# Optimal Initial Margin and Maintenance Margin for Futures Contracts

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*I provide a model that a futures clearinghouse could use to set the optimal initial margin and maintenance margin ratio that minimize traders' costs and the clearinghouse's default risk. The costs include the opportunity cost of initial margin and the funding liquidity cost of meeting a margin call. The costs and the default risk depend on the initial margin and the maintenance margin ratio, whose optimal values are therefore interrelated. I apply my model to the WTI crude oil futures contract traded on the NYMEX and show that the optimal choice of initial margin depends on the maintenance margin ratio.*

## INTRODUCTION

On November 4, 2011, the Chicago Mercantile Exchange (CME) clearinghouse sent a notification to all clearing member firms, chief financial officers, back office managers and margin managers, which stated that after the close of business on that date, the ratio of initial margin to maintenance margin would equal 1 for all its products (CME Group, 2011a). As is well known, the initial margin is the amount deposited by a futures trader in its margin account with the broker on opening a futures position. Through the process of daily settlement, daily profits (losses) are credited to (debited from) the margin account. If the balance in the account falls to the maintenance margin level, a margin call is made under which the trader is required to add enough funds (variation margin) to bring the balance in the account back to the initial margin level. If the trader fails to meet the margin call, the broker would liquidate the trader's futures position.

The CME clearinghouse's notification caused panic among futures traders who interpreted the notice as implying that the maintenance margin would be increased to the level of the initial margin (Durden, 2011). Some trader accounts which were previously held with the clearing firm MF Global had been transferred to other brokers, when that clearing firm filed for bankruptcy protection on October 31, 2011. However, in many cases, only up-to 75% of the value of the accounts was transferred. This would mean that many accounts would be instantly under-margined. It was anticipated that many traders would have to liquidate their futures positions or the brokers would liquidate their futures positions if the traders failed to meet their margin calls. The panic subsided when the CME clearinghouse issued a clarification on November 5, 2011, which stated that the intent of the previous notice was to lower the initial margin to the maintenance margin level (CME Group, 2011b).

The reaction of traders to the CME clearinghouse's initial notice underscores the importance of the initial margin and the maintenance margin ratio (ratio of maintenance margin to the initial margin) and the need to set these at appropriate values. If traders are unable to meet margin calls and are forced to liquidate their positions, this could have an adverse effect upon futures prices. Brunnermeier and Pedersen (2009) note that the funding liquidity of traders, which is related to the traders' ability to raise funds, affects the market's liquidity, which relates to the ease with which an asset can be traded. If traders are forced to reduce their positions because they cannot raise the funds necessary to support their trading positions, then market liquidity would reduce and prices would be driven by funding liquidity aspects rather than market fundamentals. A prominent example of such a situation is that of the Hunt Brothers, who in March 1980 were called upon to meet margin calls exceeding $100 million on their silver futures positions, as is related by Brimmer (1989). Brimmer (1989) also notes that on October 19, 1987, a day on which the Dow Jones Industrial Average dropped 508 points, the CME clearinghouse made margin calls for a total amount exceeding $1 billion upon several brokerage firms. A further example is that of Metallgesellschaft, whose U. S. subsidiary, MG Refining and Marketing, was called upon to meet margin calls exceeding $900 million on its energy derivatives positions in December 1993 (Edwards and Canter, 1995).

In this paper, I build upon a framework that was first introduced by Brennan (1986), and provide a model that a futures clearinghouse could use to simultaneously set the optimal initial margin and maintenance margin ratio that will minimize the traders' costs and simultaneously minimize the default risk of the clearinghouse. Brennan (1986) points out that a successful futures contract will minimize the total costs of futures trading. The costs that he considers are the opportunity cost of the margin, the liquidity cost associated with a price limit hit that interrupts trading and the costs associated with defaulting on a futures contract. However, Brennan (1986) does not consider the maintenance margin ratio in his analysis.

In my model, the trader's costs are the sum of the opportunity cost associated with the initial margin requirement and the funding liquidity cost associated with meeting a variation margin call. The opportunity cost associated with the initial margin increases with the initial margin. The funding liquidity cost associated with a margin call depends on the probability that the margin call is made, which increases with increases in the maintenance margin ratio. The funding liquidity cost, however, also depends on the magnitude of the funds that have to be raised to meet the variation margin payment, which should increase with decreases in the maintenance margin ratio. For a given initial margin, when the maintenance margin ratio is high, the probability of occurrence of a margin call is high, but the amount that has to be raised to meet the variation margin requirement is low. The funding liquidity cost should therefore be nonlinearly related to the maintenance margin ratio. For a given maintenance margin ratio, the higher the initial margin, the lower the probability of a margin call, but the higher the magnitude of the funds that have to be raised to meet the margin call. Hence, the optimal initial margin choice should depend on the maintenance margin ratio chosen. I describe the model in the second section.

Previous researchers on setting the initial margin for individual traders either ignore the maintenance margin ratio (as for example, Brennan (1986)), or assume that it is a given (as for example, Day and Lewis (2004)). To the best of my knowledge, my paper is the first to recognize the interrelationship between the initial margin and the maintenance margin ratio and to address the futures exchange's problem of setting the optimal initial margin and maintenance margin ratio simultaneously.

I apply the model to the West Texas Intermediate (WTI) crude oil futures contract traded on the New York Mercantile Exchange (NYMEX), using futures price data extending from the period January 1, 2001 through October 21, 2011. I assume that the crude oil futures price follows a stochastic process characterized by stochastic volatility. I estimate the parameters of the stochastic processes, and solve for the optimal choice of the initial margin and the maintenance margin ratio for the short futures trader and the long futures trader. I describe the methodology and results in the third section. The results confirm that the optimal initial margin requirement is significantly positively related to the maintenance margin ratio. I provide conclusions in the last section.

## THEORETICAL MODEL ON THE OPTIMAL INITIAL MARGIN AND MAINTENANCE MARGIN RATIO

I explain the model that a futures clearinghouse could use to set the optimal level of initial margin and the optimal maintenance margin ratio that will minimize the futures trader's costs of maintaining a position in the futures market, which include the opportunity costs of the initial margin and the funding liquidity cost associated with meeting a variation margin call, and simultaneously minimize the clearinghouse's default risk.

**Background to the Model**

Let day *0* and day *T* represent the current date and the delivery date of the futures contract, respectively. On day *0*, the trader opens a futures position at the futures price $F_0$ and deposits an initial margin equal to $IM_S$ ($IM_L$) into his margin account with his broker, if his initial position is short (long). The maintenance margin is represented by $MM_S$ ($MM_L$) for the short (long) trader. Let $MMR_S$ ($MMR_L$) represent the maintenance margin ratio for the short (long) trader. Then, $MMR_S$ is given by:

$$MMR_S = \frac{MM_S}{IM_S} \tag{1}$$

while $MMR_L$ is given by:

$$MMR_L = \frac{MM_L}{IM_L} \tag{2}$$

I assume that the initial margin requirement poses an opportunity cost of $k.IM_S.T$ ($k.IM_L.T$), to the short (long) futures trader, where k>0 represents the annualized percentage opportunity cost, since funds deposited as margin are not available for use for other investments. If the initial margin is deposited in the form of Treasury bills which earn interest, the initial margin requirement still poses an opportunity cost since the trader's borrowing rate would be higher than its lending rate (Baer, France and Moser, 1994).

I assume that the trader's margin account is marked-to-market at the end of each day using the futures price at the end of that day, thus daily profits are credited to and daily losses are debited from the account. Assume that on a future day *t* between the current date, time *0*, and the delivery date, *T*, the short (long) trader's cumulative loss of $F_t - F_0$ ($F_0 - F_t$) causes the account balance to drop below the required maintenance margin level of $MM_S$ ($MM_L$). In order to meet the margin call, the trader is required to add funds equal to the cumulative loss to the margin account to bring the balance back up to the level of the initial margin. In normal times funds could be raised at an annualized percentage cost *k*, the cost associated with the initial margin requirement. However, I assume that since the margin call will have to be met at short notice, the funds needed to meet the margin call can only be raised at an annualized percentage cost *l*, which is greater than *k*. This could occur if the trader's bank extracted an interest rate to advance funds at short notice that is higher than the trader's normal borrowing rate or if the trader had to sell securities to raise funds at a more disadvantageous price than in normal times.

**Model of the Optimal Initial Margin and Maintenance Margin Ratio for the Short Futures Trader**

I first explain the model to set the optimal initial margin and maintenance margin ratio for the short futures trader in detail in this sub-section.

*Minimizing the Costs of the Short Futures Trader*

My model to set the optimal initial margin and maintenance margin ratio for the short futures trader is represented by the following:

Minimize

$$k.IM_S T + l(F_t - F_0)(T - t) \tag{3}$$

with respect to $IM_S$ and $MMR_S$.

In the objective function of equation (3), the first term represents the opportunity cost of the initial margin, and the second term represents the funding liquidity cost associated with meeting the variation margin call. $t$ is the smallest value between $0$ and $T$, at which cumulative losses cause the balance in the margin account to drop below the maintenance margin level or at which $F_t - F_0 \geq IM_S - MM_S$.

*Minimizing the Default Risk of the Futures Clearinghouse*

Note that the futures clearinghouse guarantees the open futures positions of its clearing members. Each clearing member guarantees the open futures positions of its customers, which include individual traders as well as brokers who are not clearing members. A default by an individual trader or broker on its open futures position thus exposes the clearinghouse to losses. A trader could default on its futures position when a margin call is made. In addition, even if the trader does not intend to default when a margin call is made, if a grace period is provided to deposit the variation margin payment, there is a risk that losses could occur which wipe out the initial margin in the interim, thus causing the trader to default.

I therefore impose constraints which are designed to minimize the risk of default by the trader. Brennan (1986) points out that a futures exchange could make a futures contract self-enforcing, in the sense that the trader would adhere to its terms without the threat of legal action, if the margin and price limit were set such that on a limit hit, the expected loss of the trader is less than or equal to the margin that he has deposited with the broker. Day and Lewis (2004) propose two standards, the zero net present value standard, and the zero default value standard, that could be used to set an adequate level of the initial margin requirement. They assume that the exchange closes out the futures position when the account balance drops below the maintenance margin level. They note that then the position of the short (long) futures trader is like that of the holder of an up-and-out put (down-and-out call) barrier option on the futures price with an exercise price equal to the current futures price plus (minus) the initial margin requirement and a barrier equal to the futures price plus (minus) the difference between the initial margin and the maintenance margin. The trader will have less incentive to default if the amount he pays to assume his futures position, which is the initial margin requirement, is equal to the value of his position, as given by the appropriate barrier option, so that his position has a zero net present value. Correspondingly, the expected costs to the futures exchange from default by a short (long) futures trader equals the value of an up-and-in call (down-and-in put) barrier option with an exercise price and barrier price equal to those of the up-and-out put (down-and-out call), and according to the zero default value standard, the initial margin requirement should be set so that the expected costs of default to the clearinghouse or the option value equals zero.

As in Brennan (1986), and Day and Lewis (2004), I recognize that futures traders have an incentive to default on their open futures positions if their losses exceed their initial margin requirement. As in Day and Lewis (2004), I first minimize the incentive for the short futures trader to default upon his open futures position by setting the initial margin to be greater than or equal to the value of its position, which is the value $p_{uo}$ of an up-and-out put option with an exercise price equal to $F_0+IM_S$ and a barrier price equal to $F0+IM_S-MM_S$. This constraint is given by:

$$.IM_S \geq p_{uo} \tag{4}$$

Note the Day and Lewis (2004) result that the expected costs to the futures exchange from default by a short futures trader may be quantified by the value $c_{ui}$ of an up-and-in call option with the same exercise price and barrier price as the up-and-out put $p_{u0}$ of the previous paragraph. The expected costs to the exchange are essentially the same as the expected losses to the trader from its open futures position. Note Brennan's (1986) recommendation that the margin should be set so that it equals or exceeds the expected losses of the trader. Combining the implications of Day and Lewis (2004), and of Brennan (1986), I constrain the initial margin to be set such that it is greater than or equal to the trader's expected losses. Thus:

$$.IM_S \geq c_{ui} \quad (5)$$

Figlewski (1984) focuses on the probability of occurrence of losses which exceed the margin requirement. He notes that the maintenance margin should be set, such that, the probability of occurrence of a loss within the grace period which wipes out the initial margin should be set at an acceptable level. Accordingly, I impose the following constraint:

$$\Pr ob\left(F_{t+n} - F_0 \geq IM_S \middle| F_t - F_0 \geq IM_S - MM_S\right) = p_{as} \quad (6)$$

where $n$ is the length of the grace period in days and $p_{as}$ is the acceptable level of the probability that the cumulative loss within the grace period exceeds the initial margin.

**Model of the Optimal Initial Margin and Maintenance Margin Ratio for the Long Futures Trader**

Similarly, my model to set the optimal initial margin and maintenance margin ratio for the long futures trader is represented by the following:

Minimize

$$k.IM_L T + l.(F_0 - F_t)(T - t) \quad (7)$$

with respect to $IM_L$ and $MMR_L$.

The first term in equation (7) represents the opportunity cost of the initial margin while the second term represents the funding liquidity cost associated with meeting the margin call. $t$ is the day of the margin call, or, the smallest value between day $0$ and day $T$, at which the cumulative losses cause the balance in the account to drop below the maintenance margin level or at which $F_0 - F_t \geq IM_L - MM_L$.

The following constraints are imposed to minimize the futures exchange' default risk. The first constraint ensures that the trader's incentive to default is minimized by setting the initial margin so that it equals or exceeds the value of the long trader's open futures position. This is the value $c_{do}$ of a down-and-out barrier call option on the futures price with an exercise price equal to the current futures price minus the initial margin requirement or $F_0 - IM_L$ and a barrier price equal to the current futures price minus the difference between the initial margin and the maintenance margin or $F_0 - (IM_L - MM_L)$. Thus, the first constraint is:

$$.IM_L \geq c_{do} \quad (8)$$

The second constraint ensures that the initial margin is set so that it equals or exceeds the expected losses to the trader, thus again minimizing the trader's incentive to default. The expected losses to the trader or the expected costs to the exchange are quantified by the value $p_{di}$ of a down-and-in put option with the same exercise price and barrier price as the down-and-out call option of the previous paragraph. Thus, the second constraint is:

$$.IM_L \geq p_{di} \quad (9)$$

Finally, the last constraint ensures that once the margin call is made, the probability that the cumulative losses wipe out the initial margin within the grace period is set at an acceptable level $p_{al}$. Thus, if $n$ is the length of the grace period in days following the margin call on day $t$:

$$\Pr ob\left(F_0 - F_{t+n} \geq IM_L \middle| F_0 - F_t \geq IM_L - MM_L\right) = p_{al} \quad (10)$$

## METHODOLOGY AND RESULTS

### Data Used

I obtain daily data on crude oil futures prices from Datastream and calculate a time series of daily futures returns $r_t$, for the period January 1, 2001 through October 21, 2011, using the following equation:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{11}$$

where $r_t$ is the futures return, and $P_t$ is the futures price on day $t$. Panel A of Table 1 shows the mean and standard deviation of the daily futures price and return.

### Estimation of the Parameters of the Stochastic Processes Governing the Futures Price and Volatility

I assume that the futures price change on day $t$ follows a stochastic volatility model described by the following equations:

**TABLE 1**
**DESCRIPTIVE STATISTICS ON THE CRUDE OIL FUTURES PRICE AND RETURN AND THE RESULTS OF THE ESTIMATION OF THE GARCH(1,1) MODEL OF THE FUTURES PRICE VOLATILITY**

**Panel A. Mean and standard deviation of the daily futures price and return for the crude oil futures contract for the period January 1, 2001 through October 21, 2011**

| Mean/Standard deviation of variable | |
|---|---|
| **Futures price $** | **Return $r_t$ %** |
| 58.97 | 0.0419 |
| 27.21 | 2.4842 |

**Panel B. Results of the estimation of the GARCH(1,1) model for the period January 1, 2001 through October 21, 2011**
**The GARCH(1,1) model is given by:**

$$r_t = mdt + \sqrt{V_t}\sqrt{dt}\varepsilon_t \tag{14}$$

$$V_t = w + \alpha\varepsilon_{t-1}^2 + \beta V_{t-1} \tag{15}$$

**Where *m*, $w, \alpha$ and $\beta$ are constant parameters and $\varepsilon_t$ is a standard normal random variable**

| Coefficient/t statistic | | | | Number of observations | Log-likehood |
|---|---|---|---|---|---|
| ***m*** | ***w*** | **α** | **β** | | |
| 0.00094 | 0.00001026 | 0.0516 | 0.9297 | 2820 | 6700.81 |
| 2.29 | 3.2096*** | 5.8585*** | 71.3459*** | | |

**Panel C. Parameters of the stochastic volatility model of the futures price**
**The parameters are obtained from the results of the GARCH(1,1) model as:**

$$\mu = m \tag{16}$$

$$a = 1 - \alpha - \beta \tag{17}$$

$$V_l = \frac{w}{1-\alpha-\beta} \tag{18}$$

$$\xi = \alpha\sqrt{2} \tag{19}$$

| **μ** | $V_L$ | ***a*** | **ξ** |
|---|---|---|---|
| 0.00000 | 0.000547 | 0.018765 | 0.072952 |

Note *** Significant at the 99% confidence interval

$$dF_t = \mu F_t dt + \sqrt{V_t} F_t \sqrt{dt}\varepsilon_{1t} \tag{12}$$

$$dV_t = a(V_L - V_t)dt + \xi V_t \sqrt{dt}\varepsilon_{2t} \tag{13}$$

Equation (12) represents the stochastic process followed by the futures price while equation (13) represents the stochastic process followed by the futures price volatility. In equation (12), $F_t$ is the futures price on day $t$, $\mu$ is the annualized expected futures return, $V_t$ is the annualized variance of the futures price on day $t$ and $\varepsilon_{1t}$ is a standard normal random variable. In equation (13), $V_L$ represents the long run variance of the futures price, $a$ and $\xi$ are constants, and $\varepsilon_{2t}$ is a standard normal random variable. Equation (13) represents a mean-reverting process in which the variance is pulled towards its long run value at a speed given by $a$. I assume that the futures price and its volatility are uncorrelated, hence $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are uncorrelated. I estimate the parameters of the processes by applying a GARCH(1,1) model to the time series of futures returns. The GARCH(1,1) model that I employ is given by:

$$r_t = mdt + \sqrt{V_t}\sqrt{dt}\varepsilon_t \tag{14}$$

$$V_t = w + \alpha\varepsilon_{t-1}^2 + \beta V_{t-1} \tag{15}$$

$m$, $w, \alpha$ and $\beta$ are constant parameters and $\varepsilon_t$ is a standard normal random variable. This allows me to estimate $\mu$ as:

$$\mu = m \tag{16}$$

$a$, $V_L$ and $\xi$ are estimated as (Hull, 2012, p. 503):

$$a = 1 - \alpha - \beta \tag{17}$$

$$V_L = \frac{w}{1 - \alpha - \beta} \tag{18}$$

$$\xi = \alpha\sqrt{2} \tag{19}$$

Panel B of Table 1 shows the results of the estimation of equations (14) and (15). Note that $m$ is not significantly different from 0, accordingly the estimate of μ is set to 0, while the other parameters are calculated by applying equations (17) through (19). Panel C of Table 1 shows the resulting parameters of the stochastic volatility model of the futures price.

**Simulation of the Trader's Costs and Value of the Futures Position, and Default Risk of the Clearinghouse**

I assume that on day *0*, the current date, the trader takes a position in a crude oil futures contract with a delivery date in 90 days, a futures price of \$1 and an initial variance of 0.01. I assume that the trader's annualized percentage opportunity cost of initial margin $k$=5% and the annualized percentage funding liquidity cost $l$=7%. I also assume that the annualized risk free rate is 5%. I assume that the grace period allowed by the broker to the individual trader to meet margin calls is 5 days.

I vary the initial margin from 1% to 100% of the futures price in steps of 1%, and the maintenance margin ratio from 1% to 100% in steps of 1%. I use Monte Carlo simulation to simulate the futures price and variance using equations (12) and (13) and the estimated parameters in Panel C of Table 1, for each day between the current date and the delivery date. This allows me to determine if and when a margin call occurs before the delivery date. I then estimate the short (long) futures trader's costs using equation (3) (equation (7)). I conduct 1,000 simulations of the futures price path. For each combination of the initial margin and the maintenance margin ratio, I average the opportunity costs of the margin, the funding liquidity costs and the total costs across the 1,000 simulations of the futures price path.

I determine the payoff to each of the barrier options: the up-and-out put, the up-and-in call, the down-and-out call and the down-and-in put, for each simulation of the futures price path and then estimate the value of each of these options, for each combination of initial margin and maintenance margin ratio, as the average of the present value of the payoffs from the 1,000 simulations. The expected value of the short (long) futures trader's position is given by the value of the up-and-out put (down-and-out call). The expected costs of default to the clearinghouse from the short (long) futures trader is given by the value of the up-and-in call (down-and-in put). For each futures price path I determine if the balance in the margin account becomes negative within 5 days of a margin call (if any) and count the number of simulations in which this occurs to determine the probability of a negative balance.

Table 2 provides descriptive statistics of the results for the 10,000 observations (100 values of initial margin x 100 values of the maintenance margin ratio) in columns 2 and 3.

**TABLE 2**
**SUMMARY STATISTICS OF SIMULATIONS**

| | **Mean/Standard deviation of variables %** | | | |
|---|---|---|---|---|
| **Variable** | **All simulated observations** | | **Set of simulated observations which solve the model for each value of initial margin** | |
| **(1)** | **Short futures (2)** | **Long futures (3)** | **Short futures (4)** | **Long futures (5)** |
| Initial margin | 50.50 | 50.50 | 52.50 | 52.50 |
| | 28.87 | 28.87 | 27.86 | 27.86 |
| Maintenance margin ratio | 0.51 | 0.50 | 0.62 | 0.72 |
| | 0.29 | 0.29 | 0.23 | 0.20 |
| Total cost | 0.6389 | 0.6389 | 0.6609 | 0.6608 |
| | 0.3602 | 0.3602 | 0.3496 | 0.3497 |
| Opportunity cost of margin | 0.6352 | 0.6352 | 0.6604 | 0.6604 |
| | 0.3631 | 0.36311 | 0.3504 | 0.3504 |
| Funding liquidity cost | 0.0037 | 0.0037 | 0.0005 | 0.0008 |
| | 0.0060 | 0.0060 | 0.0016 | 0.0013 |
| Expected value of trader's futures position | 50.02 | 49.78 | 51.98 | 51.72 |
| | 28.53 | 28.50 | 27.52 | 27.49 |
| Expected costs of default to the futures clearinghouse | 0.05 | 0.05 | 0.01 | 0.01 |
| | 0.20 | 0.22 | 0.05 | 0.05 |
| Probability of a negative balance in the margin account within 5 days of a margin call | 1.42 | 1.39 | 0.00 | 0.00 |
| | 7.76 | 7.68 | 0.00 | 0.00 |
| Number of observations | 10,000 | 10,000 | 96 | 96 |

Note. The simulations are conducted with the following initial values: Futures price $F$ = \$1; Futures price volatility $V$ = 0.01; Stochastic process for the change in the futures price is given by $dF = \sqrt{Vdt}\varepsilon_{1t}$; Stochastic process for the change in the futures price volatility is given by $dV = a(V_L - V)dt + \xi V\sqrt{dt}\varepsilon_{2t}$, where $V_L$=0.000547, $a$=0.018765, $\xi$=0.072952, and $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are standard normal random variables, $dt$ = 1 day and the delivery date of the futures contract is in 90 days. In the simulations, the initial margin is varied from 1% to 100% in steps of 1%, the maintenance margin ratio is varied from 1% to 100% in steps of 1%, $k$=5%, $l$=7%, the

**Effect of Increases in the Initial Margin When the Maintenance Margin Ratio Is Fixed, for the Short Futures Trader**

Figures 1 through 6 illustrate the effect of increases in the initial margin for the short trader when the maintenance margin ratio is fixed at 10%. Figures 1 through 3 address the trader's costs. Both the total cost and the opportunity cost of the margin increase linearly with the margin. The funding liquidity cost increases steeply with increases in the initial margin for low values of the initial margin and reaches a peak. At low values of the initial margin, the probability of occurrence of a margin call is high, thus leading to a high liquidity cost. As the initial margin increases further, the probability of occurrence of a margin call decreases but the amount that has to be raised to meet the margin call increases, contributing to increases in the funding liquidity cost. The funding liquidity cost then decreases steeply as the initial margin increases further, due to decreases in the probability of occurrence of a margin call. The funding liquidity cost falls to zero for values of the initial margin equal to or exceeding 21%. At these values of the initial margin, the probability of occurrence of a margin call is zero. Summing up, increases in the initial margin increase the trader's opportunity cost of the margin, and decreases the funding liquidity cost beyond small values of the initial margin.

**FIGURE 1**
**THE SHORT FUTURES TRADER'S TOTAL COST VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**

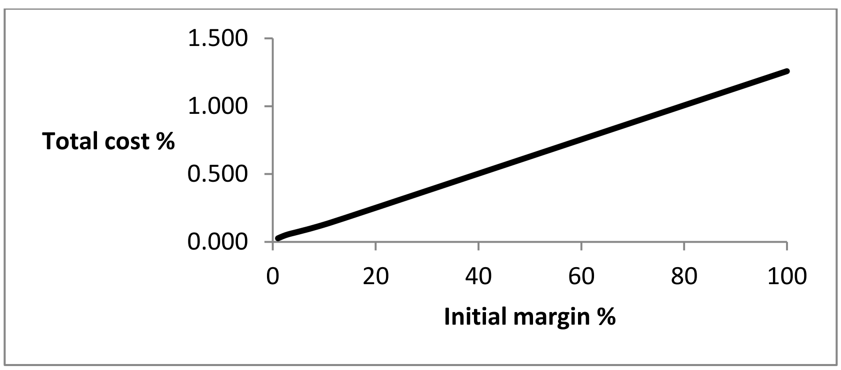


**FIGURE 2**
**THE SHORT FUTURES TRADER'S OPPORTUNITY COST OF THE MARGIN VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**

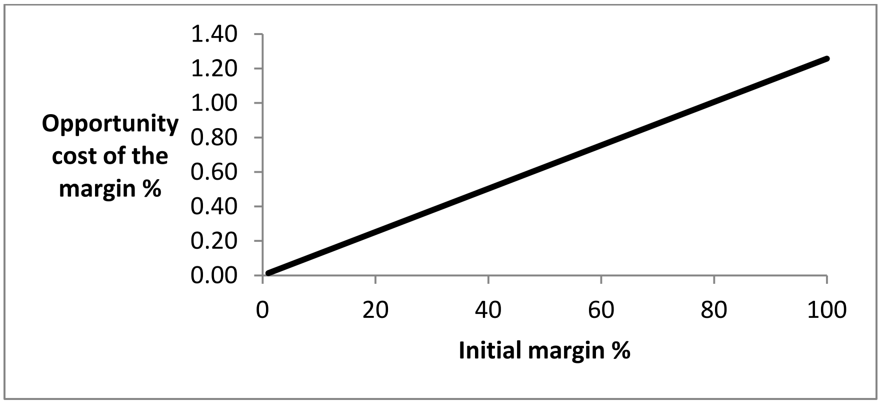
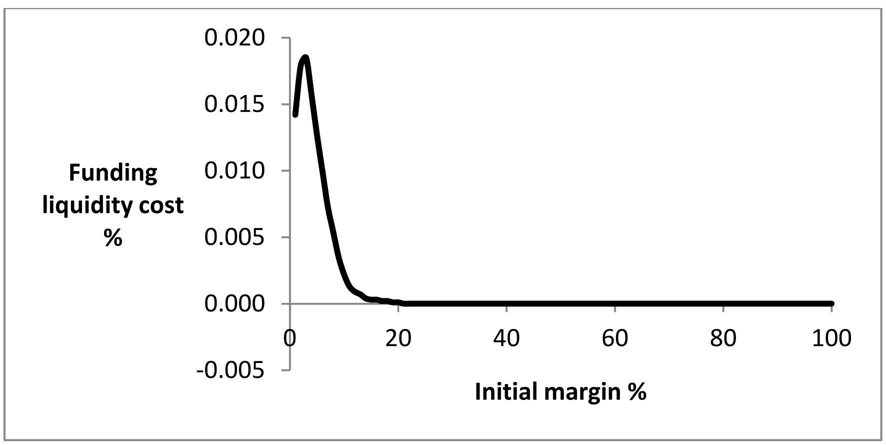



**FIGURE 3**
**THE SHORT FUTURES TRADER'S FUNDING LIQUIDITY COST VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**




Figures 4 through 6 each focus on the value of the trader's futures position, the expected costs of default to the futures clearinghouse and the probability of a negative balance in the margin account in the grace period following a margin call, respectively. The expected value of the trader's futures position increases with the initial margin. Although not evident in the figure, for values of the initial margin below 13%, the value of the trader's position exceeds the margin, while at values of the margin equal to 13% and higher, the value of the trader's position is always lower than the margin. The expected costs of default to the futures clearinghouse are large for low values of the initial margin, decrease steeply with increases in the initial margin, and fall to zero at an initial margin value of 20% or higher. Again, although not evident in the figure, the expected costs of default to the futures clearinghouse exceed the initial margin at an initial margin of 1% and are lower than the initial margin for values of the initial margin equal to 2% and higher. The probability of a negative balance in the margin account in the grace period following a margin call is approximately 75% for an initial margin of 1%, decreases steeply as the margin increases, and falls to zero for values of the initial margin equal to 19% and higher. Summing up, increases in the initial margin increase the value of the trader's position and reduce the expected costs of default to the futures clearinghouse and the probability that the trader's margin account will have a negative balance within the grace period following a margin call.

**FIGURE 4**
**EXPECTED VALUE OF THE SHORT TRADER'S FUTURES POSITION VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**

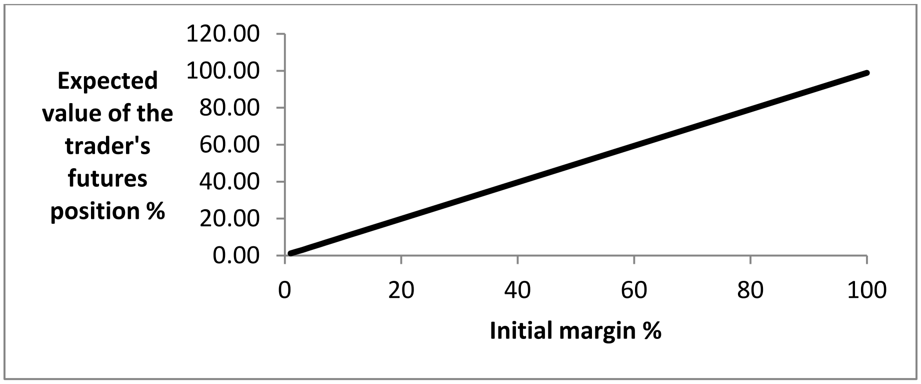



**FIGURE 5**
**EXPECTED COSTS OF DEFAULT BY THE SHORT FUTURES TRADER TO THE FUTURES CLEARINGHOUSE VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**

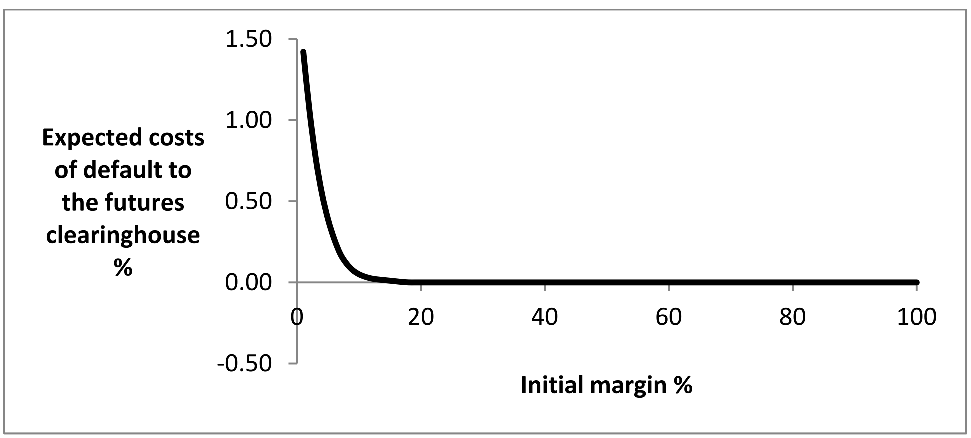
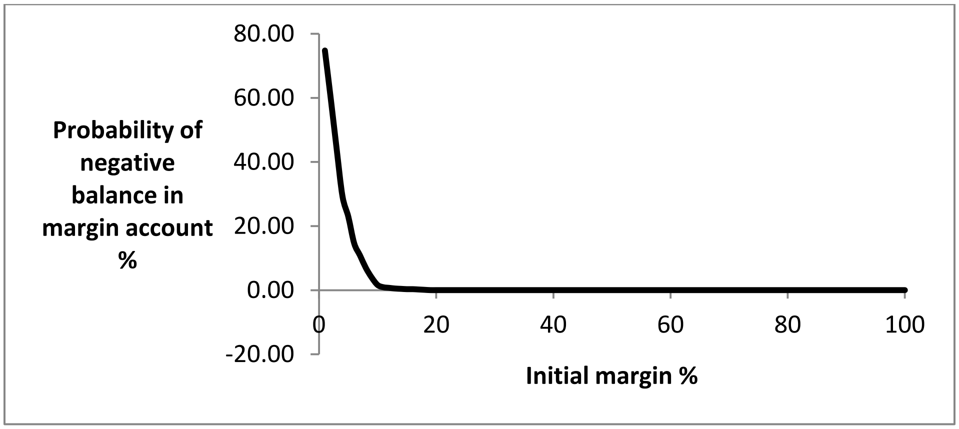


**FIGURE 6**
**PROBABILITY OF A NEGATIVE BALANCE IN THE MARGIN ACCOUNT WITHIN 5 DAYS OF A MARGIN CALL UPON THE SHORT FUTURES TRADER VERSUS THE INITIAL MARGIN FOR A MAINTENANCE MARGIN RATIO OF 10%**




### Effect of Increases in the Ratio of the Initial Margin to the Maintenance Margin When the Initial Margin Is Fixed, for the Short Futures Trader

Figures 7 through 12 illustrate the effect of increases in the ratio of the initial margin to the maintenance margin, when the initial margin is fixed at 10%. Since the ratio of initial margin to maintenance margin is the reciprocal of the maintenance margin ratio, the figures show the effects of decreases in the maintenance margin ratio. While I could have presented the graphs with the maintenance margin ratio on the x axis, I chose to present these graphs with the ratio of initial margin to maintenance margin on the x axis, in order to more effectively explain the optimal choice of the maintenance margin ratio for each value of the initial margin.

Figures 7 through 9 address the trader's costs. The trader's total cost increases steeply with the ratio of the initial margin to the maintenance margin for low values of this ratio, reaches a peak, decreases steeply with increases in this ratio up to a point, at which point the decrease is less steep. As expected,

**FIGURE 7**
**THE SHORT FUTURES TRADER'S TOTAL COSTS VERSUS THE RATIO INITIAL MARGIN/ MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**

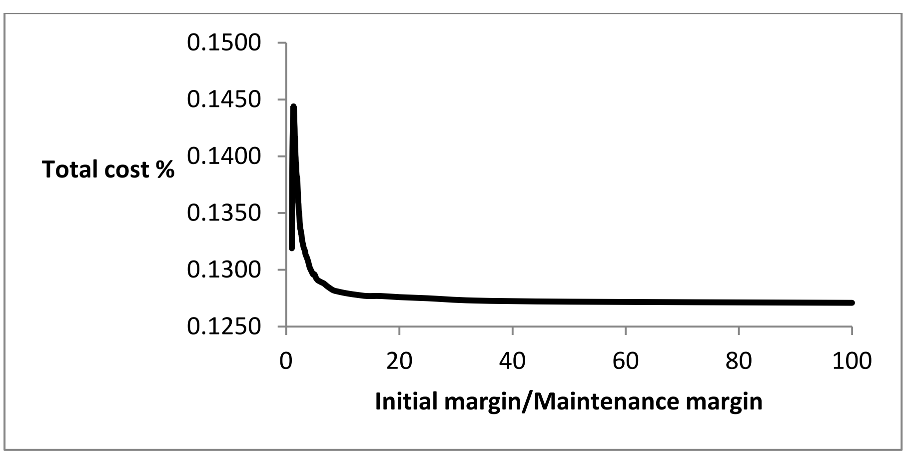


**FIGURE 8**
**THE SHORT FUTURES TRADER'S OPPORTUNITY COST OF THE MARGIN VERSUS THE RATIO INITIAL MARGIN/MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**

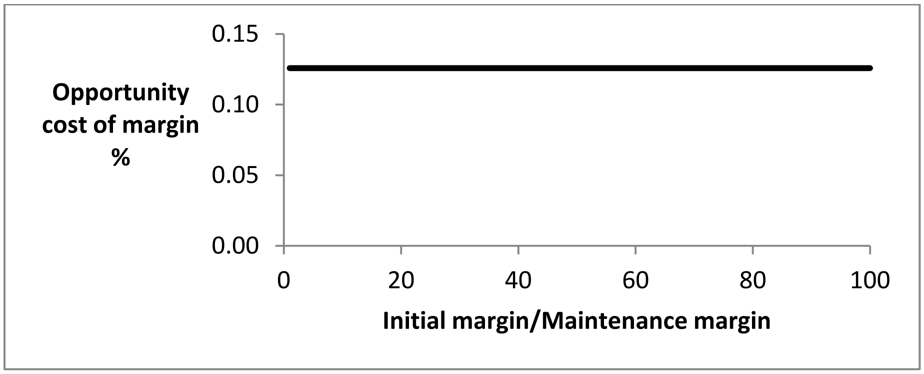
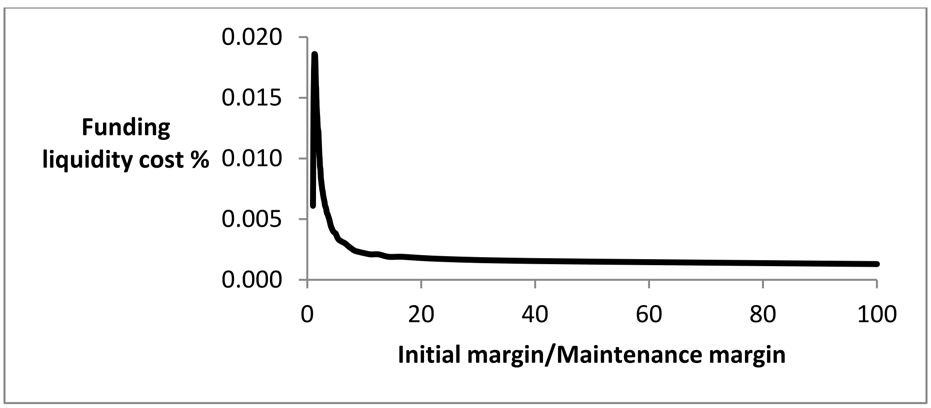



**FIGURE 9**
**THE SHORT FUTURES TRADER'S FUNDING LIQUIDITY COST VERSUS THE RATIO INITIAL MARGIN/MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**




since the initial margin is fixed at 10%, the opportunity cost of the margin does not change with changes in the ratio of the initial margin to the maintenance margin. The effect of increases in the ratio of the initial margin to the maintenance margin upon the funding liquidity cost is similar to the effect upon the total costs. When this ratio is low, given the fixed initial margin, the maintenance margin ratio is high. The probability of occurrence of a margin call is high, but the variation margin is low. As the ratio of initial margin to maintenance margin increases slightly, the probability of occurrence of a margin call decreases, but the variation margin requirement increases, contributing to an increase in the funding liquidity cost. As the ratio of initial margin to maintenance margin increases further, the probability of occurrence of a margin call decreases, leading to a decrease in the funding liquidity cost. Summing up, increases in the ratio of the initial margin to the maintenance margin has no effect on the trader's opportunity cost of the margin, and decreases the funding liquidity cost beyond small values of this ratio.

Figures 10 through 12 each focus on the value of the trader's futures position, the expected costs of default to the futures clearinghouse and the probability of a negative balance in the margin account in the grace period following a margin call, respectively. The expected value of the trader's futures position increases steeply with the ratio of initial margin to maintenance margin, for low values of this ratio and increases less steeply for higher values of this ratio. Although not evident in the figure, for values of the ratio of the initial margin to the maintenance margin below 43%, the value of the trader's position exceeds the margin, while at values of the ratio equal to 43% and higher, the value of the trader's position is always lower than the margin. The expected costs of default to the futures clearinghouse do not change with changes in the ratio of the initial margin to the maintenance margin. The probability of a negative balance in the margin account in the grace period following a margin call is zero for low values of the ratio of the initial margin to the maintenance margin which are less than or equal to 2.77 (corresponding to a maintenance margin ratio of 36%). At these values, the balance in the margin account would still be high at the time of the margin call, leading to a low probability that the balance will become negative in the grace period. As the ratio of the initial margin to the maintenance margin increases, due to decreases in the maintenance margin ratio, the balance in the account would be low at the time of a margin call making it more likely that the balance will turn negative in the grace period.

Summing up, increases in the ratio of the initial margin to the maintenance margin increases the value of the trader's position, has no effect on the expected costs of default to the futures clearinghouse and increases the probability that the trader's margin account will have a negative balance within the grace period following a margin call.

**FIGURE 10**
**EXPECTED VALUE OF THE SHORT TRADER'S FUTURES POSITION VERSUS THE RATIO INITIAL MARGIN/MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**

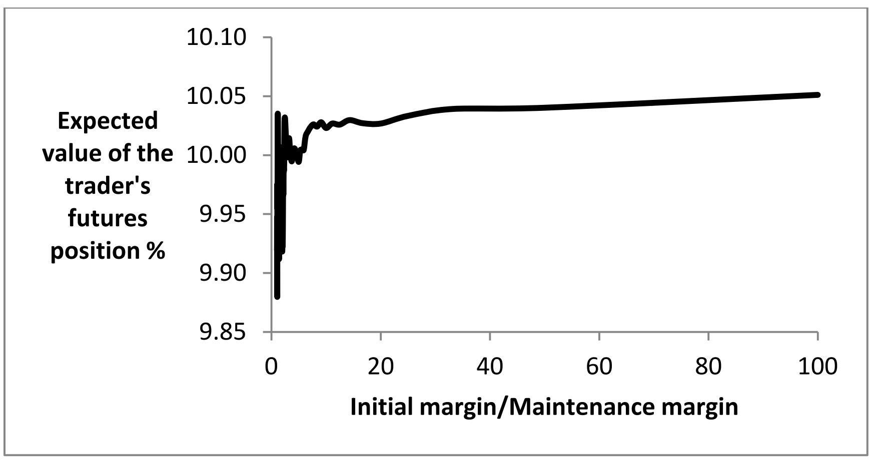



**FIGURE 11**
**EXPECTED COSTS OF DEFAULT BY THE SHORT TRADER TO THE FUTURES CLEARINGHOUSE VERSUS THE RATIO INITIAL MARGIN/MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**

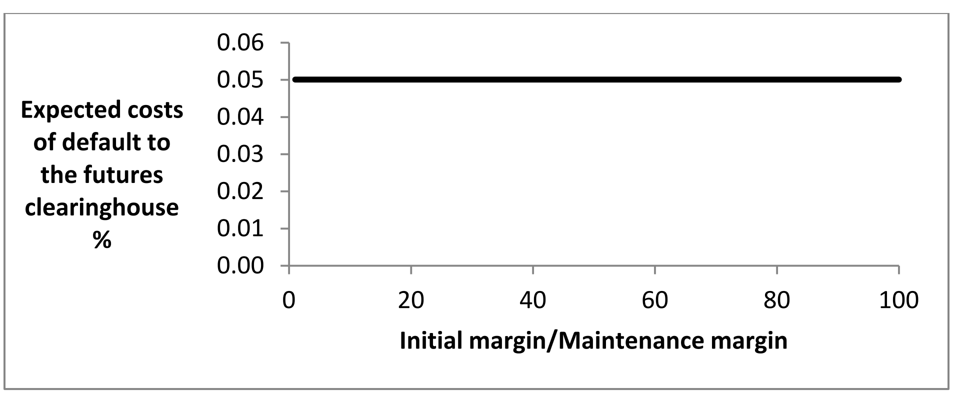
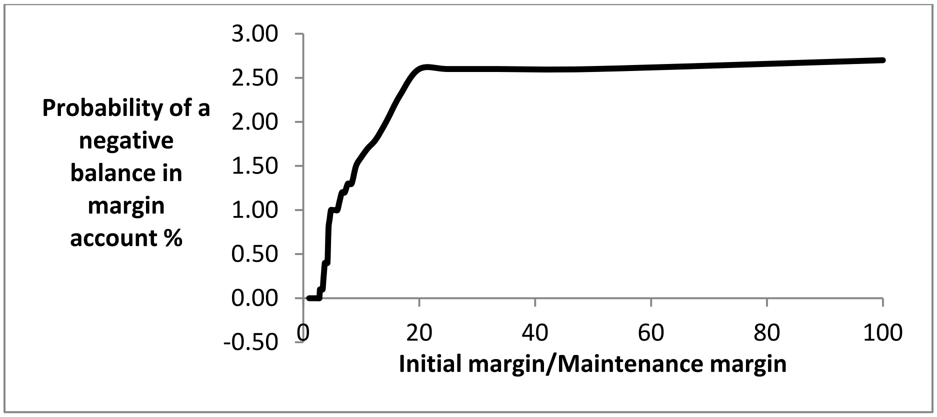


**FIGURE 12**
**PROBABILITY OF A NEGATIVE BALANCE IN THE MARGIN ACCOUNT WITHIN 5 DAYS OF A MARGIN CALL UPON THE SHORT TRADER VERSUS THE RATIO INITIAL MARGIN/MAINTENANCE MARGIN FOR AN INITIAL MARGIN OF 10%**




### Effect of Increases in the Initial Margin and the Ratio of Initial Margin to Maintenance Margin for the Long Futures Trader

The figures illustrating the effects for the long futures trader are similar to those for the short futures trader and are available from the author on request.

### Optimal Solution to the Model for Each Value of Margin

From figures 1 and 7, it is clear that minimizing the trader's total cost involves choosing a low value of the initial margin and a high value of the ratio of the initial margin to the maintenance margin. Figure 7 also shows that for a fixed initial margin, the total cost decreases as the ratio of the initial margin to the maintenance margin increases, but at a certain point, the total cost does not decrease further. Hence, for a fixed initial margin, there could be several values of the ratio of the initial margin to the maintenance margin at which the total cost is minimized. If this occurs, I choose the lowest value of the ratio of the initial margin to the maintenance margin as the optimal value of the ratio.

For each value of the initial margin ranging from 1% to 100%, I determine the optimal ratio of the initial margin to the maintenance margin that minimizes the short (long) trader's total cost, using equation (3) (equation (7)). I apply the following constraints: 1) that the initial margin is equal to or exceeds the value of the short (long) trader's futures position using equation (4) (equation (8)); 2) that the initial margin is equal to or exceeds the expected costs of the futures clearinghouse from default by the short (long) trader using equation (5) (equation (9)); 3) the probability that the balance in the margin account becomes negative in the 5 day grace period following a margin call is zero, using equation (6) (equation (10)). There are 96 combinations of the initial margin and the optimal ratio of the initial margin to the maintenance margin, which satisfy the constraints for the short futures trader and the long futures trader. The descriptive statistics corresponding to these optimal sets are presented in columns 4 and 5 of Table 2.

### Regression of the Initial Margin on the Optimal Maintenance Margin Ratio

I conduct a regression of the initial margin on the optimal maintenance margin ratio for the short futures trader and the long futures trader and provide the results in Table 3. In both regressions, the intercept term is not statistically significant, while the coefficient of the optimal maintenance margin ratio is positive and statistically significant at the 99% confidence level. The results show that the optimal initial margin and optimal maintenance margin ratio are mutually dependent, implying that the optimal choice of the two should be conducted simultaneously.

**TABLE 3**
**REGRESSION OF THE INITIAL MARGIN ON THE OPTIMAL MAINTENANCE MARGIN RATIO**

| Independent variable | Short futures | | Long futures | |
|---|---|---|---|---|
| | Coefficient | t statistic | Coefficient | t statistic |
| Intercept | 9.4760 | 1.4039 | -5.2264 | -0.6058 |
| Maintenance margin ratio | 69.1387 | 6.7141*** | 79.8782 | 6.8852*** |
| Number of observations | 96 | | 96 | |
| Adjusted R squared | 0.3192 | | 0.3376 | |

Note. *** Significant at the 99% confidence level

**Optimal Initial Margin and Maintenance Margin Ratio**

I next find the optimal combination of the initial margin and the maintenance margin ratio which provides the minimum value of the trader's total costs, while simultaneously satisfying all the constraints. The results are provided in Table 4.

*No Constraint on the Maintenance Margin Ratio*

I first address the situation when there is no constraint upon the maintenance margin ratio. I find that the minimum value of the trader's total costs is at an initial margin of 5% and a maintenance margin ratio of 1, for both the short and the long futures trader. Note that a maintenance margin ratio of 1 is used for clearing firms. Clearing firms are required to maintain the balance in their margin accounts with the futures clearinghouse at the level of the clearing margin, thus, clearing firms receive margin calls from the clearinghouse whenever they experience losses on their open futures positions. Hence, both the initial margin and the maintenance margin for a clearing firm equal its clearing margin. These values of the optimal initial margin and maintenance margin ratio correspond to those for the lowest cost trader.

*Maintenance Margin Ratio Constrained to Be Less Than 1*

However, it is impractical to use a maintenance margin ratio of 1 for individual traders, both because their number is large and they may not be as accessible to the broker when margin calls need to be made

**TABLE 4**
**OPTIMAL VALUES OF THE INITIAL MARGIN AND THE MAINTENANCE MARGIN RATIO**

| | Short futures | Long futures |
|---|---|---|
| **No constraint on the maintenance margin ratio** | | |
| Optimal initial margin % | 5 | 5 |
| Optimal maintenance margin ratio | 1 | 1 |
| **Maintenance margin ratio constrained to be less than 1** | | |
| Optimal initial margin % | 11 | 11 |
| Optimal maintenance margin ratio | 0.33 | 0.20 |

and to be complied with, as clearing firms, which are generally large corporations, are accessible to the clearinghouse. Hence, I next impose the constraint that the maintenance margin ratio has to be less than 1. The optimal initial margin is 11% for both the short and the long futures trader. However, the optimal maintenance margin ratio is 33% for the short trader and 20% for the long trader.

Hence, the results indicate that: 1) the optimal maintenance margin ratio for the short trader and the long trader are not the same and; 2) the optimal maintenance margin ratio for the short trader is higher than that for the long trader. In futures markets, the exchange's minimum maintenance margin ratio is the same for both short and long traders. However, Fortune (2000) notes that the maintenance margin requirement for stocks traded on the NYSE, according to Rule 431, is 30% for a short position and 25% for a long position. These numbers are similar to my results on the optimal maintenance margin ratios for the short and long trader, respectively.

*Comparison with the Actual Values Used by the NYMEX*

Data on the initial margin and maintenance margin in dollars for the crude oil futures contract is available from the NYMEX for the period 2008 January onwards. On average, the initial margin and the maintenance margin are $2,335 and $1,730, respectively for the period January 1, 2008 through October 21, 2011. Noting that the size of the crude oil futures contract is 1,000 barrels of crude oil, and using the futures price of the closest to maturity futures contract, I find that, on average, the initial margin as a percentage of the futures price is 3.07% for the same period. The maintenance margin ratio is maintained at 0.7407 for this period. Differences between my results on the optimal initial margin and maintenance margin ratios and those used by the NYMEX could arise if the NYMEX uses a higher percentage opportunity cost of the margin and a lower percentage liquidity cost of a margin call than I do.

## CONCLUSIONS

I provide a model that a futures clearinghouse could use to simultaneously set the optimal initial margin and maintenance margin ratio that will minimize the traders' total costs and simultaneously minimize the default risk of the clearinghouse. In my model, the trader's total costs are the sum of the opportunity cost of the initial margin and the funding liquidity cost associated with meeting a margin call. The funding liquidity cost depends on the probability that a margin call is made and the magnitude of the funds that have to be raised to meet the call. I explain the interrelationship that should exist between the optimal initial margin and the optimal maintenance margin ratio. For a given maintenance margin ratio, the higher the initial margin, the lower the probability of a margin call but the higher the magnitude of the funds that must be raised to meet the margin call. For a given initial margin, the higher the maintenance margin ratio, the higher the probability of occurrence of a margin call but the lower the magnitude of funds that need to be raised to meet the margin call. I minimize the default risk of the futures clearinghouse by minimizing the trader's incentive to default, the expected costs to the clearinghouse from default by the trader and the probability that the balance in the trader's margin account becomes negative within the grace period allowed to meet variation margin requirements following a margin call.

I apply my model to the WTI crude oil futures contract traded on the NYMEX, using data on the futures price for the closest to maturity contract for the period January 1, 2001 through October 21, 2011. I assume that the futures price and its volatility follow a stochastic volatility model. I estimate the parameters of the stochastic processes, and solve my model, using simulation. I estimate optimal levels of the initial margin and the maintenance margin ratio, and show that these are mutually dependent. The results show that the optimal initial margin is 5% and the maintenance margin ratio is 1, for both short and long traders. This level of the maintenance margin ratio is used for clearing firms, who are in effect the lowest cost traders. For individual traders, whose maintenance margin ratio is less than 1, I find that the optimal initial margin is 11%. However, the maintenance margin ratio is higher for short traders, at 33%, than for long traders, at 20%. These values are similar to those prescribed by the NYSE's Rule 431 for its securities traders. My values for the optimal initial margin and maintenance margin ratio are different from those actually used by the NYMEX. The differences imply that the NYMEX uses a higher margin cost but a lower liquidity cost than I do in my simulations.

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