OF THE NPV AND MIRR

Probability	Riverside	LaSalle
Probability NPV > 0	**74.35%**	**63.71%**
Probability MIRR > Cap Rate	**81.93%**	**71.83%**

Table 10 reports two probabilities regarding the distributions. Both projects reflect strong probabilities of positive net present values. It can be seen in Table 10 that the their respective probabilities that the MIRR exceeds the Cap Rate is approximately 82% (Riverside) and 72% (LaSalle).

In order to further clarify the risk return conflict between the two projects, an NPV comparison and resulting MIRR stratification analysis is presented in the following table:

TABLE 11
SUBSTITUTION OF MIRR INTO DCF ANALYSIS AND THE RESULTING NPV AND MIRR STRATIFICATION

Projects	NPV based on Capitalization Rate	NPV based on MIRR	SMIRR
Riverside:			
Operational Cash Flows	32,801,091	30,720,164	21.14%
Reversion Cash Flow	128,077,347	114,629,673	78.86%
Total Cash Flows	160,878,438	145,349,837	
Cost	(143,999,995)	(143,999,995)	
NPV	16,878,443	1,349,842	
LaSalle:			
Operational Cash Flows	33,533,346	31,611,813	28.78%
Reversion Cash Flow	86,828,485	78,464,174	71.28%
Total Cash Flows	120,361,831	110,075,987	
Cost	(108,749,900)	(108,749,900)	
NPV	11,611,931	1,326,087	

The process starts with the calculation of the MIRR and subsequently placing it back into the DCF model and creating a new set of cash flows. The justification for resubmitting the MIRR back into the DCF is based on its conservative traits. It can be seen in Table 10 that the Monte Carlo Simulation calculated an approximate 82% probability that Riverside's MIRR will be greater that it's cost of capital while Project LaSalle had an approximate 72% probability. The NPV differential between Riverside and LaSalle is dramatically different when using the cost of capital versus the MIRR. When cost of capital is used, Project Riverside's NPV is approximately 45% greater than that of Project LaSalle. However, when the MIRR is substituted into the NPV calculations, Projects Riverside's NPV is only approximately 2% higher than LaSalle's. The two mutually exclusive projects have now been brought into a scenario of possible indifference. The deciding factor will be an additional measurement of risk associated with the operational and terminal cash flows, accomplished through the stratification of the MIRR. At the heart of this stratification technique is the ability to find the relative proportion of the components of the cash flow payoff as influenced by the MIRR that has been substituted into the DCF process and to view the timing and/or magnitude of the resulting project's net operational and reversion cash flows. Table 11 reports that the SMIRR of Project Riverside is approximately 21% for its operational cash flows while approximately 79% for its terminal cash flow. Contrast this with Project LaSalle. Here, the SMIRR is approximately 29% for the operational cash flows and approximately 71% for the terminal value. It is anticipated that a relative greater uncertainty is associated with the timing of future cash flows beyond the initial holding period for a project (i.e., lease term agreement). The decision process has finally come down to a point where the NPV itself does not tell the definitive story and it is the SMIRR percentages with their risk implications that can decide which project to accept. While the DCF process started out with a clear winner in Project Riverside, with the introduction of the MIRR and the subsequent measurement of future risk by the SMIRR, Project LaSalle is a viable choice in the decision process.

CONCLUSION

Based on actual real estate data, this paper presents a mutually exclusive case between two similar office buildings located in metro downtown Chicago. It is posited that management/investors are actually evaluating a real put option. Specifically, they face a European Put Option, where reversion (i.e, selling or abandonment of property) can only occur at the end of the lease term. This paper introduces stochastic modeling, in the form of Monte Carlo Simulation, to supplement the static discounted cash flow (DCF) technique. Advancing the financial literature, the stratification of the modified internal rate of return (SMIRR) method is implemented to further assess the uncertainty associated with expected operating and final reversion cash flows.

Project Riverside started out as being the obvious choice in a mutually exclusive situation and evolved into a not-so-obvious choice. The MIRR is introduced as a more efficient alternative to the IRR. It is used to 'recalibrate' the DCF analysis to arrive at a more conservative NPV. Next, the stratification of the MIRR weighs the influence of both the expected future cash flows of the project and its terminal value. Specifically, Project Riverside's stratified MIRR reveals a comparatively stronger influence from its discounted terminal value. The latter cash flow is most fraught with risk as investors are forced to forecast well beyond the terminal value date. Project LaSalle has a relatively lower emphasis on discounting its terminal value and hence greater weight is associated with its expected short-term future cash flows. These immediate cash flows are inherently less risky.

The Monte Carlo methodology revealed a greater probability of a negative NPV for Riverside than LaSalle, the MIRR of LaSalle exceeds that of Riverside, the reintroduction of the MIRR into the DCF analysis found the two real estate projects comparatively close, and the subsequent stratification of the MIRR underscored Riverside's relatively higher risk terminal cash flow. Taken together, the selection of LaSalle over Riverside, the latter initially being the more likely winner, turns out to be a viable selection in this real estate put option analysis.

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