Predicting Bankruptcy with Correlated Credit Components

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In this study we suggest an original non-linear corporate credit risk model that accounts for the complete distributional properties of common observed risk modules and their corresponding estimated default threshold values. The proposed model predicts corporate bankruptcies based on the consequential interrelationships within these credit components. We illustrate the theory over a large sample of observations and further authenticate its added value when predicting business failures. Despite the relatively higher complexity involved, compared to linear credit scores or scorecards, the present scheme can serve financial institutions and other lenders to make superior lending decisions and achieve higher overall profitability.

INTRODUCTION

Customary external credit scores as well as banks' internal scorecards typically assess the credit quality of borrowing firms by examining pertinent accounting measures including size, profitability, leverage, liquidity, solvency, interest coverage, asset quality, investment activity, growth rate, dividend payout, financing results, various market quantities of equity price, return, and volatility patterns, and macroeconomic variables including unemployment rate, Gross Domestic Product (GDP) growth rate, inflation level, and credit spread over U.S. Treasury bonds.[1] These common credit estimation methodologies are normally structured through discriminant analyses, logistic regressions, or alternative linear econometric techniques. Nonetheless, all these credit forecasting schemes endure collective drawbacks, as described hereafter.

First, these linear predictive tools must refute multicollinearity, thus they all implicitly assume independent configurations of the credit components within. In reality, however, the credit elements are correlated to one another, because the same macroeconomic factors simultaneously shape many of them. For instance, reduced profitability naturally leads to depressed stock prices. Higher leverage often triggers intensified interest coverage rates. A downturned economic cycle typically prompts lower investment activity. Smaller firms usually hold inferior assets, and a poor measure of solvency is habitually associated with lower dividend payout. A more realistic credit model must consider these and other interrelationships among the relevant credit components.

Second, ordinary credit estimation methods categorize borrowers with quantifiable scores. Then, financial institutions further assign lending rates based on the mean values of these scores, regardless of other valuable statistical characteristics of these measures. Different firms, however, can reach bankruptcy thresholds at diverse credit scores, and the full distributional properties of the underlying credit components and the potential break points are essential input when predicting business failures. Moreover, the creditworthiness of two firms having the exact same credit score could be substantially different if credit components of one are more volatile than credit modules of the other. Observed variability in the fundamental credit ingredients that eventually compose the complete credit score evidently affects the firm's ultimate probability of default (PD), since the likelihoods to reach bankruptcy thresholds can vary in this case. Furthermore, these corporate break points may have their own disseminations depending on idiosyncratic as well as macroeconomic factors.

In this study, we aim to triumph over the above obstacles and present a stochastic risk model, which allows us to accommodate different statistical merits of correlated observed credit components and distributional characteristics of estimated bankruptcy threshold values in different economic circumstances. However, the current theory does not require us to explicitly compute any correlation coefficients among the underlying credit modules.[2] Instead, we utilize several dispersion properties of the underlying credit components and the likely bankruptcy threshold values. The feasible intersections of these respective distributions are the sources of dependency in our proposed credit model.

The current stochastic model is certainly more complex than existing linear credit scores and other applied scorecards in use. However, since the proposed theory relies upon observed accounting statements, it is indeed measurable, verifiable, and programmable. Thus, the suggested theory can serve a large spectrum of lenders that desire to achieve a higher predictive strength than existing linear credit scores towards likely corporate bankruptcies. The present scheme becomes exceedingly usable since it evolves from highly realistic assumptions and concrete statistical concepts. It can, therefore, assist financial institutions, banks, and other lenders to make healthier lending decisions and achieve higher overall profitability.

THE MODEL

We consider a universal paradigm where a lender utilizes a credit score, which contains n components that evidently determine the creditworthiness of the borrower.[3] The different credit modules typically include quantitative measures such as accounting ratios, market variables, and macroeconomic parameters, but in general, these credit components may also incorporate qualitative estimations of management quality, marketing strength, growth prospects, or any other quantifiable soft information.

For each credit component, we can collect its tangible measure α and further identify its failure threshold value β. Tangible measures α_i naturally comprise observed past and present records of each credit component $i \in \{1,2, \dots n\}$. The cutoff-points β_i denote estimated critical quantities that label the area in which the underlying borrowing firm would presumably file for bankruptcy Chapter 11 reorganization.[4] These appraisals are typically collected from prior bankrupt firms within the relevant industry, and when available, from defaulted firms that had (prior to their failures) similar financial characteristics to those of the borrowing firm under investigation. Nevertheless, in reality, these parameters are not fixed. Instead, we allow them to be random variables with Probability Density Functions (PDF), $\xi(\alpha)$ and $\psi(\beta)$, respectively.

By definition, bankruptcy may occur whenever a current measure of a credit module α falls below its estimated bankruptcy threshold value β, thus a credit component-related probability of default is:

$$PD_{Comp} = P(\alpha < \beta). \tag{1}$$

In this case, a credit module-related probability of default is a random variable by itself. We can further describe this probability as a function of the bankruptcy threshold value β within small intervals of width $d\alpha$ as follows:

$$P(\beta) = \int_{-\infty}^{\beta} \xi(\alpha) d\alpha. \tag{2}$$

The realization of a default probability $P(\beta)$ distinctively corresponds to a specific bankruptcy threshold value β. Furthermore, the likelihood for a credit component-related probability of default being equal to $P(\beta)$ is equivalent to the chances for a bankruptcy threshold value being equal to β. More formally we define

$$\vartheta\big(P(\beta)\big)dP = \psi(\beta)d\beta, \tag{3}$$

where $\vartheta\big(P(\beta)\big)$ is the PDF of the credit component-related probability of a default random variable $P(\beta)$. As a function of a continuous random variable, the likelihood for the credit component-related conditional probability of default is differentiable and strictly monotonic function over some interval. Furthermore, it has a PDF in the form

$$\vartheta\big(P(\beta)\big) = \psi\big(P^{-1}(\beta)\big)\frac{\partial P^{-1}(\beta)}{\partial P}, \tag{4}$$

where $P^{-1}(\beta) \stackrel{\text{def}}{=} \beta(P)$, hence it is the inverse function of $P(\beta)$.[5]

In this setting, $\int_0^1 \vartheta\big(P(\beta)\big)dP = 1$, hence $\vartheta\big(P(\beta)\big)$ is a valid PDF over the feasible domain $[0,1]$ of the probability $P(\beta)$ only when the minimum observed (actual) credit component $Min(\alpha)$ falls below the maximum of the likely (estimated) bankruptcy threshold values $Max(\beta)$. Essentially, both distributions must have an apparent intersection, otherwise $\vartheta\big(P(\beta)\big)$ is mathematically ill-defined. In the following analysis we realistically assume that all firms are subject to some bankruptcy risk and exclude the hypothetical case of which $\int_0^1 \vartheta\big(P(\beta)\big)dP < 1$.[6] Consequently, the expected credit component-related conditional probability of default is the mean value of the component failure probability random variable. We therefore obtain

$$\widehat{PD}_{Comp} = \int_{-\infty}^{\infty} P(\beta)\,\vartheta\big(P(\beta)\big)dP = \int_{-\infty}^{\infty} \psi(\beta)\left(\int_{-\infty}^{\beta} \xi(\alpha)d\alpha\right)d\beta. \tag{5}$$

At this stage, it is crucial to understand that the variability of the estimated bankruptcy threshold values is the basis for the failure dependency among the fundamental credit components, while the dispersion of the tangible measures of these credit modules weakens the dependency of breakdowns. If the bankruptcy threshold is deterministic, failure probabilities of credit components are fixed and independent of each other. Conversely, if the actual credit values are deterministic, all credit components either fail together at a given threshold, or collectively persist, hence these credit components are perfectly correlated to one another.

We therefore deduce that two borrowers could have the exact same structure of credit components, thus the same ultimate credit score, yet these two firms may convey distinct credit qualities. A necessary (and sufficient) condition for a similar creditworthiness of two borrowers having equivalent measures of credit components is that both distributions of actual credit values and estimated bankruptcy threshold values are the same across the two firms, respectively. Accordingly, existing credit scores miss critical information concerning the distributional properties of the tangible quantities of credit components and the approximated bankruptcy threshold values. We can illustrate this regular information loss with the following example of variable conversion.

We assume for now that both observed measures of credit components and estimated bankruptcy threshold values are Normally-distributed random variables with means and standard deviations denoted as μ_α, σ_α, and μ_β, σ_β, respectively. In this case, common credit scores would have routinely assessed the probability of default by creating a new variable $\delta \stackrel{\text{def}}{=} \alpha - \beta$. Under the common assumption of independency between actual credit measures and approximated bankruptcy thresholds, δ is also a Normally-distributed random variable with a mean $\mu_\delta = \mu_\alpha - \mu_\beta$ and an additive standard deviation $\sigma_\delta = \sqrt{\sigma_\alpha^2 + \sigma_\beta^2}$. Consequently, δ becomes a new random variable with PDF $\varrho(\delta)$, thus the credit component-related probability of default is:

$$\widetilde{PD}_{Comp} = \int_{-\infty}^{\infty} \varrho(\delta) d\delta. \tag{6}$$

Throughout this course, the dispersion parameters σ_α and σ_β are completely integrated to form a new standard deviation σ_δ. Subsequently, central credit information is getting lost because the new dispersion parameter σ_δ is entirely indifferent to its origins. Since the variability of the bankruptcy threshold values σ_β and the randomness of the actual credit components σ_α have totally different impacts on the ultimate failure dependency structure, the variable conversion procedure has caused the loss of valuable information. On the other hand, the current approach would assign the following credit component-related probability of default:

$$\widehat{PD}_{Comp} = \int_{-\infty}^{\infty} \frac{1}{\sigma_\beta \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\beta-\mu_\beta}{\sigma_\beta}\right)^2} \left[\int_{-\infty}^{\beta} \frac{1}{\sigma_\alpha \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\alpha-\mu_\alpha}{\sigma_\alpha}\right)^2} d\alpha\right] d\beta, \tag{7}$$

which is more sensitive to the true origins of the failure dependency structure for all the relevant credit components.

Up to this point, we have obtained the probability of default as a result of a sudden deterioration in the measure of a single credit module. Nevertheless, some high-quality borrowers are sound enough to ensure that even when a single credit measure falls, they would still continue normal operations. This economic setting represents a situation where all credit modules are currently at high-enough levels, so that albeit a sole economic shock to any of these credit components, the underlying borrowing firm maintains an acceptable credit score thus avoids filing for bankruptcy. We therefore derive the complete probability of default under various economic settings regarding the dependency structures of all credit components within a typical credit score.

When all credit components share a unified yet stochastic bankruptcy threshold, failures of these credit modules are not independent due to the joint correlations with this single dynamic cutoff point. Nonetheless, when the estimated shared bankruptcy threshold value is deterministic all the credit components are independent of each other. The latter phenomenon evolves since an individual component's conditional failure probability is uniquely determined by the distributions of the actual credit values. Thus, when the approximated bankruptcy threshold value is constant, the likelihoods of a business failure due to sudden changes in the different credit modules are also independent. In this situation, equation (1) and (2) dictate that

$$PD_{Comp} = P(\alpha < \beta) = \int_{-\infty}^{\beta} \xi(\alpha) d\alpha, \tag{8}$$

and under the simplifying assumption of Identically Distributed (ID) credit modules, the probability of default associated with failures of all n credit components is:

$$PD_{n-Comp}^{ID} = \left[\int_{-\infty}^{\beta} \xi(\alpha) d\alpha\right]^n. \tag{9}$$

In reality, however, the bankruptcy threshold value β is not fixed. It is a random variable with PDF $\psi(\beta)$, which is typically driven by idiosyncratic, industry, and macroeconomic pertinent conditions. In this case, the mean probability for failures of all n ID credit components becomes:

$$\widehat{PD}_{n-Comp}^{ID} = \int_{-\infty}^{\infty} \psi(\beta) \left[\int_{-\infty}^{\beta} \xi(\alpha) d\alpha\right]^n d\beta. \tag{10}$$

In addition, we can remove the non-trivial assumption of ID credit components and obtain the probability of default associated with failures of all n non-ID credit components as:[7]

$$\widehat{PD}^{non-ID}_{n-Comp} = \int_{-\infty}^{\infty} \psi(\beta) \prod_{i=1}^{n} \left[\int_{-\infty}^{\beta} \xi_i(\alpha) d\alpha\right] d\beta. \tag{11}$$

From equation (10) we can further derive the probability of default associated with failures of m out of n ID credit components as:

$$\widehat{PD}^{ID}_{m/n-Comp} = \binom{n}{m} \int_{-\infty}^{\infty} \psi(\beta) \left[\int_{-\infty}^{\beta} \xi(\alpha) d\alpha\right]^{m} \left[\int_{\beta}^{\infty} \xi(\alpha) d\alpha\right]^{n-m} d\beta, \tag{12}$$

where $\binom{n}{m} \stackrel{\text{def}}{\equiv} \frac{n!}{m!(n-m)!}$ represents the likely combinations of the underlying credit components.[8] Similarly, from equation (11) we can also derive the probability of default associated with failures of m out of n non-ID credit components as:

$$\widehat{PD}^{non-ID}_{m/n-Comp} = \binom{n}{m} \int_{-\infty}^{\infty} \psi(\beta) \prod_{i=1}^{m} \left[\int_{-\infty}^{\beta} \xi_i(\alpha) d\alpha\right] \prod_{j=m+1}^{n} \left[\int_{\beta}^{\infty} \xi_j(\alpha) d\alpha\right] d\beta, \tag{13}$$

where the first inner product $\prod_{i=1}^{m}[\quad]$ identifies the m failed credit components, and the second inner product $\prod_{j=m+1}^{n}[\quad]$ recognizes the $(n-m)$ non-defaulted credit modules.

Finally, we can postulate the ultimate borrower's probability of default as a random variable that accounts for failure dependency among the different credit components. We thus consider that a default event occurs whenever k or more tangible credit modules fall below their estimated bankruptcy threshold values, where k denotes the minimum required number of credit component failures that trigger corporate default, $k \in \{1,2,\dots,n\}$. In the scenario of ID credit components we obtain:

$$\widehat{PD}^{ID} = \sum_{m=k}^{n} \left\{\binom{n}{m} \int_{-\infty}^{\infty} \psi(\beta) \left[\int_{-\infty}^{\beta} \xi(\alpha) d\alpha\right]^{m} \left[\int_{\beta}^{\infty} \xi(\alpha) d\alpha\right]^{n-m} d\beta\right\}, \tag{14}$$

and in the non-ID incident we acquire the primary model as:

$$\widehat{PD}^{non-ID} = \sum_{m=k}^{n} \left\{\binom{n}{m} \int_{-\infty}^{\infty} \psi(\beta) \prod_{i=1}^{m} \left[\int_{-\infty}^{\beta} \xi_i(\alpha) d\alpha\right] \prod_{j=m+1}^{n} \left[\int_{\beta}^{\infty} \xi_j(\alpha) d\alpha\right] d\beta\right\}. \tag{15}$$

Logically, corporate creditworthiness is constructed from separate credit components, and the bankruptcy threshold values applied to these individual credit modules are different too. When the bankruptcy threshold values β_i are independent of each other, equations (14) and (15) portray a faithful solution. On some occasions, however, bankruptcy threshold values may be cross-correlated throughout the diverse credit modules. For instance, various harmful macroeconomic shocks could simultaneously shift all bankruptcy threshold values β_i to higher levels hence make lending institutions less tolerant during harsh economic environments. Alternatively, because lenders typically become more lenient during prosperous times, in extended expansionary economic cycles the number of bankruptcy threshold values may concurrently reside at lower heights. For these ordinary cases we ought to adjust the model and develop substitute derivations for a healthier assessment of probabilities of default.

We may presume that all the bankruptcy threshold values are linearly correlated random variables, hence $\beta_i = \mathbb{C}_i \beta^{(U)} + \mathbb{Q}_i$, where β_i denotes the respective default cutoff point for specific credit component i, $\beta^{(U)}$ is the unified bankruptcy threshold, and $\mathbb{C}_i$ and $\mathbb{Q}_i$ are some constants. We further assume that each bankruptcy threshold value β_i abides a Normal distribution with respective mean μ_i and variance σ_i^2. We can therefore standardize these random variables and obtain:

$$\beta^{(U)} = \frac{\beta_i - \mu_i}{\sigma_i} \sim N(0,1) \;\Rightarrow\; \beta_i = \sigma_i \beta^{(U)} + \mu_i. \tag{16}$$

Evidently, we can deploy the following transformations to better describe the probabilities of default in the ID and the non-ID credit components respective scenarios as:

$$\widehat{PD}^{ID} = \sum_{m=k}^{n} \left\{ \begin{matrix} \binom{n}{m} \int_{-\infty}^{\infty} \phi\left(\beta^{(U)}\right) \left[\int_{-\infty}^{\sigma_i \beta^{(U)}+\mu_i} \xi(\alpha) d\alpha\right]^m \times \\ \left[\int_{\sigma_i \beta^{(U)}+\mu_i}^{\infty} \xi(\alpha) d\alpha\right]^{n-m} d\beta^{(U)} \end{matrix} \right\}, \tag{17}$$

and

$$\widehat{PD}^{non-ID} = \sum_{m=k}^{n} \left\{ \begin{matrix} \binom{n}{m} \int_{-\infty}^{\infty} \phi\left(\beta^{(U)}\right) \prod_{i=1}^{m} \left[\int_{-\infty}^{\sigma_i \beta^{(U)}+\mu_i} \xi_i(\alpha) d\alpha\right] \times \\ \prod_{j=m+1}^{n} \left[\int_{\sigma_j \beta^{(U)}+\mu_j}^{\infty} \xi_j(\alpha) d\alpha\right] d\beta^{(U)} \end{matrix} \right\}, \tag{18}$$

where $\phi\left(\beta^{(U)}\right)$ denotes the PDF of the standard Normal distribution.

In addition, occasionally, the assigned distributions of the tangible credit measures α_i and the likely bankruptcy threshold values β_i are merely rough approximations. In other instances, these disseminations are completely indefinite. In these cases, we can form analogous discrete versions of the continuous models. When such empirical estimation difficulties arise, the next discretization process can allow enhanced model flexibility and better calibration of model parameters with genuine data. The following standard numerical methodology divides the feasible domains of the above parameters into a large set of small intervals, discretizes the continuous environments, and approximates the probabilities of default with respective polynomial sums as:

$$\widehat{PD}_{Disc}^{ID} = \sum_{m=k}^{n} \left\{ \binom{n}{m} \sum_i [P(\beta_i)]^m \, [1 - P(\beta_i)]^{n-m} \psi(\beta_i) \Delta\beta_i \right\}, \tag{19}$$

with $i \in \{1,2, \dots, n\}$ and

$$\widehat{PD}_{Disc}^{non-ID} = \sum_{m=k}^{n} \left\{ \binom{n}{m} \sum_i \prod_{i=1}^{m} [P(\beta_i)] \prod_{i=m+1}^{n} [1 - P(\beta_i)] \, \psi(\beta_i) \Delta\beta_i \right\}, \tag{20}$$

where according to equation (2), $P(\beta_i) \stackrel{\text{def}}{=} \int_{-\infty}^{\beta_i} \xi(\alpha) d\alpha$ is the credit component-related failure conditional probability with respect to a specific bankruptcy threshold value β_i, $\psi(\beta_i)\Delta\beta_i$ denotes the frequency with which the bankruptcy threshold value falls inside the discrete and open interval $\left(\beta_i - \frac{\Delta\beta_i}{2}, \beta_i + \frac{\Delta\beta_i}{2}\right)$, and $\Delta\beta_i$ denotes an arbitrary small change in the respective bankruptcy threshold value.

EMPIRICAL INVESTIGATION

The Data

We choose to demonstrate the functionality of the current model with five credit components of the Altman (1968) Z-score due to its solid reputation among both academics and practitioners as a predictive model for the financial health of industrial firms. To illustrate the present model, we collect 124 consecutive quarters (31 years) from the Compustat dataset. This data source provides us with company, sector, and date vital identifiers, and all the necessary variables for computing a sufficient sample of all relevant credit components within the Altman (1968) Z-scores, from January 1980 to December 2010. These required variables include current assets, non-current assets, current liabilities, long-term liabilities, other liabilities, operating income after depreciation, retained earnings, total revenues, working capital, common shares outstanding, and quarterly closing share prices. This data source further allows us to identify bankrupt firms as having S&P quality current rankings mostly tagged with '*D*' (for Chapter 11 reorganization) and a few with 'LIQ' (for Chapter 7 liquidation).

From the entire available sample we eliminate observations with missing data and these records of financials and utilities (identified through GSECTOR 40 and 55). These sectors are constantly regulated and their accounting statements are structured in different ways than in other industries. We later focus on industrial firms (identified through GSECTOR 20). To comprehend the true essence of the proposed theory and to reduce the impact of outliers we further winsorize those accounting measures that fall in the top and bottom two percentiles.

Descriptive Information

Our complete sample encloses 192,861 records scattered over 4,079 firms, where 417 of them have filed for bankruptcy over the following years: from 1980 to 1984 (67), from 1985 to 1989 (35), from 1990 to 1994 (93), from 1995 to 1999 (124), from 2000 to 2004 (59), and from 2005 to 2010 (39). Three firms having 63 observations are classified in Chapter 7 liquidation within these 417 bankrupt firms. The remaining 3,662 firms have either remained operational or were otherwise omitted from the database after a while.

The 4,079 firms in the final sample are clustered into eight different sectors as follows: energy (264), materials (309), industrials (683), consumer discretionary (817), consumer staples (213), health care (708), information technology (991), telecommunication services (69), and others (25). The 417 bankrupt firms within are further distributed across these industries: energy (29), materials (42), industrials (62), consumer discretionary (123), consumer staples (15), health care (41), information technology (83), telecommunication services (14), and others (8).

Methodologies and Findings

To begin with, we wish to validate the proposed model's basic assumptions, namely to authenticate that both tangible values of the applicable credit components and estimated bankruptcy thresholds associated with these credit modules are indeed dispersed and intersect each other. Therefore, we split the entire sample into two groups. The first subsample contains 181,212 non-bankrupt observations. The second subsample includes 11,649 bankrupt records. Since the first collection is nearly 16 times larger than the second set of observations, we balance these two subsamples while preserving the primary distributional properties by inflating the size of the latter group through a bootstrapping method. This standard procedure allows us to visually contrast the respective histograms.

We then disintegrate, for each observation, the Altman (1968) Z-score into five credit components conforming to the five accounting ratios within as follows:

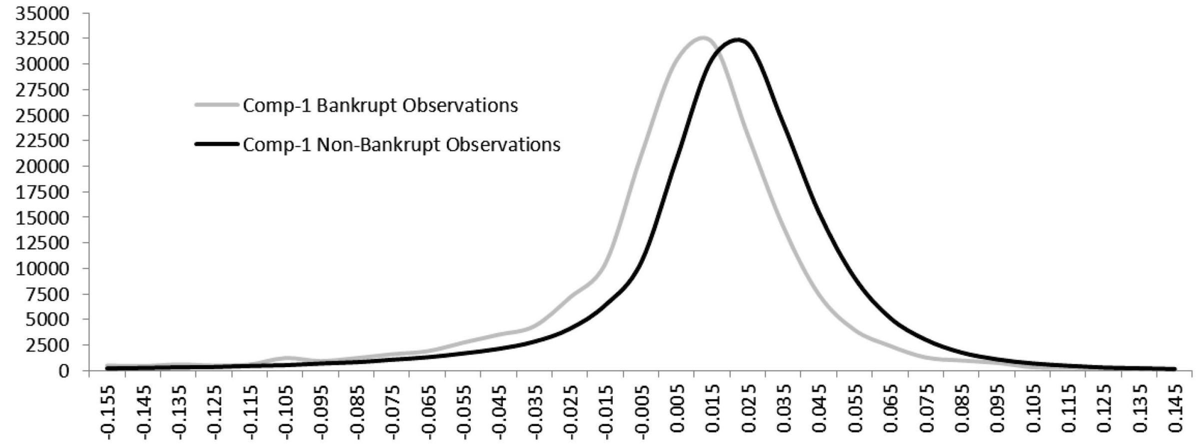
$$Z - score = 3.3 \times \frac{EBIT}{Total\ Assets} + 0.999 \times \frac{Sales}{Total\ Assets}$$
$$+0.6 \times \frac{Market\ Value\ of\ Equity}{Total\ Liabilities} + 1.2 \times \frac{Working\ Capital}{Total\ Assets} + 1.4 \times \frac{Retained\ Earnings}{Total\ Assets}, \qquad (21)$$

where $Comp\_1 \stackrel{\text{def}}{=} \frac{EBIT}{Total\ Assets}$, $Comp\_2 \stackrel{\text{def}}{=} \frac{Sales}{Total\ Assets}$, and so on.

Next, we plot the paired distributions of these five credit components: actual values among non-bankrupt observations and estimated bankruptcy thresholds among defaulted records. We then confirm that these distributions intersect within **Panels A** of **Figures 1 – 5**. Below each chart we report the minimum, the maximum, the mean, and the standard deviation of the bankrupt and non-bankrupt set of observations, respectively. These figures further assist us in substantiating the key assumptions of the proposed model.

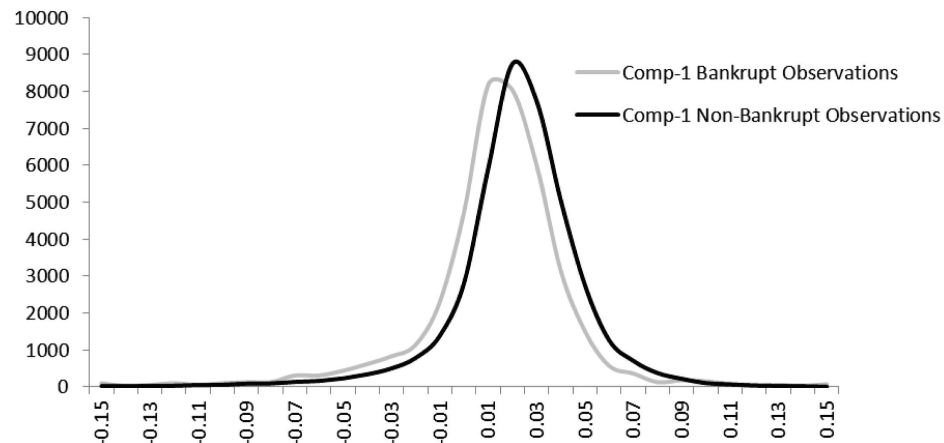
FIGURE 1
MATCHING DISTRIBUTIONS OF CREDIT COMPONENT NO. 1

Panel A: Entire Market $\frac{EBIT}{Total\ Assets}$ (Excluding Financials and Utilities)



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-1.282	0.318	0.006	0.056
Non-Bankrupt Observations	-2.103	1.834	0.017	0.046

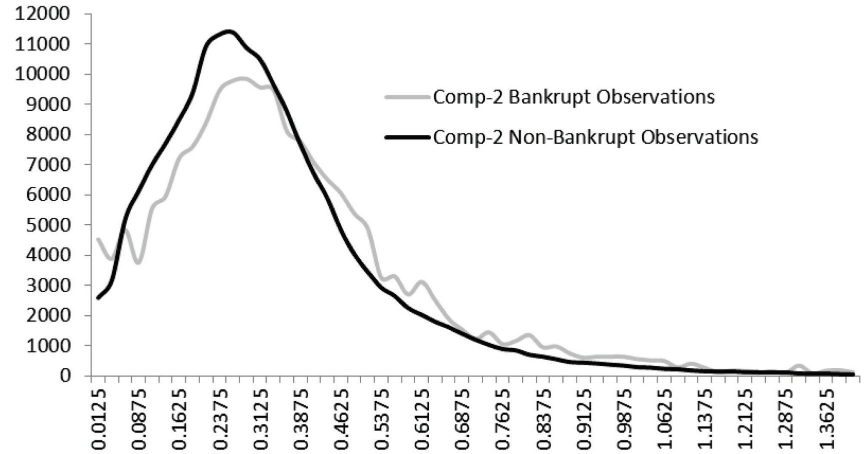
Panel B: Industrial Firms Only $\frac{EBIT}{Total\ Assets}$



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-0.277	0.174	0.014	0.035
Non-Bankrupt Observations	-0.605	0.394	0.022	0.031

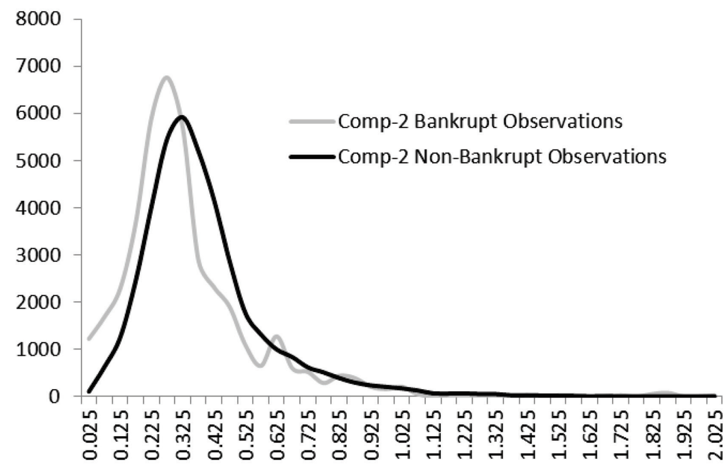
FIGURE 2
MATCHING DISTRIBUTIONS OF CREDIT COMPONENT NO. 2

Panel A: Entire Market $\frac{Sales}{Total\ Assets}$ (Excluding Financials and Utilities)



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	0.000	4.190	0.374	0.268
Non-Bankrupt Observations	0.000	8.272	0.334	0.240

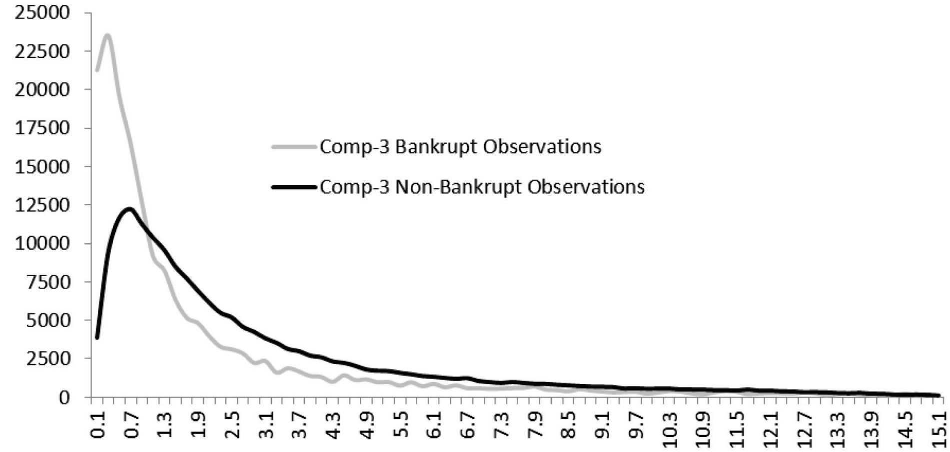
Panel B: Industrial Firms Only $\frac{Sales}{Total\ Assets}$



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	0.001	2.737	0.351	0231
Non-Bankrupt Observations	0.000	4.850	0.369	0.222

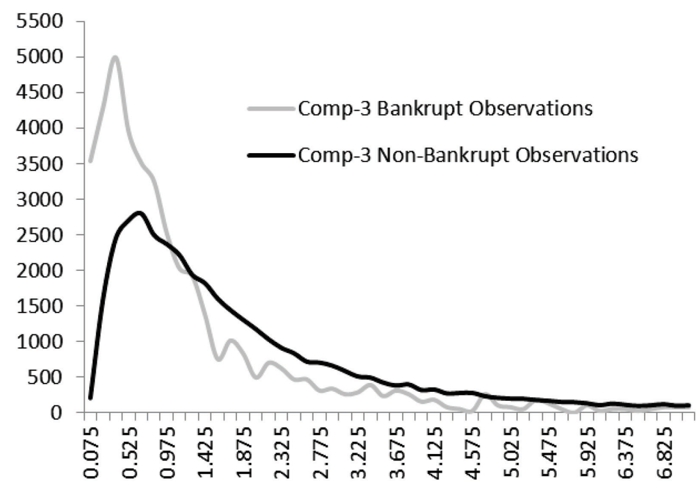
FIGURE 3
MATCHING DISTRIBUTIONS OF CREDIT COMPONENT NO. 3

Panel A: Entire Market $\frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$ (Excluding Financials and Utilities)



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	0.000	40.039	2.244	3.350
Non-Bankrupt Observations	0.006	77.617	3.244	3.531

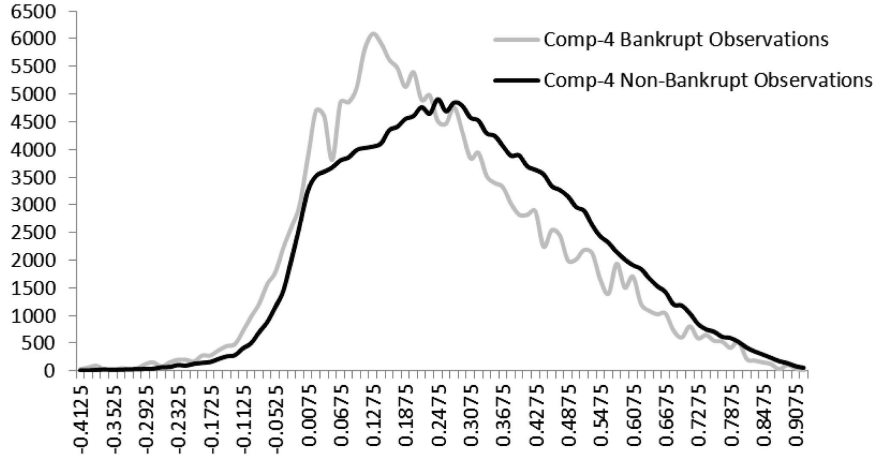
Panel B: Industrial Firms Only $\frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	0.006	15.328	0.993	1.232
Non-Bankrupt Observations	0.007	43.028	2.232	2.552

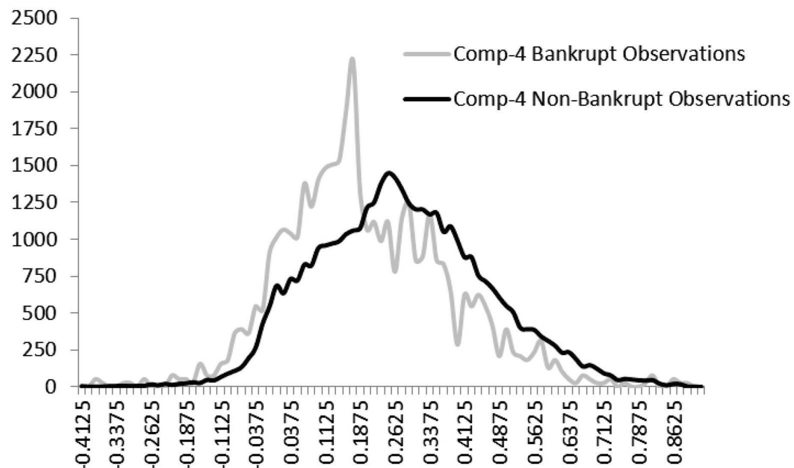
FIGURE 4
MATCHING DISTRIBUTIONS OF CREDIT COMPONENT NO. 4

Panel A: Entire Market $\frac{Working\ Capital}{Total\ Assets}$ (Excluding Financials and Utilities)



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-0.736	0.904	0.245	0.212
Non-Bankrupt Observations	-1.179	0.964	0.291	0.216

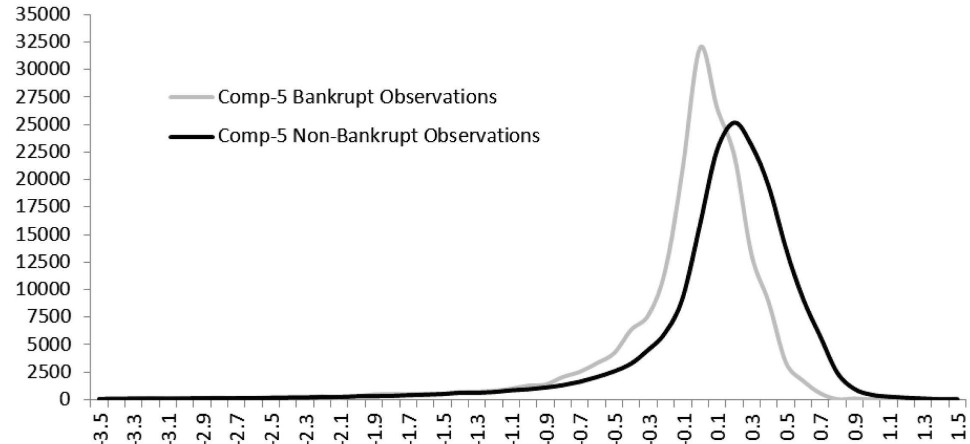
Panel B: Industrial Firms Only $\frac{Working\ Capital}{Total\ Assets}$



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-0.397	0.867	0.176	0.159
Non-Bankrupt Observations	-0.427	0.902	0.275	0.183

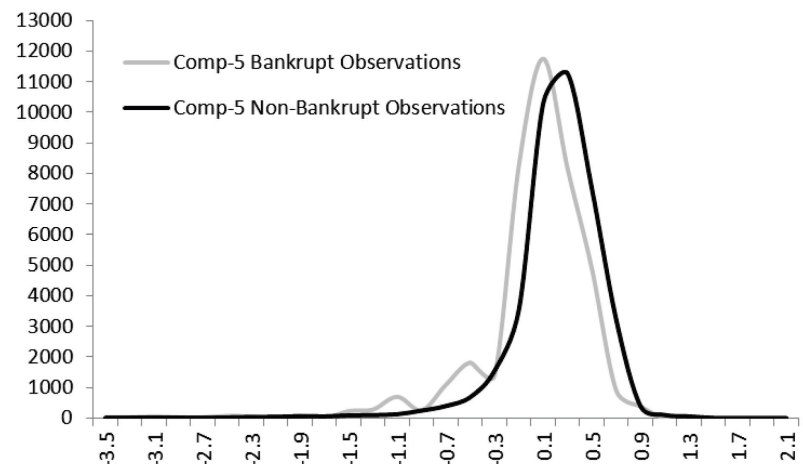
FIGURE 5
MATCHING DISTRIBUTIONS OF CREDIT COMPONENT NO. 5

Panel A: Entire Market $\frac{Retained\ Earnings}{Total\ Assets}$ (Excluding Financials and Utilities)



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-14.091	2.161	-0.070	0.667
Non-Bankrupt Observations	-28.915	2.662	0.061	0.744

Panel B: Industrial Firms Only $\frac{Retained\ Earnings}{Total\ Assets}$



	Minimum	Maximum	Mean	Standard Deviation
Bankrupt Observations	-5.328	0.676	0.011	0.458
Non-Bankrupt Observations	-14.422	1.621	0.195	0.454

We are able to witness that throughout this aggregate market analysis both credit components and the respective bankruptcy threshold values are certainly dispersed and intersect each other. Habitually, the non-bankrupt observations are stretched over larger values, while the bankrupt records are extended over lower quantities.[9] In addition, some credit modules exhibit dispersion patterns that closely resemble the Normal distribution, mainly because they draw statistics from figures that can be either positive or negative. These are credit components one ($\frac{EBIT}{Total\ Assets}$) and five ($\frac{Retained\ Earnings}{Total\ Assets}$). Other profiles of strictly non-negative accounting ratios better describe truncated Normal distributions. These are credit components two ($\frac{Sales}{Total\ Assets}$) and three ($\frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$). Credit module four ($\frac{Working\ Capital}{Total\ Assets}$), on the other hand, has few characteristics which are similar to the Normal distribution, but overall its histogram image is rather ambiguous.[10]

After corroborating the model's fundamental assumptions with the aggregate dataset we now turn to examine these assumptions over the restricted subsample of industrial firms. We recall that the Altman (1968) Z-score directly aims towards these particular companies. Similar to before, we split the industrial firms into two groups. The first set comprises 40,035 non-bankrupt observations. The second subsample includes 1,565 bankrupt records. Since the first collection is nearly 26 times larger than the second set of observations, once again we balance these two subsamples while protecting the main distributional properties by increasing the size of the second group through a standard bootstrapping method. We report the respective results for the singular credit components in **Panels B** of **Figures 1 – 5**. Below each chart we report again the minimum, the maximum, the mean, and the standard deviation of the bankrupt and non-bankrupt set of observations, respectively.

Overall, the findings within these analyses are not materially different than the results for the entire market. All five credit components and the respective bankruptcy threshold values are indisputably dispersed and intersect each other while the non-bankrupt observations are stretched over larger values, and the bankrupt records are extended over lower quantities. Moreover, similar to the prior analysis, credit components one, four, and five exhibit dispersion patterns that reasonably resemble the Normal distribution, while credit modules two and three better describe truncated Normal distributions due to their non-negative feasible domains.

A complementary appraisal of the minimum values, the maximum records, and the mean estimations for the bankrupt and non-bankrupt industrial credit components further support our early presumptions for dispersed distributions that intersect each other with universally higher quantities among the non-bankrupt observations. We therefore conclude that our model assumptions are valid and truly hold in practice.

We now turn to explore the model's functionality with individual cases of industrial firms. Since most of the five credit components under investigation practically follow Normal disseminations, yet they are not identically distributed, we deploy the next analyses through the prime model as presented in equation (15). To uncover the minimum number k of credit component failures that may trigger a corporate default we arbitrarily define $Z^* = 1.62$ as the universal bankruptcy threshold.[11] For each observation we examine whether the underlying firm remains operational under 32 different scenarios that independently alternate the separate credit components towards their respective bankruptcy thresholds. Explicitly, we inspect whether the corresponding firms default (their hypothetical Z-scores fall below Z^*) or survive (their theoretical Z-scores remain above Z^*) when zero, one, two, three, four, or all five related credit components are induced to fail (alternated to touch their respective timely threshold values).[12]

Subsequently, we compute the minimum required number k of credit component failures that can trigger corporate default, while further identifying the precise credit components that may prompt this hypothetical bankruptcy scenario. The SAS package further allows us to solve the nested integrals within equation (15) through the *'Proc IML'* procedure accompanied by the '*call quad'* routine and then to directly aggregate the ultimate probabilities of default.

To illustrate how PD diverge, we report sample results for the non-bankrupt industrial firms in **Table 1**. We also demonstrate the model's performance over bankrupt industrial firms in **Table 2**. Within these two tables and for each instance, we present the company's ticker, the year, the quarter, the S&P credit rating, the credit-related accounting measures, the complete Z-score, the five measures of credit components, the minimum number k of credit component failures that may cause a bankruptcy, and the respective probability of default. Sample cases are sorted within these two tables in consecutive order of the probabilities of default.

TABLE 1
SAMPLE RESULTS FOR NON-BANKRUPT INDUSTRIAL FIRMS 1980 – 2010

Ticker	Year	Quarter	S&P Credit Rating	EBIT	Total Assets	Sales	Market Value of Equity	Total Liabilities	Working Capital	Retained Earnings	Z-Score	Credit Comp-1	Credit Comp-2	Credit Comp-3	Credit Comp-4	Credit Comp-5	k	PD
AIR	1984	2	B-	3.435	137.228	51.374	117.136	66.142	55.823	18.387	2.195	0.025	0.374	1.771	0.407	0.134	5	0.004
AIR	1984	1	B-	2.880	132.559	44.706	112.778	61.947	55.965	17.470	2.192	0.022	0.337	1.821	0.422	0.132	4	0.025
AIR	1988	3	B-	9.823	314.885	90.007	391.414	177.537	114.763	62.567	2.427	0.031	0.286	2.205	0.364	0.199	3	0.106
ALC.	2004	2	B	13.535	378.345	136.377	249.024	125.554	67.419	241.244	2.775	0.036	0.360	1.983	0.178	0.638	3	0.116
ASEI	2005	1	B-	4.159	98.287	26.968	372.926	59.964	64.932	2.081	4.968	0.042	0.274	6.219	0.661	0.021	3	0.129
AIM	2003	4	B	0.556	18.789	7.972	38.700	14.614	2.910	3.375	2.548	0.030	0.424	2.648	0.155	0.180	3	0.147
AIR	1985	4	B-	5.731	167.363	61.040	145.104	92.182	57.965	28.068	2.072	0.034	0.365	1.574	0.346	0.168	3	0.160
CECE	**2007**	**1**	**C**	**2.494**	**81.879**	**43.463**	**143.598**	**78.223**	**17.872**	**-4.768**	**1.913**	**0.030**	**0.531**	**1.836**	**0.218**	**-0.058**	**3**	**0.165**
CECE	**2007**	**2**	**C**	**3.307**	**81.879**	**59.247**	**167.884**	**55.714**	**16.457**	**-3.623**	**2.843**	**0.040**	**0.724**	**3.013**	**0.201**	**-0.044**	**3**	**0.182**
AIR	1998	4	B-	18.901	737.416	316.825	695.739	453.208	347.314	163.991	2.311	0.026	0.430	1.535	0.471	0.222	3	0.207
APOG	1997	3	B+	28.214	494.927	246.015	608.519	409.961	119.624	134.330	2.245	0.057	0.497	1.484	0.242	0.271	3	0.229
AIM	2007	3	B	-1.734	17.422	5.292	26.375	10.822	7.286	6.675	2.476	-0.100	0.304	2.437	0.418	0.383	2	0.253
AIRT	1997	4	B	1.383	17.506	16.463	13.255	10.168	7.603	1.606	2.632	0.079	0.940	1.304	0.434	0.092	3	0.259
CECE	**2010**	**3**	**C**	**0.962**	**73.059**	**33.860**	**85.483**	**46.335**	**18.531**	**-8.934**	**1.747**	**0.013**	**0.463**	**1.845**	**0.254**	**-0.122**	**2**	**0.274**
ECOL	2005	2	B	5.862	85.106	18.779	315.684	41.070	19.317	2.339	5.370	0.069	0.221	7.686	0.227	0.027	2	0.305
AIR	1985	2	B-	5.420	155.405	60.128	113.175	78.323	56.949	24.434	2.028	0.035	0.387	1.445	0.366	0.157	2	0.325
IEX	1994	2	A	15.680	370.162	93.559	502.005	300.980	89.109	14.600	1.737	0.042	0.253	1.668	0.241	0.039	2	0.351
LGL	2010	2	C	2.351	21.296	12.535	25.245	11.241	8.862	-8.420	2.246	0.110	0.589	2.246	0.416	-0.395	2	0.354
HXL	2008	3	B-	36.600	1188.700	331.400	1318.347	759.100	295.300	7.300	1.729	0.031	0.279	1.737	0.248	0.006	2	0.387
RRD	1981	4	B-	42.413	865.797	360.081	712.682	402.693	72.346	416.477	2.413	0.049	0.416	1.770	0.084	0.481	2	0.393
CECE	**1983**	**2**	**C**	**-0.187**	**1.875**	**2.320**	**4.865**	**1.178**	**0.416**	**-0.369**	**3.376**	**-0.100**	**1.237**	**4.130**	**0.222**	**-0.197**	**2**	**0.402**
AVY	1989	3	B	34.300	1125.800	422.700	1237.236	740.600	187.400	327.100	2.085	0.030	0.375	1.671	0.166	0.291	2	0.406
CECE	**1983**	**3**	**C**	**-0.035**	**1.825**	**0.525**	**5.239**	**1.146**	**0.394**	**-0.403**	**2.917**	**-0.019**	**0.288**	**4.572**	**0.216**	**-0.221**	**1**	**0.406**
GV	2005	4	C	1.544	33.481	10.386	21.989	14.020	12.488	1.395	1.909	0.046	0.310	1.568	0.373	0.042	2	0.441
BMMI	1993	4	C	6.860	130.312	54.925	119.948	85.691	34.517	16.160	1.926	0.053	0.421	1.400	0.265	0.124	2	0.453
FSS	1985	2	B-	11.761	171.808	72.497	139.343	110.052	63.833	16.092	1.984	0.068	0.422	1.266	0.372	0.094	2	0.488
AIR	1980	1	B-	2.555	71.509	32.315	34.751	49.709	27.032	12.797	1.693	0.036	0.452	0.699	0.378	0.179	1	0.524
IKN	1983	3	B	58.452	1059.544	735.340	812.786	741.324	307.319	341.890	2.333	0.055	0.694	1.096	0.290	0.323	2	0.538
AIR	1994	1	B-	5.562	406.572	96.199	252.492	235.191	240.682	94.053	1.960	0.014	0.237	1.074	0.592	0.231	1	0.552
STS	1985	2	C	2.308	21.267	10.174	12.150	15.345	6.548	2.947	1.875	0.109	0.478	0.792	0.308	0.139	1	0.580
CECE	**2006**	**4**	**C**	**2.447**	**63.188**	**41.498**	**103.020**	**62.578**	**14.311**	**-5.258**	**1.927**	**0.039**	**0.657**	**1.646**	**0.226**	**-0.083**	**2**	**0.586**
UIC	2004	2	B	15.975	173.937	109.560	302.920	172.296	46.086	-35.410	2.020	0.092	0.630	1.758	0.265	-0.204	2	0.590
RCMT	1986	4	C	-0.032	2.992	3.806	5.883	2.185	0.730	-1.859	2.274	-0.011	1.272	2.692	0.244	-0.621	1	0.618
SPZN	1993	2	C	1.561	18.145	12.501	20.696	16.976	4.553	1.548	2.124	0.086	0.689	1.219	0.251	0.085	2	0.632
DY	1997	2	B	5.753	66.549	80.488	88.817	52.878	14.089	-3.848	2.674	0.086	1.209	1.680	0.212	-0.058	2	0.654
AIRT	2001	3	B	1.332	28.072	25.273	7.279	18.834	14.002	3.223	2.047	0.047	0.900	0.386	0.499	0.115	1	0.669
IFSIA	2007	4	B-	39.440	835.232	293.348	1009.408	640.326	238.578	-44.692	1.720	0.047	0.351	1.576	0.286	-0.054	1	0.696
YRK	1992	2	B	57.128	1222.489	517.508	1217.509	798.271	211.860	-0.203	1.700	0.047	0.423	1.525	0.173	0.000	1	0.793
IIIN	2004	2	B	26.618	142.105	96.835	54.577	102.536	50.094	1.488	2.056	0.187	0.681	0.532	0.353	0.010	1	0.841
CECE	**1982**	**2**	**C**	**0.160**	**2.249**	**3.693**	**2.058**	**2.041**	**0.492**	**-0.130**	**2.662**	**0.071**	**1.642**	**1.009**	**0.219**	**-0.058**	**1**	**0.882**

To illustrate how the proposed model's PD diverge among the non-bankrupt records, this table reports a sample of representative findings from the 41,035 non-bankrupt industrial observations. For each case, we present the company's ticker, the year, the quarter, the corresponding S&P credit rating, the credit-related accounting measures, the complete Altman (1968) Z-score, the five measures of credit components computed as: $Comp\_1 \stackrel{\text{def}}{=} \frac{EBIT}{Total\ Assets}$, $Comp\_2 \stackrel{\text{def}}{=} \frac{Sales}{Total\ Assets}$, $Comp\_3 \stackrel{\text{def}}{=} \frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$, $Comp\_4 \stackrel{\text{def}}{=} \frac{Working\ Capital}{Total\ Assets}$, $Comp\_5 \stackrel{\text{def}}{=} \frac{Retained\ Earnings}{Total\ Assets}$, the minimum number k of credit component failures that may cause a bankruptcy, and the respective probability of default PD as estimated within equation (15). These examples are sorted in consecutive order of the estimated probabilities of default. For better assessment, we highlight eight instances of the same company 'AIR' with a gray color. All share the same credit rating of 'B-.' We further highlight seven instances of the same company 'CECE' with bold fonts. All share the same credit rating of 'C.'

Some interesting inquiries arise from these sample results. For example, the company 'AIR' appears eight times in **Table 1**. In all of these eight instances (highlighted with gray color in the table), the firm gets a credit rating of 'B-,' while the Z-scores are mostly stable. Nonetheless, the respective PD substantially vary across these cases due to additional information acquired regarding the stochastic behavior of the credit components and the bankruptcy threshold values. Likewise, the company 'CECE' emerges seven times in **Table 1**. In all of these seven cases (accentuated with bold fonts in the table), the firm gets a credit rating of 'C,' while the Z-scores moderately diverge. Yet, the respective PD are considerably different across these records. We further perceive that, as expected by the theory, lower PD are continuously associated with higher minimum number k of credit component failures that may cause a bankruptcy, and vice versa.

Additionally, we also notice that the Z-scores observed within **Table 2** are not substantially different than those detected in **Table 1**. However, the ultimate probabilities of default largely exhibit higher values in the table of bankrupt industrial firms. A special attention should be given to the last two rows on **Table 2**. These remarkable examples present two bankrupt firms having market value of equity more than three times their respective total liabilities. Furthermore, these two corporations exhibit reasonable Z-scores. Nonetheless, the proposed model assigns high probabilities of default due to its ability to capture additional information regarding the stochastic behavior of the five different credit components.

TABLE 2
SAMPLE RESULTS FOR BANKRUPT INDUSTRIAL FIRMS 1980 – 2010

Ticker	Year	Quarter	S&P Credit Rating	EBIT	Total Assets	Sales	Market Value of Equity	Total Liabilities	Working Capital	Retained Earnings	Z-Score	Credit Comp-1	Credit Comp-2	Credit Comp-3	Credit Comp-4	Credit Comp-5	k	PD
MZIAQ	1980	4	D	25.833	626.696	206.021	733.882	412.108	253.923	231.289	2.536	0.041	0.329	1.781	0.405	0.369	4	0.176
FRNTQ	2000	3	D	30.593	254.177	131.083	349.894	235.307	68.013	50.580	2.404	0.120	0.516	1.487	0.268	0.199	3	0.306
3ACKH	1983	3	D	36.056	978.274	370.374	653.019	432.864	308.115	553.250	2.575	0.037	0.379	1.509	0.315	0.566	3	0.368
3ACKH	1984	2	D	48.604	993.591	408.320	615.359	421.818	301.895	588.847	2.642	0.049	0.411	1.459	0.304	0.593	3	0.370
MESAQ	1992	1	D	4.464	108.447	75.210	148.648	69.052	18.788	16.817	2.545	0.041	0.694	2.153	0.173	0.155	3	0.408
MCSIQ	1997	4	D	1.242	49.921	36.136	63.437	27.331	8.188	5.950	2.562	0.025	0.724	2.321	0.164	0.119	3	0.409
3ACKH	1987	4	D	60.500	1602.500	605.000	1491.466	813.900	255.300	931.100	2.606	0.038	0.378	1.832	0.159	0.581	3	0.411
MESAQ	1991	4	D	-0.044	91.298	73.160	94.798	54.052	15.738	13.771	2.269	0.000	0.801	1.754	0.172	0.151	2	0.509
3ACKH	1984	1	D	38.494	994.550	383.496	622.673	425.234	271.589	574.943	2.529	0.039	0.386	1.464	0.273	0.578	2	0.510
FRNTQ	2000	2	D	25.374	220.710	112.809	254.677	212.158	53.093	30.386	2.092	0.115	0.511	1.200	0.241	0.138	2	0.512
EECPQ	1990	3	D	1.722	50.009	25.058	97.456	35.834	21.693	-4.566	2.639	0.034	0.501	2.720	0.434	-0.091	3	0.519
MCSIQ	2002	2	D	21.163	447.483	123.504	280.075	150.067	158.714	-7.957	1.952	0.047	0.276	1.866	0.355	-0.018	3	0.522
GWOW	1989	1	D	0.581	87.620	21.866	67.872	36.306	44.408	58.053	2.929	0.007	0.250	1.869	0.507	0.663	3	0.523
FRNTQ	1999	3	D	13.102	152.935	85.453	167.029	152.644	43.650	3.559	1.873	0.086	0.559	1.094	0.285	0.023	2	0.576
BUTLQ	1997	3	D	3.396	103.638	106.465	109.172	62.791	32.171	-53.606	1.826	0.033	1.027	1.739	0.310	-0.517	1	0.585
FLYIQ	1997	1	D	1.037	65.177	41.114	114.858	46.723	14.662	-2.394	2.376	0.016	0.631	2.458	0.225	-0.037	2	0.660
3ACKH	1982	4	D	3.273	903.031	311.739	612.538	367.757	275.540	535.774	2.553	0.004	0.345	1.666	0.305	0.593	2	0.732
3ACKH	1989	3	D	70.300	2169.900	629.200	1931.199	1095.100	383.700	856.000	2.219	0.032	0.290	1.763	0.177	0.394	2	0.776
DZTKQ	2002	2	D	6.745	573.889	402.496	308.316	388.610	228.958	89.866	1.913	0.012	0.701	0.793	0.399	0.157	1	0.808
FLYIQ	1997	3	D	9.054	137.288	54.864	153.403	127.376	40.660	8.335	1.780	0.066	0.400	1.204	0.296	0.061	1	0.809
3ACKH	1980	4	D	25.953	891.513	331.212	352.645	346.515	297.473	525.646	2.304	0.029	0.372	1.018	0.334	0.590	1	0.809
BUTLQ	1997	4	D	3.414	104.716	105.383	111.650	79.686	33.551	-49.630	1.675	0.033	1.006	1.401	0.320	-0.474	1	0.845
LCLGQ	1985	2	D	-0.388	5.767	0.969	9.333	3.947	1.882	0.270	1.822	-0.067	0.168	2.364	0.326	0.047	1	0.869
EECPQ	1990	4	D	0.609	48.904	20.807	112.860	33.407	21.266	-3.154	2.925	0.012	0.425	3.378	0.435	-0.064	2	0.897
RUNIQ	1993	4	D	-2.298	55.007	1.520	22.824	6.917	15.790	2.228	2.271	-0.042	0.028	3.300	0.287	0.041	1	0.913

To demonstrate how the proposed model's PD diverge among the bankrupt records, this table reports a sample of representative findings from the 1,565 bankrupt industrial observations. For each case, we present the company's ticker, the year, the quarter, the corresponding S&P credit rating, the credit-related accounting measures, the complete Altman (1968) Z-score, the five measures of credit components computed as: $Comp\_1 \stackrel{\text{def}}{=} \frac{EBIT}{Total\ Assets}$, $Comp\_2 \stackrel{\text{def}}{=} \frac{Sales}{Total\ Assets}$, $Comp\_3 \stackrel{\text{def}}{=} \frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$, $Comp\_4 \stackrel{\text{def}}{=} \frac{Working\ Capital}{Total\ Assets}$, $Comp\_5 \stackrel{\text{def}}{=} \frac{Retained\ Earnings}{Total\ Assets}$, the minimum number k of credit component failures that may cause a bankruptcy, and the respective probability of default PD as estimated within equation (15). These examples are sorted in consecutive order of the estimated probabilities of default.

In **Table 3** we further contrast the main statistical properties of the ultimate probabilities of default between the two subsamples of bankrupt and non-bankrupt industrial firms as derived from the suggested model. We thus report the number of observations, the minimum, the maximum, the mean, the standard deviation, and the respective third and fourth central moments.

TABLE 3
COMPARATIVE PROPERTIES OF DEFAULT PROBABILITIES AMONG INDUSTRIAL FIRMS

	Bankrupt Observations	Non-Bankrupt Observations
Number of Measures	1,565	40,035
Minimum	0.1762	0.0042
Maximum	0.9130	0.8817
Mean	0.7874	0.3607
Standard Deviation	0.0986	0.2141
Skewness	-4.1049	-0.5889
Kurtosis	18.0680	-1.1086

To better determine the proposed model's capabilities, this table compares the main statistical characteristics of the estimated probabilities of default PD between the two subsamples of 1,565 bankrupt and 40,035 non-bankrupt industrial observations from 1980 to 2010. We report the number of valid measures, the minimum, the maximum, the mean, the standard deviation, as well as the respective third and fourth central moments.

Throughout this comparison we provide strong evidence that empirical estimations of the model's probabilities of default among the bankrupt observations stretch from a higher minimum level to a higher maximum height than for the non-bankrupt records. Furthermore, the mean PD appears to be significantly higher, while the standard deviation is considerably lower within the set of bankrupt firms. The subsample of bankrupt industrials further exhibits a lower negative skewness, which indicates a longer left tail of the distribution, and a much higher kurtosis, which designates that more observations are clustered near the peak.[13]

These numerical measures illustrate the real added value of the current model when differentiating between bankrupt and non-bankrupt candidates. The proposed scheme can clearly differentiate between failure and non-failure nominees. In addition, the current theory utilizes additional vital credit-related information, thus can accurately segregate the credit qualities of firms having similar credit ratings or parallel Z-scores.

SUMMARY

In this study we explore a stochastic model that takes a new approach to assessing corporate credit risk. The proposed theory is not a structural credit model per se, since it does not directly examine the composition of available assets and outstanding debt issuances. On the other hand, although it draws input from accessible accounting records, the current scheme cannot be considered a credit score as well. The suggested model provides precise probabilities of default and it overcomes key drawbacks in common linear credit grades. In particular, the present methodology realistically assumes a non-linear behavior of the risky modules that eventually compose these customary credit scores. Hence, the current model analyzes the complete distributional attributes and utilizes higher central moments of the relevant credit components when studying the ultimate credit quality of a borrower. Through that, our theory implicitly assumes a dependent configuration of the underlying credit components.

Essentially, the proposed model approximates the likelihoods for feasible intersections between most recent observed values of credit elements and their respective estimated bankruptcy threshold quantities. These prospects do not consider fixed appraisals. Instead, the model constantly monitors the development

of the dispersal properties of these credit components as well as their likely default cutoff points. These pragmatic presumptions assist us in capturing supplementary vital credit-related information.
When tested over an inclusive set of empirical observations, the model performs well. Not only is it able to distinguish between comparable borrowers that hold similar credit scores or equal credit ratings, but the current methodology can also predict with a good precision bankrupt instances up to two years in advance.

The current theory is indeed more complicated than external credit scores or internal scorecards in use by most financial institutions, mainly because it utilizes additional credit-related vital information. Nonetheless, the suggested model draws input from accessible accounting data hence it is surely measurable, verifiable, and programmable. Therefore, it can serve a large scale of lenders that desire to achieve a much higher predictive power than other credit models towards likely business failures. The present scheme becomes highly usable since it relies upon favorably realistic assumptions and concrete statistical concepts. It can, therefore, assist banks to attain superior lending decisions and enhanced profitability than otherwise obtained by using alternative existing credit methodologies.

ENDNOTES

1. See for example the Altman (1968) Z-score, the Ohlson (1980) O-score, the cash-flow-based score of Aziz, Emanuel, and Lawson (1988), or the Altman (2010) Z-Metrics™ methodology.
2. Although a standard variance-covariance matrix can certainly capture the interrelationships among credit components, this somewhat naïve approach misses the variability of corporate break points and their coherent links to ad hoc macroeconomic conditions.
3. For instance, the Z-score contains $n = 5$ credit components, the O-score includes $n = 9$ credit modules, the cash-flow-based score incorporates $n = 5$ credit parts, and the Z-MetricsTM score comprises $n = 14$ static and trend credit elements.
4. Alternatively, we can also set the threshold values to mark the event of Chapter 7 liquidation, yet it is somewhat more intuitive to consider threshold values for Chapter 11 bankruptcy reorganization, since common credit scores customarily refer to this realm.
5. Rice (2006) provides more explanations on this relatively modest arithmetical transition.
6. In the later empirical analysis we thoroughly corroborate this assumption.
7. The specific scaling procedure of the underlying credit score from which the current model draws inferences dictates whether the credit components are identically distributed or not.
8. When directive credit component failures are appropriate, we can extend this idea to handle ordered permutations as well.
9. The contrasted means of the second credit component makes this testimony somewhat less apparent. However, even within this specific credit module, there are significantly more low-measures among the bankrupt records, than within the non-bankrupt observations.
10. This may serve as an example in which a discretization procedure for the continuous model is beneficial.
11. For purpose of robustness we test other critical values as well, yet it appears that optimal results are obtained near the inclusive threshold Z-score of 1.62.
12. We test 32 possible scenarios since $\sum_{i=0}^{5}\binom{5}{i} \stackrel{\text{def}}{=} \binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5} = 32$.
13. It would be difficult to further contrast these subsamples with parametric or non-parametric statistical tests since both locational and distributional parameters vary across the groups.

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