The Newsvendor Problem with Pricing and Secondary Revenues

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In the newsvendor problem with pricing, the seller of homogeneous primary items decides price and inventory level before exact demand is known, with the goal of maximizing expected profit. To this, we add that if and only if a customer purchases one of the primary items, he will also spend a random amount on secondary items, giving the newsvendor a second source of revenues. We present our model of this problem and provide numerical examples.

INTRODUCTION

While the newsvendor problem (an inventory decision given uncertain demand) itself has been studied extensively in literature, the newsvendor problem with pricing (inventory level and selling price are decision variables) has received less attention. Fewer than 40 published articles in Operations Management journals have appeared on the topic, possibly due to the complexity of the problem. Adding the possibility of secondary revenues from customers who first purchase a primary item has further complicated the problem.

Here, we present models designed to address the above problems in inventory management, using performance events as the eventual setting for the newsvendor who sets price and inventory level (number of tickets to sell), with the added possibility of profits from secondary sources such as concession sales and parking fees.

FORMS OF THE NEWSVENDOR PROBLEM AND MODELS

We present different versions of the newsvendor problem, with increasing complexity, to demonstrate various approaches to address the challenge of choosing an inventory level prior to realizing exact demand.

Case 1 – The Newsvendor Problem

First, we begin with the most simple form of the newsvendor problem in which price, P, is an endogenous variable. Therefore, the only decision variable for the newsvendor is inventory level, Q. It is assumed that the newsvendor knows the distribution of reservation prices of the customers, RP, and the size of the customer base, D. Wilson and Sorochuk (2009) show that with this information, demand, $X(P)$, can be approximated as a normally-distributed random variable with parameters.

$$\mu_{X(P)} = D(1 - F_{RP}(P)) \tag{1}$$

and

$$\sigma^2_{X(P)} = D(1 - F_{RP}(P))(F_{RP}(P)) \tag{2}$$

where $F_{RP}(\cdot)$ denotes the cumulative distribution function of reservation prices.

In choosing an inventory level *Q*, the expected number of items the newsvendor will sell is given by:

$$E[Sold] = \int_{x=0}^{Q} x f_{X(P)}(x)dx + \int_{x=Q}^{\infty} Q f_{X(P)}(x)dx \,, \tag{3}$$

where $f_{X(P)}(\cdot)$ denotes the probability density function of demand. With the newsvendor incurring a unit cost, *c*, for each item he makes available for sale, the expected profit is:

$$E[\Pi] = P * E[Sold] - cQ \,. \tag{4}$$

The expected profit-maximizing solution is given by the well-known fractile solution:

$$Q^* = F^{-1}_{X(P)}\left(\frac{P - c}{P}\right). \tag{5}$$

Case 2 – The Newsvendor Problem with Pricing

Now we consider the case where the newsvendor decides not only on inventory level, but also on selling price. The problem was first addressed by Whitin (1955), and more recently by Dana (1999) and Petruzzi and Dada (1999). In their approach, Petruzzi and Dada (1999) consider demand uncertainty, ε, in both additive and multiplicative forms:

$$Y(P) = y(P) + \varepsilon \tag{6}$$

and

$$Y(P) = y(P)\varepsilon \,, \tag{7}$$

respectively. They show that in both cases, determining optimal price is straightforward, but optimal inventory levels depend on the c.d.f. of ε.

Note that in (6) and (7), the uncertainty term is independent of price. This form of uncertainty is common in the majority of the literature on the NPP, however price-dependent uncertainty is present in Wilson and Sorochuk (2009) and Sorochuk and Wilson (2009). Incorporating uncertainty in this manner has been shown to result in different solutions from those found using traditional models, yielding increases of 1-2%.

Case 3 – The Newsvendor Problem with Pricing and Secondary Revenues

Here, we consider the case where the newsvendor has an opportunity to earn secondary revenues from the customers, provided they have already purchased a primary item.

Oi (1971) addressed this form of the newsvendor problem as it pertained to Disneyland. In his approach, he noted that the secondary revenues resulted from the primary attraction – the rides, whereas the primary revenues (entrance fees) provided relatively little utility to the customer. Conversely, Marburger (1997) considered the setting of performance events, where customers gained more utility from their primary purchases (entrance fees) than from their secondary purchases (concessions, parking, etc.).

We now derive a model to calculate the newsvendor's expected profit when secondary revenues can be realized after a customer has purchased a primary item. As in Case 2, the newsvendor decides on a selling price P and inventory level Q for primary items. If a customer has a reservation price at least as high as P, and a primary item is available, the customer will pay P for a primary item, and provide an additional amount of profit on secondary items, S. It is assumed that S is a random variable that follows a normal distribution, with parameters known to the newsvendor.

Consider the secondary profits that the newsvendor will receive. If only one customer buys a ticket, the amount of secondary profit the newsvendor will receive from the customer will be random, following a normal distribution with mean μ_s and variance σ_S^2. If two customers each buy a ticket, the amount of secondary profit the newsvendor will receive from the two customers will be random, following a normal distribution with mean $2\mu_s$ and variance $2\sigma_S^2$. In general, if x customers each buy a ticket, the amount of secondary profit the newsvendor will receive from the x customers will be random, following a normal distribution with mean $x\mu_s$ and variance $x\sigma_S^2$. Therefore, the secondary profits from x customers is:

$$S_x \sim N(x\mu_S, x\sigma_S^2) \tag{8}$$

where x is the minimum of Q and the realized value of $X(P)$. Therefore, the newsvendor's profit function when including secondary profits becomes:

$$\Pi_{Total} = S_x + P * x - cQ\,, \tag{9}$$

with a corresponding expected profit of:

$$E[\Pi_{Total}] = (P + E[S]) * E[Sold] - cQ\,. \tag{10}$$

NUMERICAL EXAMPLES

To demonstrate the sensitivity of profit (and expected profit) on the various parameters in the model, we provide four numerical examples, with figures provided in the appendix. The general set of conditions is as follows:

- $D = 50$
- $c = 10$
- $RP \sim N(\mu_{RP} = 50, \sigma_{RP} = 10)$
- $S \sim N(\mu_S = 50, \sigma_S = 10)$
- $Q = 40$
- $P = 50$

Example 1 – Various Expected Reservation Prices.

In Figure 1, three functions demonstrate the effect of different expected reservation prices on profit. Here, the parameters are those listed as the general conditions, with μ_{RP} taking on values of 40, 50 and 60. At a low expected reservation price ($\mu_{RP} = 40$), profits are low, as few customers are willing to pay $P = 50$ for a ticket. The result is not only low revenues from ticket sales, but as a result fewer customers available to purchase secondary items. At a high expected reservation price (μ_{RP} = 60), profits are high, as many customers are willing to pay P = 50 for a ticket. The result is not only high revenues from ticket sales, but as a result more customers available to purchase secondary items. Note that at the high expected reservation price, the distribution appears to be normal, as is expected from (9), when both S and $X(P)$ follow a normal distribution.

Example 2 – Various Expected Secondary Profits.

In Figure 2, three functions demonstrate the effect of different expected secondary profits on profit. Here, the parameters are those listed as the general conditions, with μ_S taking on values of 40, 50 and 60. The effect of different secondary profits is not nearly as large as those seen in Example 1. While increasing the expected secondary profit does shift the profit function in the positive direction, the overall effect of increased expected profit is small, with an increased variance.

Example 3 – Various Selling Prices.

In Figure 3, three functions demonstrate the effect of different selling prices on profit. Here, the parameters are those listed as the general conditions, with P taking on values of 40, 50 and 60. At a low selling price (P = 40), profits are high, as many customers are willing to pay P = 40 for a ticket. The result is not only high revenues from ticket sales, but as a result more customers available to purchase secondary items. At a high selling price (P = 60), profits are low, as fewer customers are willing to pay P = 60 for a ticket. The result is not only low revenues from ticket sales, but as a result fewer customers available to purchase secondary items. Note that at the low selling price, the distribution appears to be normal, as is expected from (9), when both S and $X(P)$ follow a normal distribution.

Example 4 – Various Inventory Levels (Q).

In Figure 4, three functions demonstrate the effect of different inventory levels on profit. Here, the parameters are those listed as the general conditions, with Q taking on values of 30, 35 and 40. Note that for the given parameters, there is very little difference in occurrences of low profits. While lower inventory levels (Q = 30) results in smaller variance in profits, it also results in lower expected profit. The converse is true at higher inventory levels.

SUMMARY, CONCLUSIONS AND FUTURE WORK

In this paper we have provided a background on the newsvendor problem, and added complexities that lead to the newsvendor problem with price and secondary profits. For this final problem, we provided a model to be used to calculate newsvendor profits under various conditions, and gave numerical examples demonstrating the sensitivity of profits to parameters such as reservation price, individual customer expenditure on secondary items, ticket selling price and inventory level. We showed how changes in reservation price and selling price have significantly greater impact on expected profit than comparable changes in expenditure on secondary items.

Our future work includes further investigation of numerical examples applicable to the newsvendor problem with pricing and secondary revenues, and hopefully, robust analytical results that can be used to assist decision makers in setting prices and inventory levels under the given conditions.

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APPENDIX

The appendix contains figures referred to in the four numerical examples presented in the article.

FIGURE 1
SIMULATION RESULTS – FREQUENCY OF PROFIT FOR VARIOUS EXPECTED RESERVATION PRICES

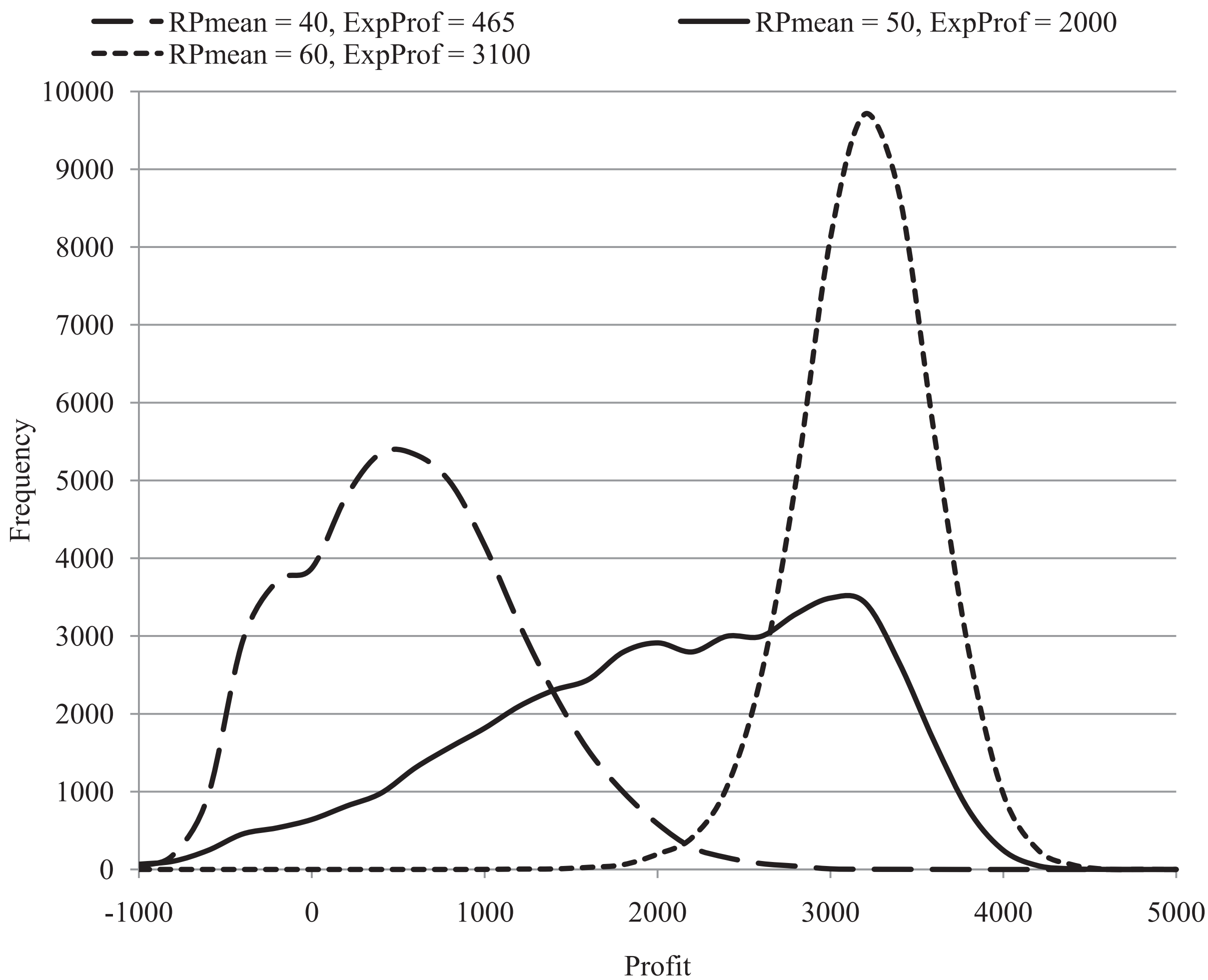

FIGURE 2
SIMULATION RESULTS – FREQUENCY OF PROFIT FOR VARIOUS SECONDARY REVENUES

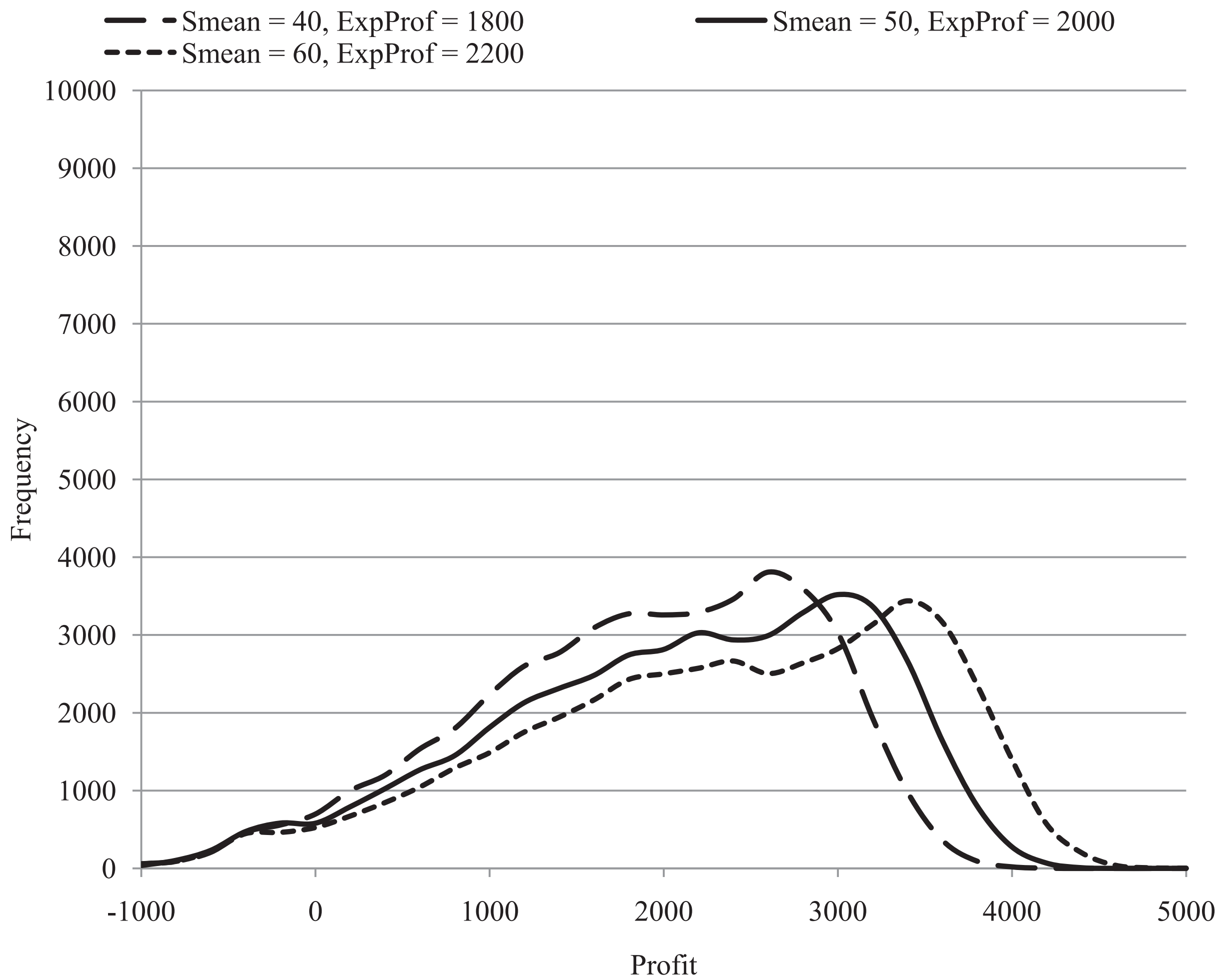

FIGURE 3
SIMULATION RESULTS – FREQUENCY OF PROFIT FOR VARIOUS SELLING PRICES

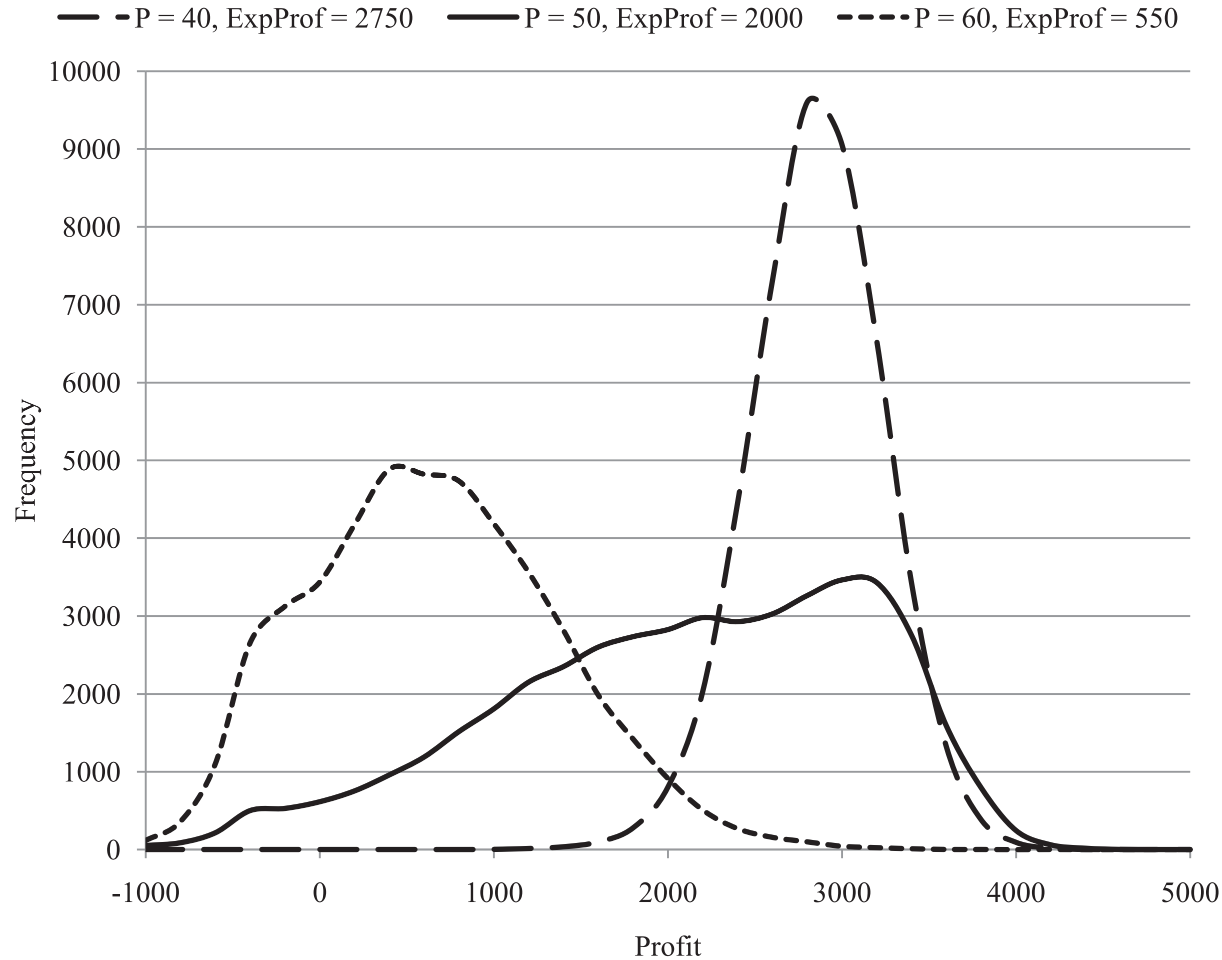

FIGURE 4
SIMULATION RESULTS – FREQUENCY OF PROFIT FOR VARIOUS QUANTITIES (INVENTORY LEVELS)