Explanation of S&P500 Index Distribution Deviation from a Gaussian Curve (Dynamic Financial Market Model)

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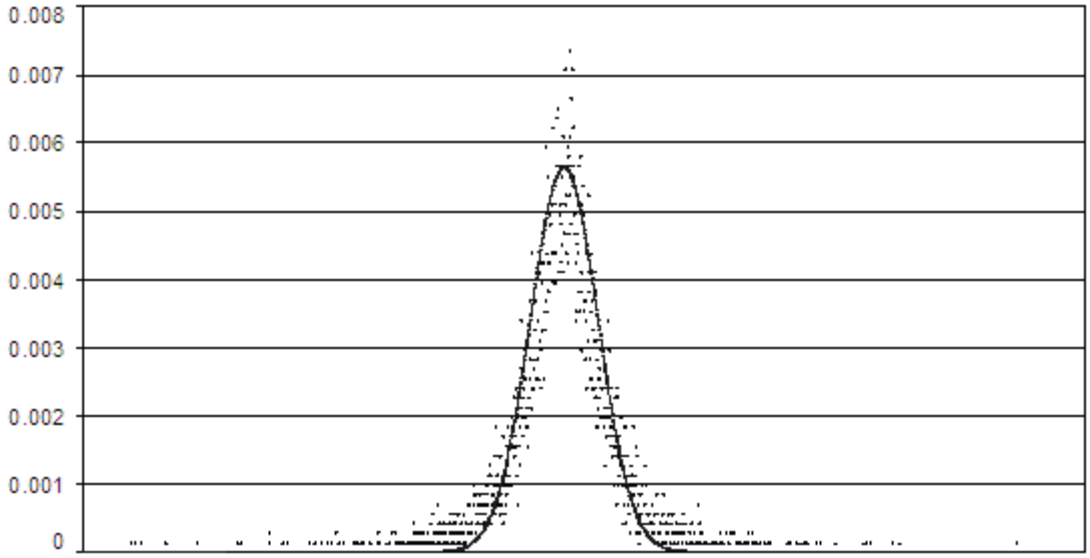
The correct model of a liquid financial market is one of the most important matters for the management of all financial market activities including for example a stock or bond porfolio or asset pricing. Clear random walk models, which consider a market price/yield development of liquid financial markets,are to be a random walk within the meaning of a symmetric normal (Gaussian) distribution, and are very useful in accurately explaining many financial market effects. If we study financial markets more closely, we recognize that such development can be partly causal and a clear random walk is only a special case of it. A Dynamic Financial Market Model considers feedback processes of financial markets which cause dependence in the probability of the next price/yield step direction and also expects a mix of random processes as a final result. Both effects cause Gaussian (normal) observations in probability distributions of financial instruments and this is why the model is also able to explain for example effects like thin or fat tails and other deformations in the probability distribution. The S&P500 index or Euro Bond futures probability distribution, on daily basis, are good examples of the diversion from normality. Basic principles of the Dynamic Financial Market Model also help explain some of the S&P 500 index, Dow Jones Industrial Average index, Euro Bond futures or EUR/USD currency development return distribution, which is a departure from a Gaussian curve.

INTRODUCTION

In the case of price/yield development of many liquid financial instruments we observe a return probability distribution (not normal/binomial) with a positive kurtosis which is characterized by fat tails at the borders and sharpness in the mid area of the distribution.

For example in the daily returns S&P500 probability distribution, picture [43b], we observe a positive kurtosis distribution wit fat tails and sharpness.

As the distribution is not a normal/binomial one, it means the process behind is not one random walk with certain average length of step but we have also reason to expect some rule inside the system, for example in a choose of a price/yield development direction.



Daily returns (1970-2010) S&P500 (dots), normal curve (line), source: B.Stádník **[43b]**

HOW TO EXPLAIN A DEPARTURE FROM GAUSSIAN CURVE ACCORDING TO DYNAMIC FINANCIAL MARKET MODEL?

There are two ways of how to explain the deviation. The first is to consider the mix of more random processes (for example two special random walks with different average length of step). The second way is to consider dynamic changes in the process (for example changes in the probability of a next step direction) which is the case of Dynamic Financial Market Model.

In the Dynamic Financial Market Model we recognize three basic presumptions:

1. Book of Orders Presumption
2. Feedback Presumption
3. Incoming Unexpected Information Presumption

Book of Orders Presumption and Its Consequences

The book of orders is the core of the situation on the exchanges (organized markets). Orders come in to the market and are placed to the book of orders. We can recognize buy orders, sell orders, stop-loss buy orders, stop-loss sell orders and also other types.

A trade is done at certain price if an investor hit an order which is already placed. The sequence of trades generates a certain price development, which can be figure for example in a chart. An order can also be cancelled before its hit.

The sequence, frequency and the volume of orders is generally unpredictable for each investor.

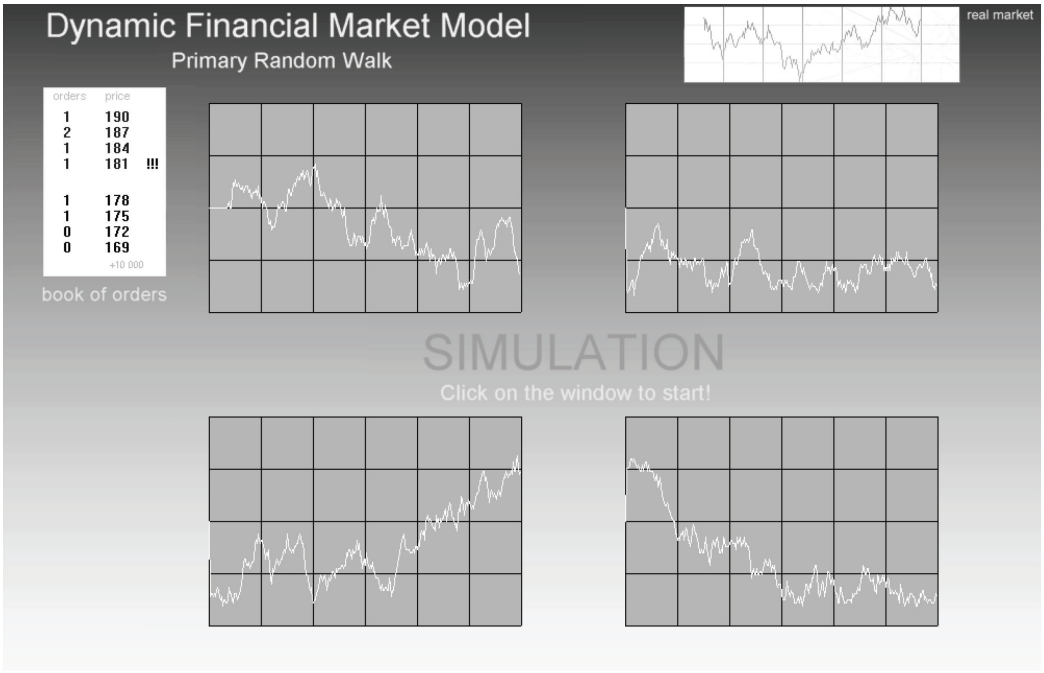
The book of orders on liquid financial markets works continuously during trading hours, collects orders and generates a price development.

The work of book of orders is a stable process if there is still certain amount of orders coming to bid and offer sides. This situation is usual for real financial markets. Stable situations are supported by the fact that investors are interested first of all in a future price development which is important for their profit or loss and they accepted very well current price in a wide price range. For investors, their opinion on future development is more important than on current price value. It also means that for each price we probably find many investors with an opposite opinion on a future price development.

The difference between the best and offer price is usually one price tick.

Market Making, OTC and other markets systems are based on the same principle as a book of orders and the same mechanism of placing and hitting of orders works there. There are some differences but not important for our purposes.

The Picture [43b-2] is an example of four basic binomial random walks, with no feedback processes. Random walks in the picture are a result of model situation where a group of buyers and sellers accept price range 10160-10340 price units. Each of investors comes in to the market with one order and places an order to the book of orders at random time. If we simulate this situation four times we get four different binomial random walks.



Outputs from book of orders simulation, source: B.Stádník **[43b-2]**

Feedback Presumption and Its Consequences

Traders don't only watch present or historical data but according to them they are placing or hit orders. There is a feedback in the financial markets which also influences a future price/yield development. Feedback can dynamically increase/decrease a frequency of incoming buy or sell orders and so change a probability of a next price step direction from 50/50% (in case of symmetric random walk) to 49/51% for example. The change depends on the past development (in case of random walk the next step is independent on past).

According to Dynamic Financial Market Model presumptions we expect many feedbacks (including technical analysis, trend stabilizing, price inertia, trading techniques, different up/down movements dynamic, market price manipulations, market regulations).

Mainly following feedback, there is departure from normality and kurtosis.

Price Inertia Feedback

Price Inertia Feedback keeps a price unchanged. This works in all periods of time, such as a minute, hour or day. If there is no new unexpected and relevant economic information, the basic random walk (from process of random incoming orders to a book of orders for example) is kept by the feedback between two narrow levels as it is shown in picture [43c]. Price inertia is basic feedback which helps to keep price unchanged.

Trend Stabilizer Feedback

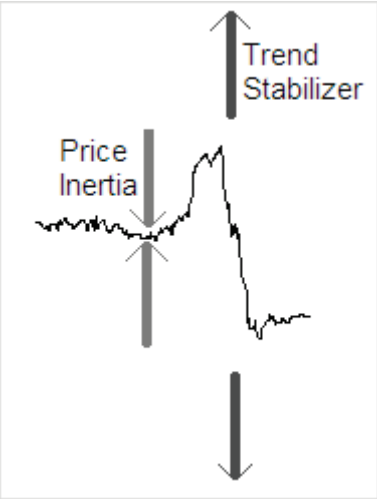
Trend Stabilizer Feedback is stabilizing trends, keeping a price development in a trend direction. This also works in all periods of time. When a trend formation appears, trend stabilizing is triggered. Trend stabilizing (supporting) has an origin in psychology with investors or with the pricing of not so important economic information coming to the market. The principle is shown in picture [43c]. Trend stabilizer feedback works against price inertia feedback and try to distribute price from the level.

Trading Techniques

For example with daily gap trading, which is popular in the case of many liquid financial instruments, traders believe that gaps opened in the morning will be closed during a trading day. They place orders to support this idea. This feedback is very close to the price inertia feedback on daily basis.

Many techniques are also based on level trading. It means if any level is broken, the movement will keep the direction. If the level is not broken the market price will return back or keep the trend. Many levels are represented with round numbers. Round numbers are price levels which are represented by the examples (10, 15,... 100, 200,...).

Trading techniques can be recognized not only ona daily basis but also during other time periods (on daily basis are the most significant).



Price Inertia and Trend Stabilizer feedbacks, source: B.Stádník **[43c]**

Impact on a Probability Distribution

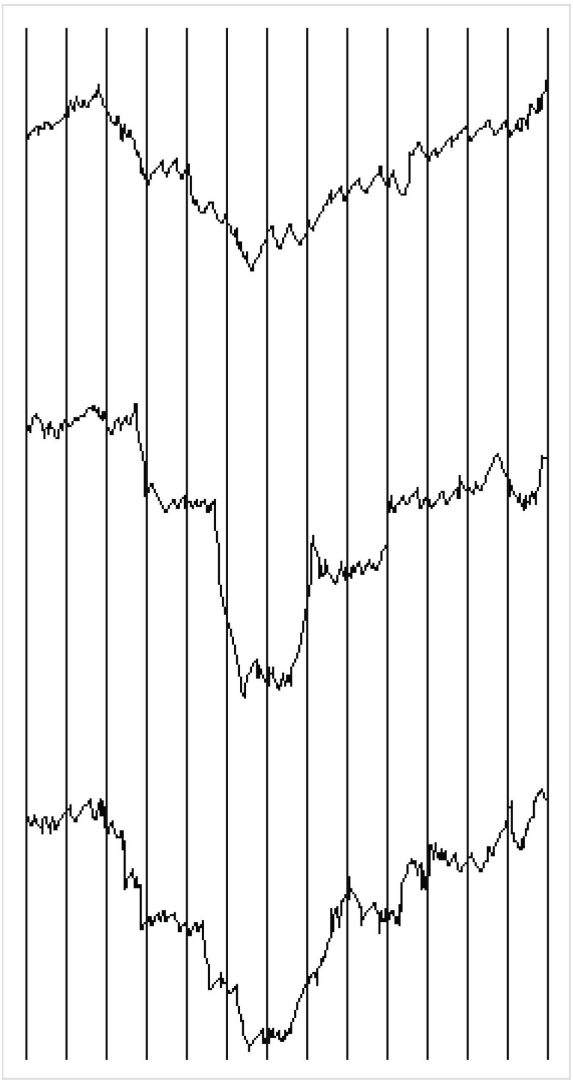
On real financial market we can observe these effects in the chart. There can be differences recognized among three charts in the picture [43d], which are explained using feedbacks [43e] (the most feed backed price development is the second one in the picture). Charts under the influence of feedbacks are more“ staircase than random walk, picture [43g] (the most feed backed development is the second one in the picture, the first one is a random walk).

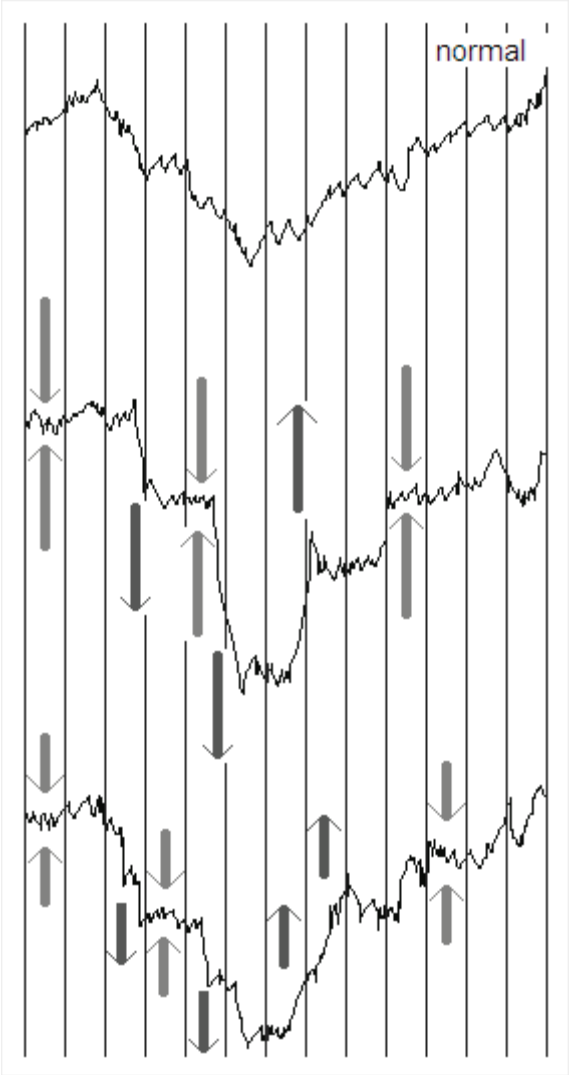
An impact on probability distribution is figured in picture [43f]. The most deformatted distribution (depends on an intensity of feedbacks) is the darkest areas in the picture. Price inertia feedback keeps price near center area of a distribution and trend stabilizer feedback distributes a price to the borders of a distribution. These processes are very similar for different time periods so we can observe described effect for example in one hour or one day distributions.

From a software application for a simulation, we get probabilities for a price step direction which cause a deformation [43h, 43i]. Probability of a step direction changes from 50.00 to 50.76% in border area, from 50.00 to 50.24% in a mid area [43h]. Probability in border-area has changed from 50.00 to 50.60 %, probability in mid-area has changed from 50.00 to 50.30 % in case of [43i].

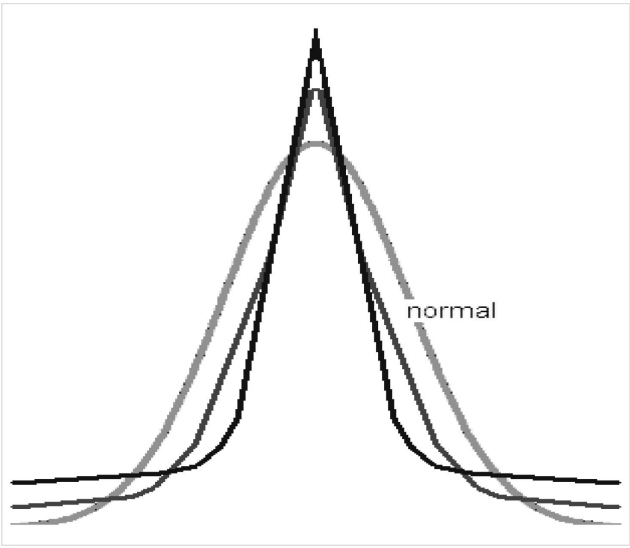
This means that less than 1 order from 100 supports the feedback. So the intensity of feedback is quite small and we cannot expect strong assistance in the predictability of a future price/yield development to make a speculative profit.

Instead of feedback basic on a random walk we expect that the final price development is influenced also by random, incoming, important, and unexpected economic information, which supports fat tails in the distribution (explained in details in Dynamic Financial Market Model).

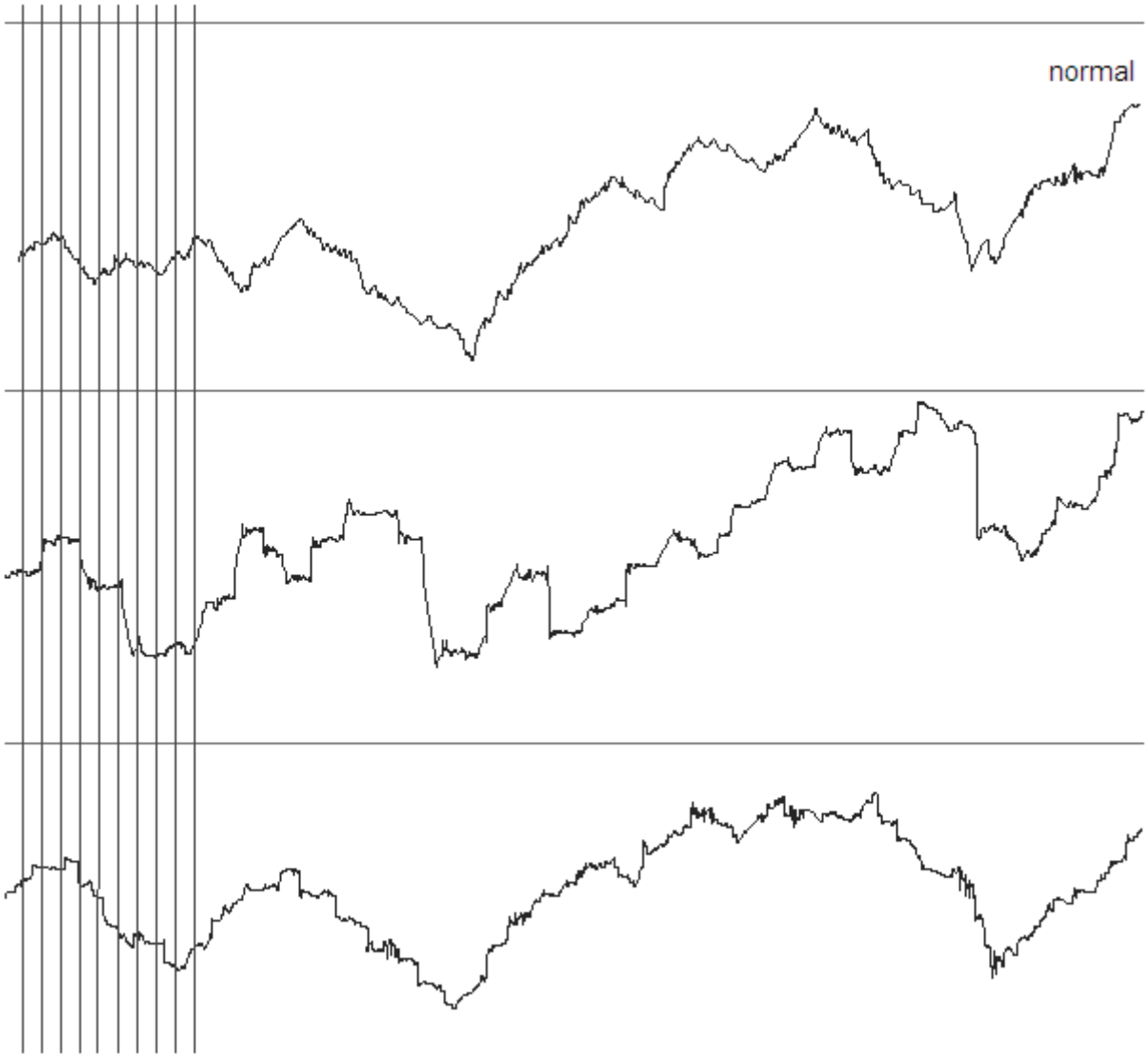




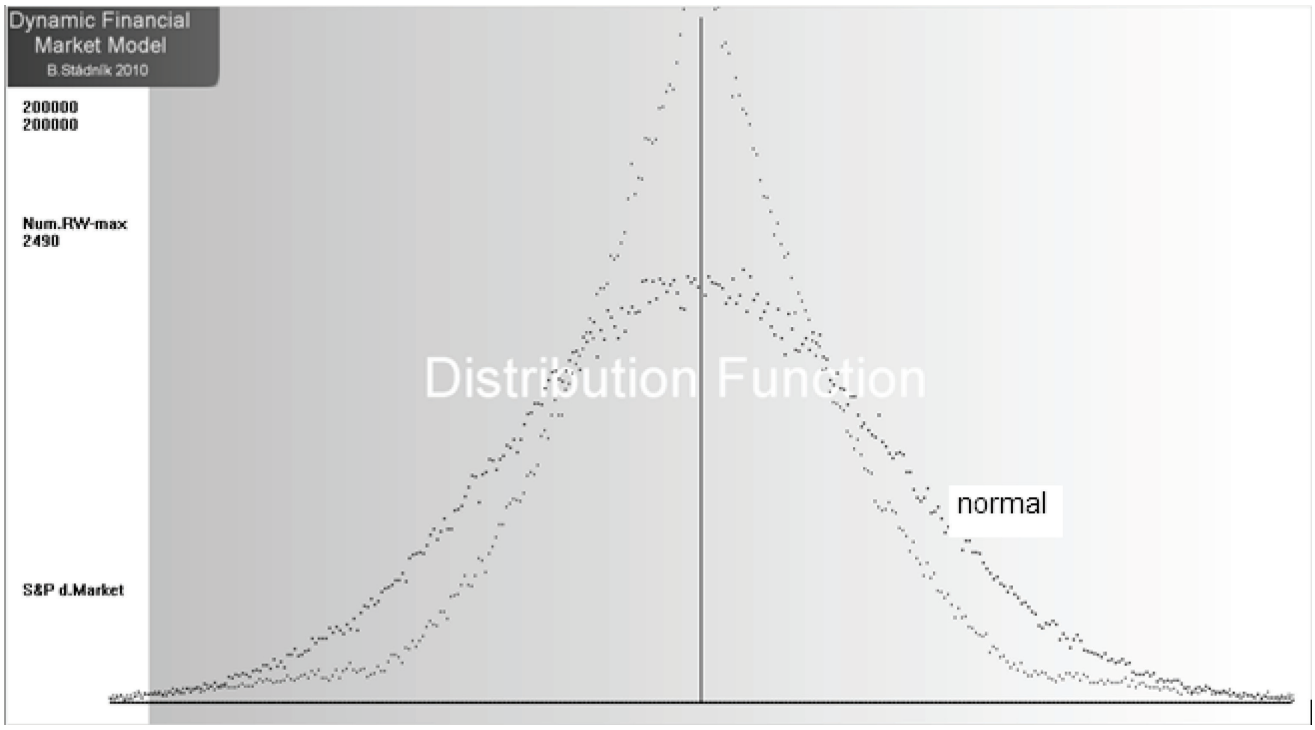
Developments with a different kurtosis, source: B.Stádník **[43d]**, **[43e]**



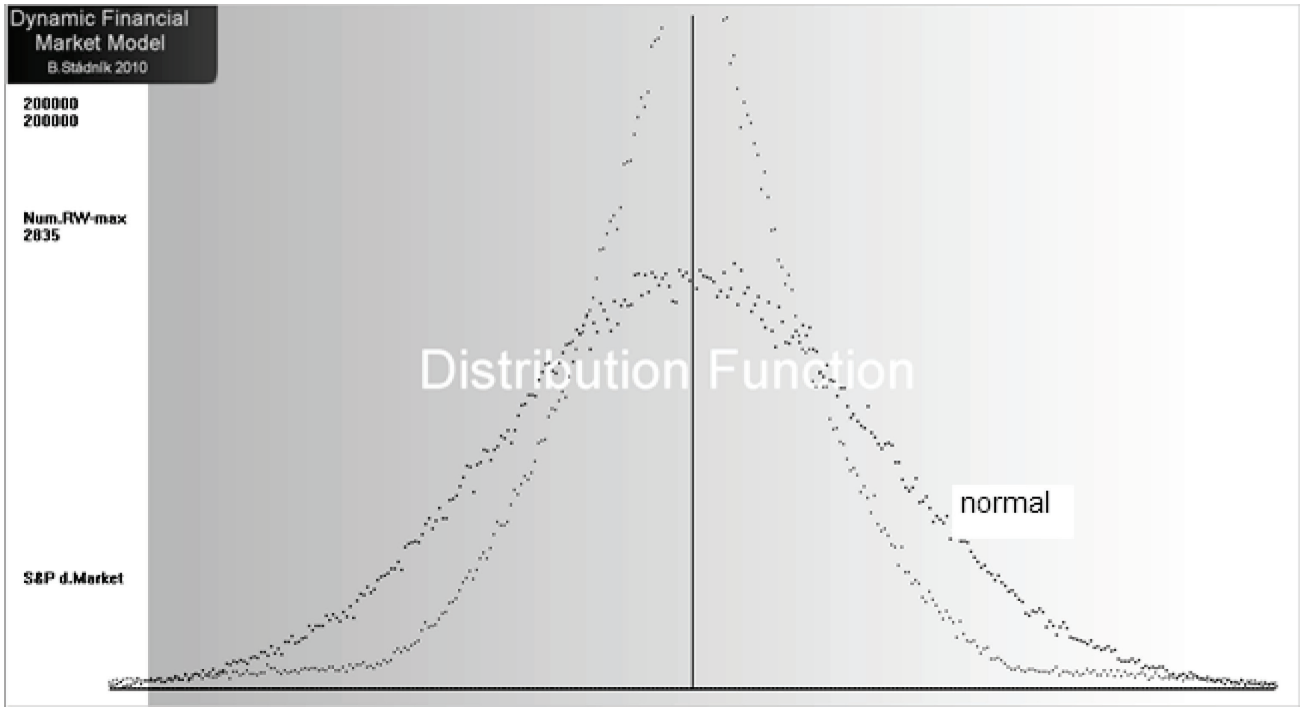
Different kurtosis, source: B.Stádník **[43f]**



Developments with a different kurtosis, source: B.Stádník **[43g]**



S&P500 simulation (author of simulation application: B.Stádník). Probability of a step direction changes from 50.00 to 50.76% in border area, from 50.00 to 50.24% in mid area, source: B.Stádník **[43h]**



Probability in border-area has changed from 50.00 to 50.60 %, probability in mid-area has changed from 50.00 to 50.30 %, source: B.Stádník **[43i]**

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