The Psychological Explanation of Asset Price Overreaction and Underreaction to New Information: Representativeness Heuristic and Conservatism Bias

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In the model of an asset market with strategic interaction among traders, this paper proves that the occurrence of asset price overreaction and underreaction to new information depends on the proportion of conservatism traders, proportion of heuristic traders, degree of conservatism bias, degree of representativeness heuristic and the number of traders in the market. Specifically, the asset price overreacts to good news and underreacts to bad news when the total representativeness heuristic minus the total conservatism bias in the market is greater than zero but less than one; otherwise, the asset price underreacts to good news and overreacts to bad news.

INTRODUCTION

The phenomena of asset price overreaction and underreaction to new information have been identified by a large body of empirical research in finance. They are not consistent with the efficient market. A few recent behavioral models presented their explanations of these phenomena. For example, Daniel, Hirshleifer and Subrahmanyam (1998) shows that self-attribution bias can raise investors' confidence through the confirmation of their private information or past success. This increased confidence pushes the prices of the past winning stocks above their fundamental values, which eventually is reversed as prices revert to their fundamentals. Barberis, Shleifer and Vishny (1998) build a model based on psychological biases (namely conservatism and representativeness heuristic) to generate the asset price overreaction and underreaction to new information. In their model, the asset price underreaction to new information is generated in a manner which is consistent with conservatism; and the asset price overreaction to new information is the result of representativeness heuristic. Hong and Stein (1999) uses the gradual diffusion of information across the population to explain the asset price underreaction to new information; and they attribute the technical traders' extrapolation based on past prices to the cause of the asset price overreaction to new information. Douks and McKnight (2005) and Jegadeesh and Titman (2001) empirically test and support the predictions of the behavioral models.

This paper extends the model of Kyle (1985) and attempts to explain the phenomena of asset price overreaction and underreaction to new information using the psychological biases, namely conservatism and representativeness heuristic. This paper views conservatism and representativeness as two popular types of psychological biases among traders. For example, the strategies of technical traders' extrapolation based on the past patterns are popularly used among traders and they are examples of representativeness heuristic. Also, a lot of other strategies used by traders resemble the characteristics of conservatism.

Hence, this paper sets up a one-period model of an asset market with one asset and one market maker. The payoff for the asset is unknown but all traders (rational, conservatism and heuristic traders) receive an informational signal about the asset's payoff before any trade takes place.

Conservatism traders exhibit conservatism, which is a type of psychological bias well documented in psychological literature. Conservatism traders are slow to update their belief about the mean and variance of the asset's payoff. As a result, conservatism traders have a smaller (larger) conditional mean about the asset's payoff than rational traders do when the informational signal is larger (smaller) than the expected asset's payoff. Heuristic traders exhibit representativeness heuristic. This is another type of psychological bias identified in the psychologists' experiments. Heuristic traders with representativeness place too much weight on the current information and too little weight on their prior knowledge. Consequently, heuristic traders have a larger (smaller) conditional mean about the asset's payoff than rational traders do when the informational signal is larger (smaller) than the expected asset's payoff.

All traders submit their market orders to the market maker. The market maker sets the asset price equal to the expected asset's payoff conditional on the observed aggregate demand. All traders are risk neutral. The traders' market orders are generated from the maximization of their expected profit after observing the informational signal, taking into their impact on the asset price and on the other traders' market orders.

The equilibrium in the asset market is essentially a Nash equilibrium. In the equilibrium, the occurrence of asset price overreaction and underreaction to new information depends on the proportion of conservatism traders, the proportion of heuristic traders, the degree of conservatism bias, the degree of representativeness heuristic and the number of traders in the market. Define the total representativeness heuristic as the total number of traders multiplying the proportion of heuristic traders and multiplying the degree of representativeness heuristic. Similarly, define the total conservatism bias as the total number of traders multiplying the proportion of conservatism traders and multiplying the degree of conservatism bias. Hence, the results of the paper can be stated as follows. The asset price overreacts to good news and underreacts to bad news when the total representativeness heuristic minus the total conservatism bias in the market is greater than zero but less than one; otherwise, the asset price underreacts to good news and overreacts to bad news.

The reasons behind the results are as follows. When the total representativeness heuristic minus the total conservatism bias in the market is less than one, rational traders' best strategy (or the equilibrium strategy) is to buy the asset in responding to good news. Furthermore, if the total representativeness heuristic is greater than the total conservatism bias in the market, then the impact on the asset price coming from heuristic traders dominates that coming from conservatism traders in the market. Due to representativeness heuristic, heuristic traders have a higher conditional mean about the asset's payoff than rational traders when the informational signal indicates good news. This causes the asset price to be higher than what it would be if the market consists of only rational traders. Since rational traders are buying the asset in responding to good news, the asset price overreacts to good news when the total representativeness heuristic minus the total conservatism bias in the market is greater than zero but less than one. On the other hand, if the total conservatism bias is greater than the total representativeness heuristic in the market, then the impact on the asset price coming from conservatism traders dominates that coming from heuristic traders in the market. Due to conservatism bias, conservatism traders have a lower conditional mean about the asset's payoff than rational traders when the informational signal indicates good news. Hence, the asset price is driven up not as high as it would be if the market consists of only rational traders, who are buying the asset in responding to good news. In other words, the asset price underreacts to good news when the total conservatism bias is greater than the total representativeness heuristic in the market.

However, when the total representativeness heuristic minus the total conservatism bias in the market is greater than one, rational traders' equilibrium strategy is to sell the asset in responding to good news. Furthermore, in this case, the impact on the asset price coming from heuristic traders dominates that coming from conservatism traders. Since heuristic traders have a higher conditional mean about the asset's payoff than rational traders when the informational signal indicates good news, the asset price will not be pushed down as low as it would be if the market consists of only rational traders. In other words, the asset price underreacts to good news when the total representativeness heuristic minus the total conservatism bias in the market is greater than one.

The similar intuitions to the above can be applied to explain the results of the asset price overreaction and underreaction to bad news.

The results of this paper suggest that the asset price more likely overreacts to good news and underreacts to bad news in a small market than in a large market; and the asset price more likely underreacts to good news and overreacts to bad news in a large market than in a small market.

The remainder of this paper consists of three sections. The next section presents the model. The analysis and the results are in Section 3. Section 4 concludes the paper.

THE MODEL

Consider a one-period model of an asset market with one asset and one market maker. The market maker supplies the liquidity to the market. The cost of doing so is assumed to be zero for simplicity. All traders submit their market orders for the asset to the market maker. It is common belief that the payoff of the asset is normally distributed with the mean of $\bar{\theta}$ and variance of σ_θ^2. No trader knows the payoff of the asset but they receive an informational signal about the asset's payoff before any trade occurs. This informational signal is modeled as $S = \theta + \in$ where $\in$ is normally distributed with the mean of zero and variance of $\sigma_\in^2$. The random variables θ and $\in$ are independent. The informational signal is considered as good news if $S > \bar{\theta}$ and it is considered as bad news if $S < \bar{\theta}$.

There are three types of traders: rational traders, conservatism traders and heuristic traders. After receiving the informational signal about the asset's payoff, rational traders update their conditional mean about the asset's payoff according to the following:

$$E(\theta \mid (S, r)) = \bar{\theta} + \eta(S - \bar{\theta}), \tag{1}$$

where r indicates rational traders and $\eta = \dfrac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\in^2}$. Since random variables θ and S are jointly normally distributed, equation (1) follows from the result of Theorem 1 in the appendix.

Conservatism traders exhibit conservatism bias. Conservatism is the type of behavioral bias identified in psychological experiments (see Edwards (1968)). Traders with the conservatism bias are slow to update their belief about the asset's payoff relative to rational traders. Hence, conservatism traders' conditional mean for the payoff of the asset is modeled as the summation of their prior knowledge plus the partial adjustment towards the rational traders' conditional mean about the asset's payoff. Specifically,

$$E = (\theta \mid (S, c)) = \bar{\theta} + m_c\left(E_r(\theta \mid S) - \bar{\theta}\right) = \bar{\theta} + m_c\eta(S - \bar{\theta}), \tag{2}$$

where the parameter c indicates conservatism traders and $m_c \in (0,1)$. If the parameter m_c equals to one, then conservatism traders become rational traders. The degree of conservatism bias is measured by $1 - m_c$. Hence, the lower the parameter m_c, the greater is the traders' conservatism bias. In addition,

$$E_c(\theta \mid S) \le E_r(\theta \mid S) \text{ if } S \ge \bar{\theta}\text{; if } S < \bar{\theta}\text{, then } E_c(\theta \mid S) > E_r(\theta \mid S)$$

Heuristic traders exhibit representativeness. The representativeness heuristic is a type of behavioral bias. It is well documented in the psychologists' experiments (see Kahneman and Tversky (1973), Tversky and Kahneman (1974), and Grether (1980)). Traders with representativeness place too much weight on the current information and too little weight on their prior knowledge when they update their beliefs about the asset's payoff. Hence, heuristic traders' updated conditional mean of the asset's payoff is modeled as the following:

$$E = (\theta \mid (S,h)) = \bar{\theta} + m\left(E_r(\theta \mid S) - \bar{\theta}\right) = \bar{\theta} + m_h \eta (S - \bar{\theta}), \tag{3}$$

where h indicates heuristic traders and $m_h > 1$. If the parameter m_h equals to one, then heuristic traders become rational traders. The degree of representativeness is measured by $m_h - 1$. Hence, the higher above 1 the parameter m_h, the greater degree of representativeness heuristic. Furthermore, equation (3) suggests the following: The conditional mean of the asset's payoff for heuristic traders is larger than that for rational traders when the informational signal indicates good news (i.e., $S > \bar{\theta}$); and it is smaller than that for rational traders when the informational signal indicates bad news (i.e., $S < \bar{\theta}$). Here, heuristic traders place too much weight on the informational signal and not enough weight on their prior beliefs. This characterization of heuristic traders' behavior is consistent with representativeness.

There are N traders in total in the market. Denote the proportion of traders being conservatism traders and heuristic traders as f_c and f_h, respectively, where $f_c \in [0,1]$ and $f_h \in [0,1]$.

The market maker behaves competitively. After receiving the aggregate demand of all traders, he sets the asset price equal to the expected asset's payoff conditional on the observed aggregate demand for the asset. The asset price is denoted as P and the aggregate demand is denoted as D. Hence, the asset price is determined by the following equation:

$$P = E(\theta \mid D) \tag{4}$$

The equilibrium is characterized by the following: (1) Given the asset pricing rule stated in equation (4) and taken into account the impact of his market order on the asset price and on other traders' market orders, trader i, where $i \in \{1,2,..,N\}$, of type j, j = r, c, h, chooses his market order (denoted as X_{ij}) to maximize his expected profit

$$\max_{X_{ij}} \left[E(\theta \mid (S,j)) - E(P \mid (S, X_{ij}))\right] X_{ij}, \tag{5}$$

where $E(\theta \mid (S,j)) = E(\theta \mid (S,r))$ if j=r; $E(\theta \mid (S,j)) = E(\theta \mid (S,c))$ if j=c; $E(\theta \mid (S,j)) = E(\theta \mid (S,h))$ if j=h; and $E(P \mid (S, X_{ij})) = E(\theta \mid D)$. (2) Given all the market orders coming from all traders, the market maker sets the asset price equal to the expected asset's payoff conditional on the observed aggregate demand according to equation (4).

Note that all traders are risk neutral in this model. The equilibrium here is essentially a Nash equilibrium.

The following assumes that the equilibrium strategies for rational, conservatism and heuristic traders are linear functions of their informational signal. Then later they are solved and confirmed to be equilibrium strategies for the traders.

Denote the total number of rational, conservatism and heuristic traders as N_{r}, N_{c} and N_{h}, respectively. Hence, $N = N_r + N_h + N_C$. Assume that the equilibrium strategies for rational, conservatism and heuristic traders are linear functions of their informational signal and they are as follows: for i = 1, 2, .., N_r,

$$X_{ir} = a_{ir} + b_{ir} S \ , \tag{6}$$

for i=1,2,..,N_c,

$$X_{ic} = a_{ic} + b_{ic} S \ , \tag{7}$$

and for i=1,2,..,N_h,

$$X_{ih} = a_{ih} + b_{ih} S \ . \tag{8}$$

Also assume that the equilibrium asset price follows the linear pricing rule:

$$\mathrm{P} = \mu + \lambda \mathrm{D}, \tag{9}$$

where $D = \sum_{i=1}^{N_r} X_{ir} + \sum_{i=1}^{N_c} X_{ic} + \sum_{i=1}^{N_h} X_{ih}$; and all the coefficients μ, λ; a_{ir} and b_{ir} for (i=1,2,..,N_r); a_{rc}, and b_{ic} (for i=1,2,..,N_c); a_{rh}, and b_{ih} (for i=1,2,..,N_h) are to be determined later.

Substituting equations (1) through (9) into the optimization problem (5), it follows that the first order condition for the optimization problem (5) is as follows:

$$\overline{\theta} + \eta(S - \overline{\theta}) - \mu - \lambda(2X_{ir} + \sum_{\substack{n=1 \\ n \neq i}}^{N_r} (a_{nr} + b_{nr} S) + \sum_{n=1}^{N_h} (a_{nh} + b_{nh} S) + \sum_{n=1}^{N_c} (a_{nc} + b_{nc} S)) = 0 \tag{10}$$

$$\overline{\theta} + m_c \eta(S - \overline{\theta}) - \mu - \lambda(2X_{kc} + \sum_{\substack{n=1 \\ n \neq k}}^{N_c} (a_{nc} + b_{nc} S) + \sum_{n=1}^{N_r} (a_{nr} + b_{nr} S) + \sum_{n=1}^{N_h} (a_{nh} + b_{nh} S)) = 0 \tag{11}$$

and

$$\overline{\theta} + m_h \eta(S - \overline{\theta}) - \mu - \lambda(2X_{lh} + \sum_{\substack{n=1 \\ n \neq k}}^{N_h} (a_{nh} + b_{nh} S) + \sum_{n=1}^{N_r} (a_{nr} + b_{nr} S) + \sum_{n=1}^{N_c} (a_{nc} + b_{nc} S)) = 0 \tag{12}$$

Again, substituting equations (6) through (8) into equation (10), (11) and (12) respectively, it follows that

$$a_{ij} = \frac{\overline{\theta} - \mu - m_j \eta \overline{\theta}}{\lambda} - A \tag{13}$$

and

$$b_{ij} = \frac{m_j \eta}{\lambda} - B \ , \tag{14}$$

where $A = \sum_{n=1}^{N_r} a_{nr} + \sum_{n=1}^{N_c} a_{nc} + \sum_{n=1}^{N_h} a_{nh}$; $B = \sum_{n=1}^{N_r} b_{nr} + \sum_{n=1}^{N_c} b_{nc} + \sum_{n=1}^{N_h} b_{nh}$; $m_j = 1$ if j = r; $m_j = m_c$ if j=c; and $m_j = m_h$ if j=h.

Notice from equations (13) and (14), that for $i' \neq i$, $a_{ij} = a_{i'j}$ and $b_{ij} = b_{i'j}$ for the same $j \in \{r, h\}$ (the same type of traders). Hence, let $a_{ir} = a_r$, $b_{ir} = b_r$ when $m_j = 1$; and $a_{ic} = a_c$, $b_{ic} = b_c$ when

$m_j = m_c$; and $a_{ih} = a_h$, $b_{ih} = b_h$ when $m_j = m_h$. Equations (13) and (14) imply the following four equations are true:

$$a_r = \frac{\bar{\theta} - \mu + \eta\bar{\theta} + Nf_c\eta\bar{\theta}(m_c - 1) - Nf_h\eta\bar{\theta}(1 - m_h)}{\lambda(N+1)}, \tag{15}$$

$$a_c = \frac{\bar{\theta} - \mu + \eta\bar{\theta} + Nf_c\eta\bar{\theta}(m_c - 1) - Nf_h\eta\bar{\theta}(1 - m_h)}{\lambda(N+1)} - \frac{\eta\bar{\theta}(m_c - 1)}{\lambda} \tag{16}$$

$$a_h = \frac{\bar{\theta} - \mu + \eta\bar{\theta} + Nf_c\eta\bar{\theta}(m_c - 1) - Nf_h\eta\bar{\theta}(1 - m_h)}{\lambda(N+1)} + \frac{\eta\bar{\theta}(1 - m_h)}{\lambda} \tag{17}$$

$$b_r = \frac{\eta - Nf_c\eta(m_c - 1) - Nf_h\eta(m_h - 1)}{\lambda(N+1)} \tag{18}$$

$$b_r = \frac{\eta - Nf_c\eta(m_c - 1) - Nf_h\eta(m_h - 1)}{\lambda(N+1)} + \frac{\eta(m_c - 1)}{\lambda} \tag{19}$$

and

$$b_h = \frac{\eta - Nf_c\eta(m_c - 1) - Nf_h\eta(m_h - 1)}{\lambda(N+1)} + \frac{\eta(m_h - 1)}{\lambda} \tag{20}$$

Using equation (4),

$$P = E\big(\theta \mid A + BS + x = D\big) = \bar{\theta} + \frac{B\sigma_\theta^2}{B^2\sigma_S^2 + \sigma_x^2}(D - A - B\bar{\theta}) \tag{21}$$

Using equations (9) and (21) along with the definitions of A and B, one can show that

$$\mu = \bar{\theta}, \tag{22}$$

and

$$\lambda = \frac{B\sigma_\theta^2}{B^2\sigma_S^2 + \sigma_x^2} \tag{23}$$

Where $\sigma_S^2 = \sigma_\theta^2 + \sigma_\in^2$.

Note that λ is determined by equation (23) and the positive root from equation (23) is used to ensure that the second order condition of the optimization problem (5) holds and ensure that the equilibrium price is increasing in the total demand for the asset.

Denote the equilibrium market orders for rational, conservatism and heuristic traders as X_r, X_c and X_h, respectively. Using equations (15) through (23), and equations (6) through (9), the equilibrium strategies for rational, conservatism and heuristic traders and the equilibrium asset price for the market maker are computed as the following:

$$X_r = \frac{\eta\big(1 + Nf_c(1 - m_c) - Nf_h(m_h - 1)\big)(S - \bar{\theta})}{\lambda(N+1)} \tag{24}$$

$$X_r = \frac{\eta\big(1 + Nf_c(1 - m_c) - Nf_h(m_h - 1)\big)(S - \bar{\theta})}{\lambda(N+1)} + \frac{(m_c - 1)\eta(S - \bar{\theta})}{\lambda} \tag{25}$$

$$X_r = \frac{\eta\left(1 + Nf_c(1-m_c) - Nf_h(m_h - 1)\right)(S - \overline{\theta})}{\lambda(N+1)} + \frac{(m_h - 1)\eta(S - \overline{\theta})}{\lambda} \tag{26}$$

and

$$P = \overline{\theta} + \frac{\eta\left(N + Nf_h(m_h - 1) - Nf_c(1 - m_c)\right)\left(S - \overline{\theta}\right)}{N+1} \tag{27}$$

where λ is determined in equation (23).

Here, the terms $Nf_h(m_h - 1)$ and $Nf_c(1 - m_c)$ measure the total representativeness heuristic in the market and the total conservatism bias in the market, respectively. They both affect the demand coming from rational, conservatism and heuristic traders, and the asset price. According to equation (24), if the total representativeness minus the total conservatism bias in the market exceeds one, then rational traders will sell (buy) the asset in responding to good news (bad news); otherwise, rational traders will buy (sell) the asset in responding to good news (bad news).

If rational traders buy the asset in responding to good news, then equation (25) implies that in responding to good news, conservatism traders will sell or buy the asset less aggressively than rational traders. This is due to the fact that the conditional mean about the expected asset's payoff for conservatism traders is smaller than that for rational traders when the informational signal is greater than the expected asset's payoff. If rational traders sell the asset, conservatism traders will sell the asset more aggressively than rational traders (due to equation (25)). This is because the conservatism traders' lower conditional mean about the expected asset's payoff causes conservatism traders to sell the asset more aggressively than rational traders do. On the other hand, in responding to bad news, conservatism traders will buy the asset more aggressively if rational traders buy the asset; and conservatism traders will buy or sell the asset less aggressively if rational traders sell the asset. This is because conservatism traders have higher conditional mean about the asset's payoff than rational traders do when the informational signal is smaller than the expected asset's payoff.

In addition, note from equation (26) that in responding to good news, heuristic traders will buy the asset more aggressively when rational traders buy the asset; and heuristic traders will buy or sell the asset less aggressively when rational traders sell the asset. This is because heuristic traders have a higher conditional mean about the expected asset's payoff than rational traders do when the informational signal is greater than the expected asset's payoff. On the other hand, in responding to bad news, equation (26) implies that heuristic traders will sell or buy the asset less aggressively when rational traders buy the asset; and heuristic traders will sell the asset more aggressively when rational traders sell the asset. This is due to the fact that heuristic traders have a lower conditional mean about the asset's payoff than rational traders do when the informational signal is smaller than the expected asset's payoff.

Finally, note from equation (27) that the asset price is affected by the total conservatism bias and total representativeness heuristic in the market.

The following section analyzes the impact of the psychological biases on the occurrence of the asset price overreaction or underreaction to good news or bad news.

THE RESULTS

This section presents detailed analysis relating the occurrence of the asset price overreaction or underreaction to conservatism and representativeness heuristic.

In this framework, the asset price overreaction to new information occurs when the asset price, in responding to new information, is higher (lower) than what it would be if the market consists of only rational traders ($f_c = 0$ and $f_h = 0$), who are buying (selling) the asset; otherwise, the asset price underreaction to new information occurs.

Using equation (27), the asset price (denoted as P_r) for the asset market with only rational traders (i.e., $f_c = 0$ and $f_h = 0$) is computed as

$$P_r = \overline{\theta} + \frac{N\eta(S - \overline{\theta})}{N + 1} \tag{28}$$

Using equation (28), equation (27) is rewritten as the following:

$$P - P_r = \frac{\eta\left(Nf_h(m_h - 1) - Nf_c(1 - m_c)\right)\left(S - \overline{\theta}\right)}{N + 1} \tag{29}$$

Hence, if the total representativeness heuristic (i.e., $Nf_h(m_h - 1)$) is larger than the total conservatism bias in the market (i.e., $Nf_c(1 - m_c)$), then the total representativeness heuristic in the market impact the asset price more than the total conservatism bias in the market. Hence, in responding to good news, $P > P_r$; and in responding to bad news, $P < P_r$. This is due to the fact that, when the informational signal indicates good news (bad news), heuristic traders have a higher (lower) conditional mean about the asset's payoff than rational traders; while conservatism traders have a lower (higher) conditional mean about the asset's payoff than rational traders. However, the asset price overreaction to new information occurs only when $P > P_r$ while rational traders are buying the asset; or $P < P_r$ while rational traders are selling the asset.

On the other hand, if the total representativeness heuristic (i.e., $Nf_h(m_h - 1)$) is smaller than the total conservatism bias in the market (i.e., $Nf_c(1 - m_c)$), then, the total conservatism bias in the market impacts the asset price more than the total representativeness heuristic. Hence, in responding to good news, $P < P_r$; and in responding to bad news, $P > P_r$. This is due to the fact that when the informational signal indicates good news (bad news), conservatism traders have a lower (higher) conditional mean about the asset's payoff than rational traders; while heuristic traders have a higher (lower) conditional mean about the asset's payoff than rational traders. However, the occurrence of the asset price underreaction to new information requires either $P > P_r$ while rational traders are selling the asset or $P < P_r$ while rational traders are buying the asset.

The following discusses rational traders' best strategies in responding to good news, then it examines how the asset price responds to good news ($S > \overline{\theta}$).

In respond to good news, equation (25) implies that conservatism traders will sell or buy the asset less aggressively if rational traders buy the asset; alternatively, conservatism traders will sell the asset more aggressively if rational traders sell the asset. While equation (26) implies that, in responding to good news, heuristic traders will buy the asset more aggressively if rational traders buy the asset; alternatively heuristic traders will buy or sell the asset less aggressively if rational traders sell the asset. However, in responding to good news, when $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$, rational traders' best replying strategy to the strategies of conservatism and heuristic traders is to sell the asset (see equation (24)). The reason for this is as follows. Since in responding to good news, if rational traders buy the asset, then heuristic traders will buy the asset more aggressively (see equation (26)) while conservatism traders will sell or buy the asset less aggressively. Given the fact that the impact on the asset price coming from heuristic traders is larger than that coming from conservatism traders, this means that the asset price will be driven up excessively high, which will in turn hurt rational traders. On the other hand, if rational traders sell the asset, heuristic traders will buy or sell the asset less aggressively (see equation (26)) although conservatism traders sell the asset more aggressively (see equation (25)), with the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$ implying that the impact on the asset price coming from heuristic traders is larger than that coming from conservatism traders. Hence, there is still room for rational traders to sell the asset and do better than the alternative strategy of buying the asset. In other words, in responding to good news, rational traders' best strategy in the equilibrium is to sell the asset when $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$.

However, when $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$, rational traders' best replying strategy to the strategies of both conservatism and heuristic traders is to buy the asset instead of selling the asset (see equation (24)). The reason for this is as follows. If rational traders sell the asset, then conservatism traders will sell the asset more aggressively while heuristic traders will buy or sell the asset less aggressively. This, together with the fact that the impact on the asset price coming from conservatism traders is either larger or slightly smaller than that coming from heuristic traders (i.e., $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$), implies that the asset price can be pushed down excessively low, which will in turn hurt rational traders. However, if rational traders buy the asset price, then conservatism traders will sell or buy the asset less aggressively. This will drive up the asset price but it will not hurt rational traders as much as selling the asset as discussed in the previous case. Hence, in responding to good news, rational traders' best strategy in the equilibrium is to buy the asset when $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$.

Now, the following further examines how the asset price in the equilibrium behaves in responding to good news.

As discussed above, when the total representativeness heuristic in the market minus the total conservatism bias in the market is less than one (i.e., $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$), rational traders' best strategy is to buy the asset in responding to good news (also, see equation (24)); in addition, when the total representativeness heuristic in the market is greater than the total conservatism bias in the market (i.e. $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 0$), the asset price $P > P_r$ (see equation (29)). Therefore, if the total representativeness heuristic in the market minus the total conservatism bias in the market is greater than zero but less than one (i.e. $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$,), then the asset price is driven up higher than what it would be when the market consists of only rational traders (along with the market maker). In other words, the asset price overreacts to good news. The alternative inequality of $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$ includes $Nf_h(m_h - 1) < Nf_c(1 - m_c)$ and $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$. With the inequality of $Nf_h(m_h - 1) < Nf_c(1 - m_c)$, rational traders' best strategy is to buy the asset in responding to good news (as discussed above) but the asset price $P < P_r$ (see equations (24) and (29)). This means the asset price is not driven up as high as it would be if the market consists of only rational traders who are buying the asset. In other words, the asset price underreacts to good news. In addition, with the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$, rational traders' best strategy is to sell the asset in responding to good news (as discussed above) and the asset price $P > P_r$ (see equations (24) and (29)). This means that the asset price is not pushed down as low as it would be when the market consists of only rational traders. That is, the asset price underreacts to good news.

The above results are summarized in the following proposition.

Proposition 1

The asset price overreacts to good news when $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$. And the asset price underreacts to good news when $f_h(m_h - 1) - f_c(1 - m_c) < 0$ or $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$.

The results of Proposition 1 basically say the following: The asset price overreacts to good news when the total representativeness heuristic minus the total conservatism bias in the market lies in the interval of (0,1); otherwise, the asset price underreacts to good news.

In addition, the results of Proposition 1 suggest some empirical implications. Note that it is more likely for $f_h(m_h - 1) - f_c(1 - m_c)$ to fall into the interval $(0,(\frac{1}{N}))$ if the number of traders in the market is smaller (or $\frac{1}{N}$ is larger). Hence, the results in Proposition 1 suggest that the asset price more likely overreacts to good news in a small market than in a large market; the asset price more likely underreacts to good news in a large market than in a small market.

The remaining section examines rational traders' best strategies in responding to bad news, then it discusses how the asset price reacts to bad news.

In responding to bad news, equation (25) implies that conservatism traders will buy the asset more aggressively if rational traders buy the asset; alternatively conservatism traders will buy or sell the asset less aggressively if rational traders sell the asset. Also, equation (26) implies that heuristic traders will sell or buy the asset less aggressively if rational traders buy the asset; alternatively heuristic traders will sell the asset more aggressively if rational traders sell the asset. When $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$ holds, rational traders' best replying strategy to the strategies of conservatism and heuristic traders is to buy the asset. The reasons for this are as follows. Since the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$ implies that the impact of heuristic traders on the asset price dominates that of conservatism traders, if rational traders sell the asset, then heuristic traders will sell the asset more aggressively while conservatism traders will buy or sell the asset less aggressively. This, together with the fact that the impact on the asset price coming from heuristic traders dominates that coming from conservatism traders, implies that the asset price will be pushed down excessively low, which will in turn hurt rational traders. However, if rational traders buy the asset, then heuristic traders will sell or buy the asset less aggressively while conservatism traders will buy the asset more aggressively. Since the impact on the asset price coming from heuristic traders is larger than that coming from conservatism traders, this means that there is still room in the market for rational traders to buy the asset and not to push the asset price excessively high to hurt themselves as much as selling the asset. Hence, rational traders' buying strategy does better than the alternative strategy of selling the asset. In other words, in responding to bad news, rational traders' best strategy in the equilibrium is to buy the asset when $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$.

However, if the alternative inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$ holds, then rational traders' best replying strategy to the strategies of conservatism and heuristic traders is to sell the asset. The reasons behind this are as follows. With the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$, the impact on the asset price of heuristic traders either is dominated by that of conservatism traders (i.e., $f_h(m_h - 1) < f_c(1 - m_c)$) or slightly dominates that of conservatism traders (i.e., $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$). Under either case, if rational traders buy the asset, then conservatism traders will buy the asset more aggressively while heuristic traders will sell or buy the asset less aggressively. This, together with the fact that the impact on the asset price coming from conservatism traders is larger or slightly smaller than that coming from heuristic traders, implies that the asset price can be pushed excessively high. This will in turn hurt rational traders. Alternatively, if rational traders sell the asset, then conservatism traders will buy or sell the asset less aggressively while heuristic traders will sell the asset more aggressively. This will not push the asset price excessively low to hurt rational traders as much as buying the asset since conservatism traders impact the asset price more or slightly less than heuristic traders. Hence, rational traders' selling strategy does better than the alternative buying strategy. In other words, rational traders' best strategy in the equilibrium is to sell the asset in responding to bad news when $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$.

Now, the following discusses how the resulting equilibrium asset price responds to bad news.

Assuming that the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$ holds. This means that rational traders' best strategy is to buy the asset in responding to bad news as discussed from the above. This further implies that $P > P_r$ (see equation (29)). This means that the asset price is driven up higher than what it would be if the market consists of only rational traders. In other words, the asset price overreacts to bad news when $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$.

On the other hand, if $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$ holds, then as discussed above, rational traders' best strategy is to sell the asset in responding to bad news. In this case, if $f_h(m_h - 1) > f_c(1 - m_c)$, equation (29) implies that $P > P_r$. This means that the asset price is not pushed down as low as it would when the market consists of only rational traders who are selling the asset. In other words, the asset price underreacts to bad news when $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$. Alternatively, if $f_h(m_h - 1) < f_c(1 - m_c)$ holds (naturally, the inequality of $Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$ holds), then equation (29) implies that $P < P_r$. This means that the asset price is pushed down lower than what it would be when the market consists of only rational traders who are selling the asset. In this case, the asset price overreacts to bad news.

The above results are presented in the following proposition.

Proposition 2

The asset price overreacts to bad news when $Nf_h(m_h - 1) - Nf_c(1 - m_c) > 1$ or $f_h(m_h - 1) - f_c(1 - m_c) < 0$. And the asset price underreacts to bad news when $0 < Nf_h(m_h - 1) - Nf_c(1 - m_c) < 1$.

The results of Proposition 1 can also be interpreted as follows. If the total representativeness heuristic minus the total conservatism bias in the market lies in the interval of (0,1), the asset price underreacts to bad news; otherwise, the asset price overreacts to bad news.

In addition, the results of Proposition 2 suggest some empirical implications. Note that the $f_h(m_h - 1) - f_c(1 - m_c)$ is more likely to exceed ($\frac{1}{N}$) when the number of traders in the market is larger (or $\frac{1}{N}$ is smaller). Hence, the asset price more likely overreacts to bad news in a large market than in a small market. Also, since $f_h(m_h - 1) - f_c(1 - m_c)$ is more likely to fall into the interval [0, $\frac{1}{N}$] when the market is smaller (or $\frac{1}{N}$ is larger), the asset price more likely underreacts to bad news in a small market than a large market.

CONCLUSION

This paper attempts to explain the phenomena of asset price overreaction and underreaction to new information using the psychological biases, namely conservatism and representativeness. In this one-period model of an asset market, allowing for strategic interactions among traders, the paper shows that the occurrence of asset price overreaction and underreaction to new information depends on the proportion of conservatism traders, the proportion of heuristic traders, the degree of conservatism bias, the degree of representativeness heuristic and the number of traders in the market. Specifically, the asset price overreacts to good news and underreacts to bad news when the total representativeness heuristic minus the total conservatism bias in the market is greater than zero but less than one; otherwise, the asset price underreacts to good news and overreacts to bad news.

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APPENDIX

Theorem 1
If the random variables X* and Y* are jointly normally distributed, then

$$E(X^* \mid Y^* = Y) = EX^* + \frac{Cov(X^*, Y^*)}{Var(Y^*)}(Y - EY^*)$$

and

$$Var(X^* \mid Y^* = Y) = Var(X^*) - \frac{[Cov(X^*, Y^*)]^2}{Var(Y^*)}$$ (See Hoel, p.200).