

Reciprocal Cost Allocations for Many Support Departments Using Spreadsheet Matrix Functions

**Dennis Togo
University of New Mexico**

The reciprocal method for allocating costs of support departments is the only method that recognizes all services provided to other departments. Yet, even as the number of support departments and their costs increase, the adoption of the reciprocal method has been hampered since it requires solving simultaneous equations for reciprocated costs of each support department. Matrix functions in spreadsheets will solve for reciprocated costs of many support departments. The Sasha Case illustrates the use of matrices to model services among support and operating departments, to solve simultaneous equations for the reciprocated costs of support departments, and to allocate the reciprocated costs to other departments.

INTRODUCTION

The objective of support department cost allocations is to have accurate product, service and customer costs. For many businesses in which the number of support departments and their costs and services provided are increasing, the selection of an allocation method is critical in tracking costs. Textbook authors (e.g., Hansen and Mowen, 2011; Horngren, Datar and Rajan, 2012) identify the reciprocal method as the most accurate support department cost allocation because it captures all services provided to other support and operating departments. However, most accounting practitioners continue to use the oversimplifying direct and step-down methods even with their shortcomings of not recognizing all services provided to support departments. The continued use of cost allocation methods not conducive to current business practices is perpetuated when accounting textbooks and instructors do not emphasize the reciprocal method.

Spreadsheet matrix functions facilitate the use of the reciprocal method as they easily model and solve the set of independent simultaneous equations formulated by the reciprocal method. Furthermore, the computed reciprocated costs of each support department can be allocated to the other support and operating departments using matrices. The following section briefly summarizes the benefits and disadvantages of the direct, step-down and reciprocal methods for support department cost allocations. In another section, the Sasha Company illustrates the facilitative matrix functions for performing reciprocal cost allocations of support departments.

COST ALLOCATION METHODS

Horngren et al. (2012) present the direct, step-down, and reciprocal methods to allocate support department costs to operating departments. The direct method is the most popular and simplest to use as support department costs are allocated only to operating departments based on services consumed. Hence,

the direct method does not recognize support services provided to other support departments. The step-down method improves cost allocations with partial recognition of support services to other support departments. The step-down method allocates costs at each “step” based on services consumed by remaining support and operating departments. A drawback to the step-down method is that previously closed support departments cannot receive cost allocations when closing the remaining support departments. Another drawback to the step-down method is that the order of closing support departments may vary depending on the criterion used (e.g., largest percentage of services provided to other support departments, or largest cost of support departments). Hence, cost allocations using the step-down method will vary depending on the criterion selected.

The reciprocal method should be used because it recognizes all services among support and operating departments. The reciprocated cost of a support department is defined as its own department cost and costs allocated from all the other support departments. Horngren et al. (2012) recognized that while a service department may provide services to itself, the reciprocated costs of a service department include the costs for services provided to its own department. There are n reciprocated cost variables for n support departments. The reciprocal method requires solving n simultaneous linear equations modeling the interrelationships of support departments. Adopters of the reciprocal method often incur difficulty having to solve for reciprocated costs for many support departments. Spreadsheet matrix functions facilitate the reciprocal method with their ability to model and solve complex scenarios of support services. The following Sasha Company case illustrates spreadsheet matrix techniques for the reciprocal method.

SASHA COMPANY: A CASE FOR MATRIX-BASED RECIPROCAL COST ALLOCATIONS

Background

Sasha Company is a large manufacturer of popular electronic games for children. It incurs significant costs in support Departments A, B, C, D and E. In the past, management has used the direct method to allocate support department costs to operating Departments X, Y and Z. Sasha Company expects to increase its product line and incur more support services; consequently, there is a need to have better product cost information. After a review of the accounting literature, management concludes the company should adopt the reciprocal cost allocation method and use matrices available in spreadsheets to a) model relationships among support and operating departments, b) solve the simultaneous equations for support departments’ reciprocated costs, and c) allocate the reciprocated costs of support departments to the operating departments.

Input Data for Support Services

The costs for the five support departments A, B, C, D and E and three operating departments X, Y, and Z are presented below. In addition, each support department percentages of services provided to other departments are displayed. For example, Department A has costs of \$1,400,000 and services can be traced to Departments B, C, D, E, X, Y and Z in the amounts of 0.15, 0.15, 0.10, 0.10, 0.25, 0.15 and 0.10, respectively. Each support department costs will be allocated to the other departments and that is represented with a -1.00. Hence, the total of support services for each department after its allocation is equal to 0.00. The array for percentages of services provided by departments A, B, C, D and E is also presented as a 5x8 |P| matrix to be discussed in a following section.

	Dept A	Dept B	Dept C	Dept D	Dept E	Dept X	Dept Y	Dept Z	Total
Costs:	1,400,000	1,200,000	1,000,000	800,000	600,000	3,000,000	2,500,000	2,000,000	12,500,000

Services:	P								
Dept A	-1.00	0.15	0.15	0.10	0.10	0.25	0.15	0.10	0.00
Dept B	0.20	-1.00	0.15	0.05	0.05	0.25	0.15	0.15	0.00
Dept C	0.15	0.05	-1.00	0.10	0.10	0.20	0.25	0.15	0.00
Dept D	0.10	0.10	0.05	-1.00	0.10	0.25	0.25	0.15	0.00
Dept E	0.05	0.10	0.05	0.10	-1.00	0.30	0.35	0.05	0.00

Linear Equations of Reciprocated Costs

Sasha Company generates algebraic equations for reciprocated costs A, B, C, D and E of the five support departments. For example, Equation 1 for Department A has the reciprocated cost +1.00A equal to its own cost of \$1,400,000 plus 0.20B, 0.15C, 0.10D and 0.05E. An equivalent linear expression Equation 1a for Department A is better suited for matrix algebra. Similarly, equations for reciprocated costs of support Departments B, C, D and E are expressed as Equations 1b, 1c, 1d, and 1e.

Dept A	+1.00A = 1,400,000 + 0.20B + 0.15C + 0.10D + 0.05E	Equation 1
Dept A	+1.00A – 0.20B – 0.15C – 0.10D – 0.05E = 1,400,000	Equation 1a
Dept B	–0.15A + 1.00B – 0.05C – 0.10D – 0.10E = 1,200,000	Equation 1b
Dept C	–0.15A – 0.15B + 1.00C – 0.05D – 0.05E = 1,000,000	Equation 1c
Dept D	–0.10A – 0.05B – 0.10C + 1.00D – 0.10E = 800,000	Equation 1d
Dept E	–0.10A – 0.05B – 0.10C – 0.10E + 1.00D = 600,000	Equation 1e

Matrix Relationship for Reciprocated Costs

The matrix relationship $|S| \times |X| = |K|$ is presented as Equation 2 for the set of five simultaneous Equations 1a, 1b, 1c, 1d and 1e. For example, Equation 1a is equivalent to multiplying the first row of the $|S|$ matrix with the first and only column of the $|X|$ matrix and then setting it equal to 1,400,000. The 5x5 $|S|$ matrix represents the reciprocated services among support departments, the 5x1 $|X|$ matrix represents the reciprocated costs as unknown variables A, B, C, D and E, and the 5x1 $|K|$ matrix represents the individual cost of each department. Each value within a matrix may be identified by its row and column; for example, $(s_{1,2})$ is equal to -0.20 of the $|S|$ matrix at row 1 and column 2. An array of numbers is noted as $(s_{1,1}:s_{5,5})$, which is equivalent to the $|S|$ matrix. The percentages of services presented previously can be represented as an array of numbers noted as $(p_{1,1}:p_{5,8})$ or the 5x8 $|P|$ matrix.

The EXCEL function TRANSPOSE may be used to obtain the $|S|$ matrix. After selecting a (5x5) area in the spreadsheet for the $|S|$ matrix, enter the EXCEL formula =-transpose(p_{1,1}:p_{5,5}) where the 5x5 array within the $|P|$ matrix represents services among support departments. Then enter Ctrl Shift Enter keys together.

$$\begin{array}{c|ccccc|c|c|c|c|c}
 & & & S & & & x & | & X & = & | & K & | & \text{Equation 2} \\
 \hline
 & +1.00 & -0.20 & -0.15 & -0.10 & -0.05 & & & A & & & 1,400,000 & \\
 & -0.15 & +1.00 & -0.05 & -0.10 & -0.10 & & & B & & & 1,200,000 & \\
 & -0.15 & -0.15 & +1.00 & -0.05 & -0.05 & x & & C & = & & 1,000,000 & \\
 & -0.10 & -0.05 & -0.10 & +1.00 & -0.10 & & & D & & & 800,000 & \\
 & -0.10 & -0.05 & -0.10 & -0.10 & +1.00 & & & E & & & 600,000 & \\
 \hline
 \end{array}$$

Solving Reciprocated Costs Using Matrix Functions

The solution for reciprocated costs of each support department is computed mathematically below by multiplying both sides of the matrix Equation 2 with the inverse of |S| or |S⁻¹|. The |S⁻¹| matrix multiplied with the |S| matrix equals the identity matrix |I|. The identity matrix |I| multiplied with the |X| matrix equals just the |X| matrix. Therefore, Equation 3 solves for |X|, the reciprocated costs of each support department, by multiplying the |S⁻¹| matrix with |K|. The related EXCEL formula for multiplying the two matrices is =mmult(minverse(|S|),|K|) or =mmult(minverse(s_{1,1}:s_{5,5}),k_{1,1}:k_{5,1}). The solution matrix for the reciprocated costs for departments A, B, C, D and E is shown below.

$$|S| \times |X| = |K| \quad \text{Equation 2}$$

$$|S^{-1}| \times |S| \times |X| = |S^{-1}| \times |K|$$

$$|I| \times |X| = |S^{-1}| \times |K|$$

$$|X| = |S^{-1}| \times |K| \quad \text{Equation 3}$$

$$|X| = \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} = \begin{matrix} 2,244,439 \\ 1,889,686 \\ 1,752,809 \\ 1,417,808 \\ 1,235,990 \end{matrix}$$

Allocating Reciprocated Costs Using Matrix Functions

The final step of the reciprocal method is to allocate the reciprocated costs of each support department to the other departments. Equation 4 is |D| x |P| = |A|, which multiplies the 5x5 |D| matrix with the 5x8 |P| matrix of service percentages found in a previous section. The diagonal |D| matrix is used to facilitate matrix multiplication and it has the reciprocated costs of each support department along the diagonal with all other values being zero. The resultant 5x8 allocated |A| matrix has reciprocated costs of each support department allocated to all other departments. The EXCEL formula for this matrix multiplication is =mmult(|D|,|P|) or =mmult((d_{1,1}:d_{5,5}),(p_{1,1}:p_{5,8})).

$$|D| \times |P| = |A| \quad \text{Equation 4}$$

			D			x	P	=	A
2,244,439	0.00	0.00	0.00	0.00	0.00	x	P	=	A
0.00	1,889,686	0.00	0.00	0.00	0.00				
0.00	0.00	1,752,809	0.00	0.00	0.00				
0.00	0.00	0.00	1,417,808	0.00	0.00				
0.00	0.00	0.00	0.00	1,235,990	0.00				

The resultant |A| matrix is placed in the cost allocation found below. All reciprocated costs of the support departments have been allocated leaving zero balances for Departments A, B, C, D and E. All services performed by support departments have been recognized, and the operating departments X, Y and Z now account for the total costs of \$12,500,000.

	Dept A	Dept B	Dept C	Dept D	Dept E	Dept X	Dept Y	Dept Z	Total
Costs:	1,400,000	1,200,000	1,000,000	800,000	600,000	3,000,000	2,500,000	2,000,000	12,500,000

Dept Allocations:

	A								
A	-2,244,439	336,666	336,666	224,444	224,444	561,109	336,666	224,444	0
B	377,937	-1,889,686	283,453	94,484	94,484	472,422	283,453	283,453	0
C	262,921	87,640	-1,752,809	175,281	175,281	350,563	438,202	262,921	0
D	141,781	141,781	70,890	-1,417,808	141,781	354,452	354,452	212,671	0
E	61,800	123,599	61,800	123,599	-1,235,990	370,796	432,596	61,800	0
Total	0	0	0	0	0	5,109,342	4,345,369	3,045,289	12,500,000

SUMMARY

The reciprocal method is the best approach for allocating support department costs, especially in complex business organizations. Yet, its adoption and instruction has been hampered by math skills necessary to model, compute and allocate reciprocated costs of support departments. Matrix functions found in spreadsheets remove the difficulties associated with the reciprocal method. The Sasha Company case illustrates the matrix-based reciprocal method for allocating support department costs.

REFERENCES

Hansen, D. & Mowen, M. (2011). *Cornerstones of Cost Accounting*, Mason, Ohio: South-Western Cengage Learning.

Horngrén, C.T., Datar, S.M. & Rajan, M. (2012). *Cost Accounting: A Managerial Emphasis*, Upper Saddle River, New Jersey: Prentice-Hall.