An Excel Model of Mortgage Refinancing Decisions for Sensitivity Analysis and Simulation

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This paper advances the use of algebraic formulae in place of amortization schedules in calculating annual interest expenses. The formulae are of great value in alleviating computational burdens in mortgage refinancing analysis. The methodology presented here can be readily imparted to business undergraduates and MBA students taking managerial finance courses. The new formula approach also resolves computational difficulties which appear to have been one of the major reasons why the use of sensitivity analysis and simulation has not become popular in refinancing analysis.

INTRODUCTION

Computing interest expenses in mortgage refinancing by means of traditional amortization schedules is practically impossible unless professional financial engineers are available for assistance.

The purpose of this work is twofold. The first purpose is the use of algebraic formulae in calculating old and new mortgage loan balances, as well as their annual interest expenses, without using the annual amortization schedules. The second purpose is to show how to conduct sensitivity and simulation analysis on Excel without using the advanced features that are not well known to Excel neophytes. Under the new approach, undergraduates and MBA students can attain a deeper understanding of mortgage refinancing analysis because they can solve many exercise problems without much hardship, and can learn to readily implement refinancing analysis in the applied settings.

Mortgage refinancing analysis is a subject in real estate finance. See Valachi (1982), G-Yohannes (1988), and Rose (1992). However, this topic should belong in a financial management course, as is clear from the fact that it is nothing but an application of capital budgeting analysis. However, mortgage refinancing analysis has not been discussed in introductory finance texts; see, for instance, Brealey, Myers, and Allen (2009), Brigham and Ehrhardt (2010), Keown, Martin, Petty, and Scott, Jr. (2010), Gitman (2006), Moyer, McGuigan and Kretlow (2009), and Ross, Westerfield, and Jaffe (2008).

One of the reasons for the exclusion of this issue from introductory finance texts seems in our view to be that calculating interest expenses is too overwhelming, especially for beginning students who are still in the process of learning the basic steps of how to conduct refinancing analysis. This is because it is necessary to prepare amortization schedules, for example for 15-year or 30-year mortgages, to solve realistic refinancing exercises. Notice that there are hundreds and hundreds of entries to compute in completing these amortization schedules.

It is true that the computational burden in mortgage refinancing decisions was lessened considerably thanks to advances in computer technology and the invention of spreadsheet software. Yet, computational burden is still onerous.

Randle and Johnson (1996) are the early workers who discussed the use of Lotus 1-2-3, Quattro Pro, and Excel in mortgage refinancing. However, special credit in this regard should be given to Chen (1997) who presented a complete spreadsheet program written in Lotus 1-2-3. Johnson and Randle (2003) utilized an enhanced version of Excel rather than Lotus 1-2-3 to solve their mortgage refinancing problem on Window 95.

Thus, the use of spreadsheet software per se in refinancing analysis is not news that has academic significance today. Therefore, the logical question to pose is why this paper should be of interest to the finance community. The answer is that the method of refinancing analysis to be presented in this paper has extraordinary features that we believe are new advances which are truly noteworthy.

In recent years, teaching an introductory finance course by using Excel became a new trend in finance pedagogy. They can experiment with any new techniques, such as sensitivity and simulation analysis, which they were not able to try in the past. It is easily possible for an instructor to design an option pricing exercise problem in an applied setting and let students solve it numerically with Excel. They can generate the probability distribution of the net advantage of refinancing (henceforth, NAR), for instance, by constructing a refinancing worksheet on their laptop computers. With Excel, a finance course is no longer just a plethora of abstract theories, but has become a concrete subject which they can intuitively grasp.

Excel advocates maintain that Excel should be integrated into teaching finance because finance students can attain much deeper insight into hard-to-digest finance theories, thus becoming more competent financial managers. See MacDougall and Follows (2006), Bauer, Jr. (2006), and Whitworth (2010).

The first book to introduce Excel as a new pedagogy of teaching finance was *Principles of Finance with Excel* by Benninga (1997). The second was *Advanced Modeling in Finance with Excel VBA* by Jackson and Staunton (2001), which is more advanced than Benninga's. This was followed by Benninga's *Financial Modeling* (2005) published by MIT Press.

Let us provide a brief overview of past studies in refinancing analysis to see how other studies stand in relation to this study. There are many variables involved in mortgage refinancing decisions such as an old and new mortgage interest rate, the life of an older mortgage as well as that of a new mortgage, a variety of origination or settlement costs, tax treatment of discount points, a home owner's income tax bracket, and so forth. See Timmons and Betty (1997) for these variables. See also Bird and McCraw (1993) and Stanton and Wallace (1998) with regard to discount points. A variety of different approaches to mortgage refinancing decisions have been examined by past researchers. The capital budgeting techniques used in refinancing analysis included the net present value method, the internal rate of return method, the payback period method, a variety of breakeven analyses about whether or not to refinance an old by a new, and so forth. Fortin, Michelson, Smith, and Weaver (2007). Hoover noted that various forms of breakeven analysis were utilized in past refinancing analyses. For instance, he considered a payback period to just cover the cost of refinancing by interest tax savings from switching from the higher interest rate on an old mortgage to the interest rate on a new mortgage. The lower the new interest rate the faster the payback period. Hoover (2003).

Application of an option theoretic approach to mortgage refinancing was a new way stimulated by advances in optional price theories. The first attempt in this line in refinancing analysis came from Kau and Keenan (1995). Agarwal, Driscoll, and Laibson (2007) also consider another option theoretic approach to mortgage refinancing. Virmani and Murphy (2010) conclude that a 1% drop in the interest rate differential is a rule of thumb for refinancing in line with the guideline from the option pricing models. Keep in mind that the rule of thumb suggested by many financial advisers used to be 2% in the 1980s. Hence, a rule of thumb is not an absolute figure. See Agarwal, Driscoll, and Laibson (2007).

Several financial economists focused on empirical behavior in mortgage financing decisions. Some of the issues considered were the following. Competitive structures in mortgage markets were found important in refinancing. Home owners' propensity to refinance were found to be greater as a result of competition in markets. Their credit ratings were also found to be another significant variable, along with mortgage rate declines or increases. It was found that home owners cashed out or cashed in refinancing their homes due to a variety of reasons. Mortgage interest rates alone could not fully explain their behavior. Some chose to cash in their home mortgage to upgrade, improve, or expand their homes, or liquefy their homes in response to stock market activities. See Bennett, Peach, and Peristiani (2001) and He and Casey (2010).

An interest rate differential is a well-known decision variable in mortgage refinancing. At the same time, whether or not to refinance also depends on how long a home owner will keep the new mortgage loan. Kalotay, Yang, and Fabozzi (2008) refer to it as the borrowing horizon. A longer borrowing horizon should often permit home owners to enhance their NAR.

Refinancing analysis is no longer just a question of whether to refinance or not to refinance involving computing the NAR once. A sensitivity analysis is necessary in investigating the range of the NAR where borrowing horizons, interest differential, discount points, and so forth are also critical variables which influence refinancing decisions.

Economists used to point out that simulation was not well received in business despite its long history. See Brealey et al (2009). See also Hertz (1964) in this regard. However, simulation is not dead. For instance, Graham and Harvey (2001) report that 15% of major firms utilize sophisticated operations research techniques. A well-known example of such a major firm is Merck. See Nichols (1994).

It seems that simulation might be not utilized by many firms, perhaps due to the fact that they lack in financial and personnel resources rather than the fact that simulation per se is not a useful tool of analysis.

Indeed, Zhang, Gan, Feng, and Xie (2012) present an application of simulation to mortgage refinancing analysis when key variables are regarded as stochastic. Many articles on simulation appeared in the last decade in pedagogic journals in finance. See Ammar, Kim, and Wright (2008), Dow and Newsom (2004), Longstaff and Schwartz (2001), and Whitworth (2008). It appears that simulation is a renewed subject in finance. In our view, the revival of simulation is no accident. One of the reasons is that teaching simulation is no longer as difficult as it used to be. Thanks to Excel, finance professors today can impart to students how simulation can be conducted in classroom settings.

Several final comments are in order. First, this work differs from others in that its focus is on computational and pedagogic issues which have been long-ignored in mortgage refinancing analysis, rather than on economic theories and empirical behavior in mortgage refinancing which has attracted past researchers in financial economics. The numerical efficiency of our algebraic formula approach to mortgage refinancing is a key which enables finance students to learn mortgage refinancing analysis with Excel without undue computational hardship.

To many traditional professors of finance, sensitivity and simulation analysis might be viewed as too arcane to be taught to business undergraduates and MBAs, since some of these professors' backgrounds are non-technical and hence they were less inclined to discuss simulation in introductory finance courses. This appears to be one of the factors which made simulation analysis unpopular in the past.

However, in our view, business students today no longer view simulation as an esoteric technique because they have been well acquainted with it. This is because they are required to take courses in management science and business application software as business core requirements. So, they are properly prepared to learn this subject today.

The algebraic formula method to be introduced in this paper combined with Excel is a major pedagogic advance because finance undergraduates as well as MBAs with a standard knowledge of Excel can be readily trained to conduct sensitivity and simulation analysis in mortgage refinancing decisions.

The organization of this paper is as follows. Section II presents the algebraic formula tables to compute the loan balance and monthly payment of a mortgage loan for any arbitrary time period. A numerical example is provided to illustrate how to use the formulae. Section III presents the algebraic formula to compute annual interest expenses. Also, a numerical example is provided to show how to compute annual interest expenses concretely. Section IV explains how to conduct a mortgage refinancing analysis implemented on an Excel worksheet and how sensitivity analysis can be conducted by using the same worksheet. Section V demonstrates how to conduct a simulation analysis in mortgage refinancing by Excel without using its advanced programming features. The final Section VI is for the summary and concluding remarks.

A NEW APPROACH TO LOAN AMORTIZATION

Matsumoto, Hull, Vineyard, and Kisuule (2010) showed how to derive the loan balance for an arbitrary month for a home mortgage as well as how to compute annual interest expenses on a mortgage loan. Table 1 below presents a mortgage loan balance formula for an arbitrary month t.

TABLE 1
LOAN BALANCE FORMULAE

t	Loan types B_t	the t-th loan balance formulae
1	Ordinary term loan B_t=	$\mathscr{B}(0)\dfrac{\left[(1+i)^n-(1+i)^{t-1}\right]}{\left[(1+i)^n-1\right]}$

Notations:
t=line number
$\mathscr{B}(0)$ =a loan face value where $\mathscr{B}(0)$ is equal to B_1 for all loans except an immediate term loan
B_t=loan balance for the t-th month
i=interest rate per month
h=term of a loan used to determine a monthly payment on a balloon loan
m=number of months deferred
n=term of a loan
t=month t
Note that the table lists all formulae for popular term loans as a matter of information, though they are not utilized in this work.

Table 1 will be followed by Table 2, which is a companion table presenting a monthly payment formula.

TABLE 2
LOAN PAYMENT FORMULAE

t	Loan types	B_t=the t-th loan balance
1	Ordinary term loans	$\mathcal{B}(O)\dfrac{\left[i(1+i)^n\right]}{\left[(1+i)^n-1\right]}$

Notations:
t=line number
$\mathcal{B}(O)$ =the face value of a loan which is equal to B_1 all except an immediate term loan
B_t=loan balance for the t-th month
i=interest rate per month
h=term of a loan used to determine a monthly payment on a balloon loan
m=number of months deferred
n=term of a loan
t=the t-th month

The two tables are of great value since any arbitrary row of an amortization schedule can be generated, once the monthly loan balance and its monthly payment are known. The implication is that an amortization schedule is no longer needed to generate a loan balance and interest expense in mortgage refinancing analysis. It appears useful to provide an exposition of how the algebraic formulae in Tables 1 and 2 can be utilized.

Example 1

Joe obtains a two-year 12% ordinary term loan for $10,000 from the Bank of St. James in March. In this work, the initial loan balance of $10,000 is denoted by $\mathcal{B}(O)$. The 12% here is the annual percentage rate APR. The periodic rate i is 1% per month and with the term to maturity n of 24 months.♦[1]

The amortization schedule of the loan with annual interest expenses is presented in Table 3.

Substitute $\mathcal{B}(O)$ =10,000, i=0.01 and n=6 into the payment formula for an ordinary term loan in Table 2 as follows:

$$P = 10,000\frac{\left[0.01(1.01)^{24}\right]}{\left[(1.01)^{24}-1\right]} = 470.73472 \tag{1}$$

which is the monthly loan payment of this loan.

The next step is to compute the loan balance on the last month using the loan balance formula in Table 1. Substitute $\mathcal{B}(O)$ =10,000, i=0.01,t =24, and n=24 to the mortgage loan balance formula in the row 3rd column of Table 1.

$$B_{24} = 10,000\frac{\left[1.01^{24}-1.01^{(24-1)}\right]}{\left[1.01^{24}-1\right]} = 466.07398 \tag{2}$$

which is exactly equal to its last loan balance in the amortization schedule presented in Table 3. Multiply the above loan balance by 0.01 to derive the interest payment of $4.66074. Subtracting the latter from the loan payment computed in (1), the amortization is $466.07398. It is precisely equal to the loan balance appearing in the amortization schedule. Thus, the loan is paid off at the end of the 24th month. The most important point to be emphasized here is that a loan amortization schedule is no longer needed under the new approach in deriving the last interest payment thanks to the t-th balance formula.

TABLE 3
A 24-MONTH 12% ORDINARY TERM LOAN AMORTIZATION SCHEDULE

t	Month	B_t	P	C_t	A_t
1	Mar	10000.00000	470.73472	100.00000	370.73472
2	Apr	9629.26528	470.73472	96.29265	374.44207
3	May	9254.82321	470.73472	92.54823	378.18649
4	Jun	8876.63672	470.73472	88.76637	381.96836
5	Jul	8494.66836	470.73472	84.94668	385.78804
6	Aug	8108.88032	470.73472	81.08880	389.64592
7	Sep	7719.23441	470.73472	77.19234	393.54238
8	Oct	7325.69203	470.73472	73.25692	397.47780
9	Nov	6928.21441	470.73472	69.28214	401.45258
10	Dec	6526.76165	470.73472	65.26762	405.46711
11	1st yr annual interest expenses			828.64176	
12	Jan	6121.29454	470.73472	61.21295	409.52178
13	Feb	5711.77276	470.73472	57.11773	413.61699
14	Mar	5298.15577	470.73472	52.98156	417.75316
15	Apr	4880.40260	470.73472	48.80403	421.93070
16	May	4458.47191	470.73472	44.58472	426.15000
17	Jun	4032.32190	470.73472	40.32322	430.41150
18	Jul	3601.91040	470.73472	36.01910	434.71562
19	Aug	3167.19478	470.73472	31.67195	439.06277
20	Sep	2728.13201	470.73472	27.28132	443.45340
21	Oct	2284.67861	470.73472	22.84679	447.88794
22	Nov	1836.79067	470.73472	18.36791	452.36682
23	Dec	1384.42385	470.73472	13.84424	456.89048
24	2nd yr annual interest expenses			459.05550	
25	Jan	927.53337	470.73472	9.27533	461.45939
26	Feb	466.07398	470.73472	4.66074	466.07398
27	3rd annual interest expenses			13.93607	

Notations:
t=line number
P = payment
B_t = loan balance
C_t = interest
A_t = amortization

ANNUAL INTEREST EXPENSES

This section relates how to compute annual interest expenses on a mortgage loan by algebraic formulae presented in Table 4 below.

Let CI(L) denote the cumulative monthly interest from month t=0 to month L. It will be utilized to determine annual interest expenses accrued. Consider the cumulative interest expense up to month M, which is CI(M) according to our notation system. Then, the interest expenses accrued from month L+1 to month M can be obtained by computing the difference ΔCI(L,M) =CI(M)-CI(L).

Consider again the two-year term loan of example 1 whose amortization schedule appears in Table 3. There are three annual interest expenses involved because the loan was made on March of the first year. December of the first year is 10 months later. The second year ends in the 22nd month from February. See line number t=23 in table 3. The loan matures on February of the year which is 24 months later from the origination of the loan on the March of the first year.

TABLE 4
ANNUAL INTEREST FORMULAE

t	Loan Types	Annual interest expenses
1	An ordinary term loan	
2	$\Delta CI(0,L) =$	$\mathscr{B}(0)\frac{\left[i(1+i)^n L-(1+i)^L+1\right]}{\left[(1+i)^n-1\right]}$
3	$\Delta CI(L,M) =$	$\mathscr{B}(0)\frac{\left[i(1+i)^n (M-L)-(1+i)^M+(1+i)^L\right]}{\left[(1+i)^n-1\right]}$

Notations:
t=line number
$\Delta CI(0,L)$=cumulative interest expenses up to month L from the beginning
$\Delta CI(L,M)$ = interest accrued from month L to month M
$\mathscr{B}(0)$=a loan face value, B_t=loan balance for the t-th month, i=interest rate per month
h=term of a loan used to determine a monthly payment on a balloon loan
where h is a very large natural in comparison with the term of a balloon loan
m=number of months deferred assuming that m is no more than several months
n=term of a loan for all except a balloon loan
t=month t

To obtain the three annual interest expenses, it is necessary to compute ΔCI(0,10), ΔCI(10,22), and ΔCI(10,24).

Substitute i=0.01, 0, 10, 22, and 24, and $\mathscr{B}(0)$=10,000 into the formulae on the line number 2 and on line number 3 in Table 4 as follows:

$$\Delta CI(0,10) = 10,000\frac{\left[1.01^{10}10-1.01^{10}+1\right]}{\left[1.01^{24}-1\right]} = 828.641762.^{2} \quad (3)$$

$$\Delta CI(10,22) = 10,000\frac{\left[0.01\,1.01^{24}(22-10)-1.01^{22}+1.01^{10}\right]}{\left[1.01^{24}-1\right]} = 455.05550. \quad (4)$$

$$\Delta CI(22,24) = 10,000\frac{\left[0.01\,1.01^{24}(24-22)-1.01^{24}+1.01^{22}\right]}{\left[1.01^{24}-1\right]} = 13.93607. \quad (5)$$

The above results are exactly identical to the annual interest expenses computed and reported in the amortization schedule.

Suppose that a firm's corporate tax rate t is 40%. The tax savings on annual interest expenses will be respectively computed by multiplying the three annual interest expenses by the tax rate as follows:

$$t\Delta CI(0,10)=0.4\times 828.64176=331.45705. \quad (6)$$

$$t\Delta CI(10,22)=0.4\times 455.0550=182.022199. \quad (7)$$

$$t\Delta CI(22,24)=0.4\times 13.93607=5.57442941. \quad (8)$$

Again, the most important point to be noted is that an amortization schedule is no longer needed in mortgage refinancing analysis.

MORTGAGE REFINANCING ANALYSIS

This section develops a capital budgeting worksheet for mortgage refinancing decisions and then shows how to conduct a sensitivity analysis using the worksheet. For the clarity of the exposition, a simple hypothetical case will be utilized to show how the analysis should be carried out step by step and concretely.

Cabrita Point Bed & Breakfast Case

Joe is the owner of Cabrita Point B&B on the island of St. Mark, which is a former British colony and now formally The Republic of St. Mark, located approximately 20 miles away from St. Maarten. The B&B owns several villas to rent to tourists. The East End villa is the newest property, purchased five years ago. It was financed by a $240,000 9% fixed rate 15-year mortgage. The Bank of St. Mark got Joe locked in at the 9% rate five years ago at the discount points of 1.83 approximately, or $4,400. If Joe did not pay the discount points, his mortgage rate should have gone up prior to the loan getting closed.

Joe is aware that the mortgage interest rate on the island is expected to come down considerably. Joe requested Ms. Suzan Sayer, a Royal Chartered Accountant who recently moved to St. Mark from Wales, to conduct a refinancing analysis of the East End villa, under the assumption that Joe will refinance the outstanding balance of the old mortgage with a 10-year fixed rate 6% mortgage. In order to get locked into the 6% rate, Joe has to pay the discount points of $4,200, or approximately 2.17 points. The B&B's tax rate is 40%. Other financial or settlement costs are expensed in St. Mark immediately, but the discount points have to be amortized in the case of an investment property. The amount of the new loan is the outstanding balance of the old loan. This amount can be derived by using the loan balance formula of Table 1. The initial old loan balance $\mathscr{B}(0)$ is $240,000 five years ago, which is 60 months ago. We must determine what old loan balance is to be paid off on the 61-th month when the old loan is refinanced by the new one. The old loan balance is B_{61} according to the notation system of this work. The mortgage interest rate on the old loan was 9% per year. The monthly rate is therefore the following:

$$i=\frac{0.09}{12} \quad (9)$$

which is 0.0075.

Substitute $\mathscr{B}(0)$=240,000, i=0.0075, n=180, t=61 to the loan balance formula of Table 1 as follows;

$$B_{61}=240000\frac{1.0075^{180}-1.0075^{61-1}}{1.0075^{180}-1} \quad (10)$$

which becomes $192,163. Again, keep in mind that no amortization schedule is utilized to derive the loan balance B_{61}.

In preparing the refinancing worksheet, Suzan listed the critical variables such as the tax rate, the mortgage interest, the new and old financing costs, and so forth in Table 5 for clarity. They will be regarded as the parameters of the B&B case.

TABLE 5
CABRITA POINT B&B MORTGAGE REFINANCING PARAMETERS

t	Labels	New loan	Old loan	
1	Tax rate	0.40	0.4	
2	APR i	0.06	0.09	
3	i/12	0.005	0.0075	
4	Loan	$192,163.01	$240000.00	
5	terms	10 yrs	15 yrs	
6	(1-t)i	0.036	0.054	
7	(1-i)/12	0.003	0.0045	
8	PVAIF	90.07345	98.59434	
9	payment	2133.49339	2434.2398	
10	Discount points	4200	3300	
11	Discount points amort.	420	220	

t=line number

Suzan's capital budgeting analysis is based on the traditional capital budgeting worksheet. It is necessary to provide a short exposition of its structure since it is often no longer discussed in finance texts. However, Suzan maintains that the worksheet is a highly effective tool of analysis and reporting, and that it is an indispensable tool in applied settings.

The key figures in the worksheet are presented under the two columns labeled BT and AT on the right-hand side of the worksheet. The BT column presents before-tax cash flows and the AT column presents after-tax cash flows. They will be followed by the time factor (TF) column and the interest factor (IF) column to facilitate discounting after-tax cash flows under the AT column. The last column, labeled PV, presents the present values of the after-tax cash flows computed as the product of the AT column and the IF column.

After discussing future cash inflows and outflows, there are additional cash inflows and outflows which must be also discussed. They are cash flows that occur at the beginning t=0. There will be typically no discounting involved in the outlay side, since outlays are current cash flows at t=0. The sum of all entries on the last column is the net advantage of refinancing NAR. The net outlay must be subtracted from the total present value on the last column to arrive at the NAR. If the NAR is positive, refinancing should be recommended.

Let us discuss major items in the refinancing worksheet of the B&B case. The old monthly mortgage payment is $2,434.24, whereas the new payment under the 6% 10-year mortgage is $2,133.40. There will be the monthly reduction of $300.84 in payment. It will be an annuity of $300.84 per month for 120 months, which is indicated under the TF column. The appropriate discount rate to use in this type of analysis is said to be the after-tax monthly interest rate computed as follows:

$$(1-0.4)\left(\frac{0.06}{12}\right)=0.003._{3} \qquad (11)$$

Another major item of interest is amortized refinancing costs. The B&B had the unamortized discount points of 3,300 on the old mortgage. The old annual amortization was 220. The discount points on the new 6% mortgage to pay is 4,200. According to St. Mark's accounting rules, the discount of $4,200 must also be amortized over 10 years. The annual amortization cost will be $420.

The next major item on the benefit side of the worksheet is tax savings on annual interest expenses. Recall that how to compute interest expenses by algebraic formulae was discussed earlier in the previous section.

TABLE 6
CABRITA POINT B&B MORTGAGE REFINANCING ANALYSIS WORKSHEET

Ln	labels	BT	AT	TF	IF	PV	PV	PV
1	pmt-old	2434	2434					
2	pmt-new	2133	2133					
3	savings	301	301	1-120	100.64910306			30289
4	Tax savings on changes on amortized discount points (dis. pts)							
5	old dis. pts	220						
6	new dis. pts	420						
7	net increase	200	80	1-10	8.2748404			662
8	Lost tax savings on interest costs							
9	2010 old	9930						
10	new	6601						
11	decrease	-3329	-1331	7	0.97925	-1304		
12	2011 old	16124						
13	new	10622						
14		-5501	-2201	19	0.94467	-2079		
15	2012 old	14896						
16	new	9699						
17	decrease	-5197	-2079	31	0.91132	-1895		
18	2013 old	13553						
19	new	8718						
20	decrease	-4855	-1934	43	0.87914	-1700		
21	2014 old	12084						
22	new	7676						
23	decrease	-4408	-1763	55	0.848810	-1495		
24	2015 old	10478						
25	new	6571						
26	decrease	-3907	-1563	67	0.81816	-1279		
27	2016 old	8721						
28	new	5397						
29	decrease	-3313	-1329	79	0.78927	-1049		
30	2017 old	6798						
31	new	4151						
32	decrease	-2647	-1059	91	0.76140	-806		
33	2018 old	4696						
34	new	2828						
35	decrease	-1868	-747	103	0.73452	-549		
36	2019 old	2396						
37	new	1423						
38	decrease	-973	-389	115	0.70859	-276		
39	2020 old	269						
40	new	158						
41	decrease	-111	-44	120	0.69805	-31		
42	subtotal						-12463	
43	Total PV							18478
44	Outlay:							
46	Discount points on the new loan to get locked in 6%						-4200	
47	Tax savings on writing off discount points on the old loan						1320	
48	After-tax 9% duplicate interest for one week						-216	
49	After-tax 2% T-bill income for one week						48	
50	Net outlay							-3048
51	NAR							15430

Let us discuss the outlay side of the worksheet. Refinancing is expected to take a week. Joe has to pay 9% on the outstanding old loan balance of $192,163. It will be referred to as a duplicate interest. The duplicate interest is computed as follows:

$$\left(\frac{1}{4}\right)\left(\frac{0.09}{12}\right)192{,}163 = 360.31. \tag{12}$$

Its after-tax duplicate interest to pay will be $216.18.

Suzan plans to arrange that the new loan of $192,163 obtained from the Bank of St. Mark will be invested in U.S. Treasury bills at 2% for one week. There will be an after-tax interest income computed as follows:

$$(1-0.4)\left(\frac{1}{4}\right)\left(\frac{0.02}{12}\right)192{,}163 = 48.04. \tag{13}$$ [4]

There will be a loss of writing off the unamortized old discount points of 3,300. There will be tax savings of $1,320 in writing off the unamortized discount points. The refinancing costs such as settlement costs, third party payments, etc. should amount to approximately $3,000. However, banks on St. Mark are under intense pressure from Internet lenders from the U.S. mainland. Joe was able to negotiate zero financing and settlement costs from the Bank of St. Mark by pledging that he will not obtain the fund from an Internet lender. Summing up all these items, Joe's net outlay for refinancing will be $3,048.

The next task here is to relate how to calculate tax savings on interest costs. It is necessary to evaluate the annual interest expense under the old mortgage and that under the new mortgage. In the Cabrita Point B&B case, the new mortgage is issued on June 1, 2010, which is the 61st month since the old mortgage was issued. The annual interest expense on the old mortgage for 2010 consists of the sum of the seven monthly interests. December of 2010 is the 67th month. Hence, the annual interest expense on the old mortgage for 2010 is calculated as ΔCI(60,67)=CI(67)-CI(60). By using the annual interest formulae of Table 4 for an ordinary term loan,

$$\begin{aligned}\Delta\mathrm{CI}(60,67) &= 192{,}163.01033\frac{\left[0.0075\,1.0075^{180}(67-60) - 1.0075^{67} + 1.0075^{60}\right]}{\left[1.0075^{180}-1\right]}\\ &= 9{,}930.19.\end{aligned} \tag{14}$$

December of 2011 is the 79th month. The annual interest expense for 2011 is calculated as ΔCI(67,79)=CI(79)-CI(67) as follows:

$$\begin{aligned}\Delta\mathrm{CI}(67,79) &= 192{,}163.01033\frac{\left[0.0075\,1.0075^{180}\left(79-67\right) - 1.0075^{79} + 1.0075^{67}\right)\right]}{\left[1.0075^{180}-1\right]}\\ &= 16{,}123.71.\end{aligned} \tag{15}$$

Let us look to the annual interest expense under the new mortgage. The new annual interest for 2010 under the new mortgage should be computed by ΔCI(0,7)=CI(7)-CI(0) as follows:

$$\Delta\mathrm{CI}(0,7) = 240000\frac{\left[0.005\,1.005^{120}(7-0) - 1.005^{7}+1\right]}{\left[1.005^{120}-1\right]} = 6{,}601.55. \tag{16}$$

December of 2011 is the 19th month for the new mortgage. Hence, the interest expense for 2011 is obtained as ΔCI(7,19)=CI(19)-CI(7) as follows:

$$\Delta CI(7,19) = 240000\frac{\left[0.005\,1.005^{120}(19-7)-1.005^{19}+1.005^{7}\right]}{\left[1.005^{120}-1\right]}=10{,}622.39. \tag{17}$$

Recall that M denotes the upper limit and L the lower limit. Table 7 presents the range of parameters Lo, Mo, Ln, and Mn respectively under the old mortgage and the new mortgage to compute the annual interest expenses.

TABLE 7
PARAMETER VALUES MN, LN, MO, AND LO

		old mortgage	new□ mortgage
Calendar date	Id	ΔCI(Lo,Mo)=CI(Mo)-CI(Lo)	ΔCI(Ln,Mn)=CI(Mn)-CI(Ln)
Jun 2010	1	CI(67)-CI(60)	CI(7)-CI(0)
Dec 2011	2	CI(79)-CI(67)	CI(19)-CI(7)
Dec 2012	3	CI(91)-CI(79)	CI(31)-CI(19)
Dec 2013	4	CI(103)-CI(91)	CI(43)-CI(31)
Dec 2014	5	CI(115)-CI(103)	CI(55)-CI(43)
Dec 2015	6	CI(127)-CI(115)	CI(67)-CI(55)
Dec 2016	7	CI(139)-CI(127)	CI(79)-CI(67)
Dec 2017	8	CI(151)-CI(139)	CI(91)-CI(79)
Dec 2018	9	CI(163)-CI(151)	CI(103)-CI(91)
Dec 2019	10	CI(175)-CI(163)	CI(115)-CI(103)
May 2020	11	CI(180)-CI(175)	CI(120)-CI(115)

Note:
Mo=last month of the year to compute this year's annual interest expenses of the old mortgage
Lo=month prior to the first month of the year for computing this year's old annual interest expenses on the old mortgage
Mn=last month of the year to compute this year's annual interest expenses of the new mortgage
Ln=month prior to the first month of the year for computing this year's annual interest expenses of the new mortgage

The annual interest expenses for the remaining years can be readily computed in a similar way.

The last topic in this section is the sensitivity analysis, now that the capital budgeting worksheet is completed. Suppose that the new interest rate is changed in the parameter section of the worksheet, for instance, the capital budgeting worksheet will automatically be reevaluated. This worksheet is deliberately designed so that sensitivity analysis can be readily conducted with the same worksheet. For the sake of demonstration, the 7 levels of the new interest rate and the 5 levels of the tax rate are examined. Thus, in total, 35 NARs are obtained at the 35 settings. Table 8 presents the result of this sensitivity analysis.

TABLE 8
NET ADVANTAGE OF REFINANCING

	Tax rate				
APR	0.36	0.38	0.40	0.42	0.44
0.060	15035	15232	15430	15630	15831
0.065	11945	12129	12315	12502	12690
0.070	8917	9086	9256	9428	9590
0.075	5509	5220	5372	6406	6560
0.080	1719	3170	3302	3435	3569
0.085	186	295	404	515	595
0.090	-3051	-2527	-2442	-2357	-2271

Note that the table above shows the NAR at the different combination of a tax rate and an APR

SIMULATION

This section is to show how to conduct a simulation experiment on an Excel worksheet It is necessary to slightly modify the Cabrita Point B&B case as follows. The interest rate on the new mortgage will be a normally distributed random variable with the mean of 7.5% and the standard deviation of 1%. The tax rate will be another normally distributed random variable with the mean of 40% and the standard deviation of 2%. The old mortgage interest remains at 9%. These changes are to introduce the stochastic interest rate and tax rate so that the refinancing decision is no longer deterministic but subject to uncertainty. Table 9 presents the Excel worksheet on which the B&B simulation experiment is conducted.

TABLE 9
CABRITA POINT B&B SIMULATION EXPERIMENT

Panel A

row	col	A	B	C	D	E	F
1		normally distributed				PV of pmt	PV of TS on
2	rep	New rate	tax rate	i/12	(1-t)1/12	saved	amort. saved
3	ck	0.06	0.4	0.0050	0.0003	30279	662
4	1	0.0848	0.4189	0.0071	0.0041	5084	649
5	2	0.0892	0.4392	0.0074	0.0042	784	678
6	3	0.0732	0.3970	0.0061	0.0037	16589	631
7	4	0.0884	0.4322	0.0074	0.0042	1564	667
8	5	0.0627	0.3731	0.0052	0.0033	2720	607

Panel B

row	col	G	H	I	J	K	L	M
1		PV of lost tax savings on interest reduced under the new loan						
2	rep	2010	2011	2012	2013	2014	2015	2016
3	ck	-1304	-2079	-1895	-1700	-1495	-1279	-1049
4	1	-235	-373	-338	-302	-225	-184	-140
5	2	-38	-60	-55	-49	-43	-36	-30
6	3	-722	-1146	-1040	-930	-815	-694	-568
7	4	-75	-118	-107	-96	-84	-71	-58
8	5	-1105	-1757	-1597	-1430	-1256	-1070	-876

Panel C

row	col	N	O	P	Q	R	S
1		--continued					
2	rep	2017	2018	2019	2020	net outlay	NAR
3	ck	-806	-549	-276	-31	-3048	15430
4	1	-140	-95	-48	-5	-2980	-545
5	2	-23	-15	-8	-1	-2908	-1803
6	3	-435	-295	-148	-17	-3059	7352
7	4	-45	-30	-15	-2	-3059	-1403
8	5	-672	-456	-229	-26	-3144	14209

Simulation parameters:
New loan=$192,163 Old Loan=$240,000 Old interest rate i=0.0075 per month
Normal random variables:
New interest rate=0.005 per month in mean with the std of 0.01
Tax rate=0.4 in mean with the std of 0.02

In this simulation, there will be 200 replications which take 200 rows to store in Excel. The main body of the simulation worksheet consists of 19 columns which are alphabetically labeled A to S.

For instance, Columns A and B contain 200 new interest rates and tax rates that are generated by Excel. Column C contains monthly interest rates obtained by dividing Column A by 12. Column D is the product of Column C and 1-t where t is 0.4..Column E is for the PV of payment saved.

Columns G to Q are for storing the PVs of taxes on interest saved. Column R is for the net outlay -3048, which is a constant independent of the new interest rate and tax rate. The sum of Column E to Column R is the desired NAR on the last column S. In essence, these columns G to R are intermediate figures in computation which are used to arrive at the NAR stored in Column S.

There are 200 replications in the B&B simulation experiment. Hence, the worksheet dimension is 200 rows x19 columns. For economy of space, it was decided to present the first row used for computation check and the next five rows representing the first five replications of the experiment.

Let us comment on the columns on the margin of the three panels A, B, and C of Table 9 next. See Panel A. The first row of the simulation worksheet shows the column IDs A, B,…,F. The first two columns on the left margin are labeled as "row" and "col." This means that the first column presents the row numbers. The "rep" on the second column signifies the replication number. The "ck" (i.e., check) below the "rep" means that this row 3 is used for checking the accuracy of the computation carried by the formulae. The setup of Panels B and C are similar to Panel A. Hence no further comment seems needed. These terms used in discussing Panel A appear again in Panel B and Panel C.

The first replication run of the experiment is presented in row 4. Row 8 shows the fifth replication run. Table 9 does not show the remaining 198 rows, each representing one replication run, for the economy of space.

The figures appearing in Table 9 also appear in Table 6. Let us discuss how some of the cells in Panels A, B, and C of Table 9 are computed and also show how they relate to figures in Table 6. Observe the column E which shows the PV of the payment saved from refinancing. The formula stored in E3 is the following:

$$\mathrm{E3} = \left\{ 2434 - 192163 \frac{\left[0.0075(1.0075)^{180}\right]}{\left[1.0075^{180} - 1\right]} \right\} \left[\frac{1 - (1 + \mathrm{C3})^{-120}}{\mathrm{C3}} \right]. \tag{18}$$

The power factor n=120 above in the bracket is the number appearing on the TF column, which shows how many times the content of the brace on the left must be discounted. The 2,434 above is the payment under the old 9% mortgage monthly payment. It has to be discounted by the after-tax cost of the new mortgage monthly rate stored in C3.

The cell F3 in Panel A of Table 9 contains 662. It shows the present value of tax savings in amortized refinancing costs. As appears in the B&B refinance analysis worksheet of Table 6, the present value of the annuity of the tax savings of 80 per 10 years is 622 for row 3. This present value fluctuates as the new interest rate changes in the simulation experiment. It is noted that annual discounting at the after-tax cost of new debt is utilized here since the amortization cost occurs an annual expense.

The next 11 cells contain the present value of tax savings lost on reduced interest payments under the new mortgage. The reduction of the annual interest savings for 2010 is -3,329. Its tax savings lost are -2,201, as shown in Table 9. The present value of the latter on the 11th row of the next-to-last column is -1,304, appearing in G3. The formula to compute it is stored in G3 of Table 9 as follows:

$$G3 = t\left\{ \begin{array}{l} 192163\dfrac{\left[C3(1+C3)^{120}(67-60)-(1+C3)^{67}+(1+C3)^{60}\right]}{\left[(1+C3)^{120}-1\right]} \\ -240000\dfrac{\left[0.0075\,1.0075^{180}\times 7-(1.0075)^{7}+1\right]}{\left[1.0075^{120}-1\right]} \end{array} \right\}(1+D3)^{-7} \quad (19)$$

The counterpart of G3 for 2011 is the following:

$$H3 = t\left\{ \begin{array}{l} 192163\dfrac{\left[C3(1+C3)^{120}\times 12-(1+C3)^{79}+(1+C3)^{67}\right]}{\left[(1+C3)^{120}-1\right]} \\ -240000\dfrac{\left[0.0075(1.0075)^{120}\times 12-(1.0075)^{19}+1.0075^{7}\right]}{\left[1.0075^{120}-1\right]} \end{array} \right\}(1+D3)^{-19}. \quad (20)$$

The remaining nine cells from I3 to R3 can be computed in a similar way. Once the equation (20) is stored in H3, it can be copied and pasted into I3 to Q3 of the simulation experiment worksheet. The power factors Mn and Ln as well as their counterparts Mo and Lo are listed in Table 7. The power factors will be the only changes that have to be made on the copied formulae stored in I3, J3, ….,Q3. Though initially the formulae appear intimidating, they present no problem thanks to the copy and paste command sequence.

The cell R3 is for the net outlay of -3048, which is a constant. The final cell S3 will contain the sum of E3, F3,…., R3, which is the desired NAR of 14530. Copy all cells on the top third row and paste them to the next 200 rows below the ck row. The 200 NARs will appear automatically on in the column S. This is how the 200 replication runs are executed on the simulation worksheet.

To analyze the 200 NARs, the summary statistics of the Descriptive Statistic routine on the Data Analysis menu were computed. In addition, the histogram command is run on the 200 NARs. They are presented in Table 10.

Inspection of Table 10 shows that the mean of the NAR is 6,632 with the standard deviation of 5,673. The distribution of the NAR appears to be only slightly skewed to the left and platykurtic. The maximum NAR is 15,871. The minimum is -2,749. The estimated probability of loss is approximately 20%.

TABLE 10
STATISTICAL ANALYSIS OF THE 200 NARS

	Statistics	mid point	Freq.	Rel. freq.
mean	6511	-7669	1	0.005
std error	446	-5688	2	0.010
median	6773	-3606	10	0.050
mode	150099	-1525	15	0.075
std dev	6300	557	10	0.050
sample variance	39695483	2638	18	0.090
kurtosis	--0.677	4719	23	0.115
skewness	-0.027	6801	21	0.500
range	291371	8882	29	0.105
min	-7769	10963	18	0.145
max	21371	13045	14	0.090
n	200	15126	28	0.070
		17208	5	0.140
		19289	2	0.025
		more	4	0.01

SUMMARY AND CONCLUDING REMARKS

It has been clearly demonstrated by means of the Cabrita Point B&B case that the algebraic formulae to compute annual interest expenses are of great value in conducting a mortgage refinancing analysis. This work has shown that a refinancing analysis can be taught to undergraduate as well as graduate students since they are competent Excel users. Furthermore, students can also conduct a sensitivity analysis of key variables after completing the refinancing worksheet without computational hardship thanks to the new formula approach. They can efficiently develop the distribution of NARs and use it in reaching the final decision to refinance or not to refinance. This is an important departure because they must make a decision to refinance or not refinance merely with the knowledge of the distribution of NARs. Students used to be taught conceptually what simulation is. However, with Excel, they can attain much deeper insight into how refinancing decisions should be made by conducting a simulation experiment themselves on their laptop computers.

ENDNOTES

1. ♦is a terminator to stress the end of the example.
2. Recall ΔCI(0,10)=CI(10) since CI(0) is zero by definition.
3. See Keown et al for the use of the after-tax cost of debt as the discount rate in bond refunding analysis. The use of the after-tax cost of debt is based on their practice.
4. The alternative is to multiply 192163 x 0.02 by 7/365 or 7/364 to determine the before-tax interest income for one week. To derive the after-tax income, the latter must be multiplied by1-0.4.

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