

Constrained Nonlinear Models for Optimal Allocations of Resources in Business Operations

Ji Li

California State University-Bakersfield

Di Wu

California State University-Bakersfield

Given limited business resources, including monetary capital, raw materials, and labor, management often needs to make an optimal allocation of these resources so that its company as a whole can achieve the maximum contribution margin or profit. This paper focuses on applying constrained nonlinear models to study the optimal solutions of contribution margin and profit in businesses when multiple products are manufactured with limited business resources. The paper offers examples using these quantitative models as well as computer technology approaches, which can be incorporated into any cost accounting curriculum with the aim of enhancing students' critical reasoning and quantitative skills.

INTRODUCTION

From the perspectives of diversifying business risks and targeting different groups of customers, many businesses in nearly all industries choose to manufacture differentiated products or provide different services. Through the process of allocating business resources and capital in a way that reduces the exposure to any particular business risk and volatility, companies can hence have a better chance to survive in unforeseen economic or market turmoil.

In general, when manufacturing multiple products or providing different services, companies need to address one important issue: how to allocate limited business resources—including materials, supplies, labor and capital—to multiple production or service lines in order to achieve the best effective resource use and maximum benefits, which are often measured in terms of contribution margin and operating profit (Horngren, 2015). In order to find the optimal allocation, management typically needs to apply quantitative models to study business objectives and constraints. In some cases, quantitative models can be as simple as linear models with linear constraints (Togo, 2008). Therefore, linear programming approaches—including the simplex method, integer programming, binary programming, quadratic programming, and mixed integer programming—can be implemented in order to resolve linear models with linear constraints (Chvatal, 1983). Computer technology can facilitate the applications of these linear models and let cost accountants and managers easily find optimal allocations of business resources without going through detailed and complicated quantitative steps.

However, using only linear models with linear constraints can be shortsighted and may mislead business decisions, particularly when operating records and data do not support linear models (Wu, 2016). For instance, purchasing a larger amount of materials may result in a greater discount offered by

suppliers, workers may become more productive and spend less labor hours as they manufacture more products (Horngren, 2015) with a learning curve model, and a less expensive price may induce more sales in units, all of which sometimes should not be modeled in linear equations and constraints (Landsburg, 2013). As a result, some research and data analysis suggest nonlinear models, such as polynomial, power, exponential, and logarithmic functions in these cost accounting issues.

With the development of computer technology, especially the availability of many user-friendly modeling software packages, more sophisticated models, such as constrained nonlinear models, can be used by cost accountants and managers without having much background in the field of quantitative sciences. Hence, with the aim of enhancing students' critical reasoning and quantitative skills, accounting education in undergraduate and graduate levels should incorporate more content and examples in nonlinear and practical models, in addition to linear and simplified models. This paper focuses on how to model business operations and find optimal allocations of business resources when multiple products are manufactured and when constrained nonlinear models need to be used in data analytics. With more implementation and involvement of big data concepts and techniques, business professionals expect more of an impact from big data in the business world (Warren, 2015). Therefore, this paper may provide useful tools to accounting professionals for big data subjects.

MATHEMATICAL MODELS AND OPTIMIZATION METHODS

Many cost and managerial accounting issues always enjoy linear models and constraints when collected operating records support linearity among variables. Therefore, using linear regression methods can easily determine variable coefficients and suggest linear models for later decision-making, business forecasting, and performance evaluations.

Definition 1. A linear model is a function that can be expressed in the following form:

$$f(x) = c_1x_1 + c_2x_2 + \dots + c_nx_n, \text{ where } x = (x_1, x_2, \dots, x_n), x \in D, D \subseteq R^n \quad (1)$$

Any function that cannot be expressed in the form of Equation (1) is a nonlinear model. Nonlinear models may include high-order polynomial, power, exponential, radical, and logarithmic functions.

Definition 2. A linear inequality is an expression that can be expressed in the following form:

$$f(x) \leq b, \text{ where } f(x) \text{ is linear, } x = (x_1, x_2, \dots, x_n), x \in D, D \subseteq R^n, b \in R. \quad (2)$$

Nonlinear inequality is a similar expression as in Equation (2) except that $f(x)$ is nonlinear.

Definition 3. An unconstrained objective function is a linear or nonlinear function that needs to be maximized or minimized on its domain, such that

$$\max_{x \in D} / \min_{x \in D} f(x), \text{ where } x = (x_1, x_2, \dots, x_n), x \in D, D \subseteq R^n. \quad (3)$$

Definition 4. A constrained objective function is a linear or nonlinear function that needs to be maximized or minimized on its domain with a set of constraints, such that

$$\begin{aligned} & \max_{x \in D} / \min_{x \in D} f(x) \\ & \text{subject to } g_i(x) \leq b_i, \text{ where } x = (x_1, x_2, \dots, x_n), x \in R^n, i = 1, \dots, m, b_i \in R \end{aligned} \quad (4)$$

Normally, when $f(x)$ and all $g_i(x)$ are linear, the problem is called a linear programming problem, which may be solved by simplex method, integer programming, binary programming, quadratic programming, and mixed integer programming.

However, if Equation (4) is nonlinear, then its solutions may require advanced optimization methods, such as method of Lagrange multipliers and iterative numerical methods (Bertsekas, 1996). This paper focuses on how to solve constrained nonlinear models in cost accounting issues.

Definition 5. Relevant range is a specific operation or activity level with a lower bound and upper bound due to the resource constraints.

Definition 6. x^* is said to be an absolute maximizer (minimizer) to the constrained problem in Equation (4), $f(x)$ if $f(x^*) \geq f(x)$ ($f(x^*) \leq f(x)$) for any $x \in D$, where $x = (x_1, x_2, \dots, x_n)$, $D \subseteq R^n$ and all constraints are satisfied.

When management needs to budget or plan business operations for the purposes of maximizing contribution margin or profit, historical records or budgeted values are often collected or prepared for subsequent analysis. Then management will need to identify operating resources, which are normally constraints in business operations and to study relationship between variables, such as sales volume VS price (Landsburg, 2013) and input of resources VS output of products. Although linear models and constraints work in some cases, with more and more sophisticated production processes, product designs, and marketing strategies, collected data may support nonlinear correlations and relationships between variables of interest. For instance, it is often found that variable costs of director labor per unit may change as workers pick up the learning curve, and these changes may not be directly proportional to the cost drivers at a constant rate. Sales volumes in units and prices are highly correlated in many situations, but scatterplots of collected data may show nonlinear correlations. All these observations may suggest using the more advanced nonlinear models.

Let us consider that a company sells n products, and each product has a relationship between unit selling price and units to be sold. For instance, let x_1, x_2, \dots, x_n be the number of units sold for each of n products, $p_1(x_1), p_2(x_2), \dots, p_n(x_n)$ be the corresponding price, D_1, D_2, \dots, D_n be the relevant range for each product manufacturing line, $vc_1(x_1), vc_2(x_2), \dots, vc_n(x_n)$ be the total variable cost function for each product manufactured, and finally fc be the total fixed costs that are constant within the relevant range.

Therefore, the contribution margin and profit function can be modeled, respectively, as

$$CM(x) = \left[\sum_i (x_i p_i(x_i) - vc_i(x_i)) \right], \text{ where } x = (x_1, x_2, \dots, x_n) \in D = (D_1, D_2, \dots, D_n), D \subset R^n \quad (5)$$

$$f(x) = \left[\sum_i (x_i p_i(x_i) - vc_i(x_i)) \right] - fc, \text{ where } x = (x_1, x_2, \dots, x_n) \in D = (D_1, D_2, \dots, D_n), D \subset R^n \quad (6)$$

Meanwhile, the company also has identified a set of constraints of business resources, such as labor hours, machine hours, and raw materials. Hence, these constraints should be incorporated into the models as follows:

$$g_i(x) \leq b_i, \text{ where } x = (x_1, x_2, \dots, x_n), x \subseteq R^n, i = 1, \dots, m, b_i \in R \quad (7)$$

Therefore, management will need to determine the optimal solution to either Equation (5) or (6) subject to the constraints in Equation (7). In order to find solutions to a constrained nonlinear model, advanced optimization methods will be employed.

NUMERICAL METHODS

One method to find the optimal solutions to the constrained nonlinear problems in Equation (4) is using LaGrange multipliers in the scope of multivariate calculus and optimization theory. To be more specific, given the following constrained maximization problem:

$$\begin{aligned} & \max_{x \in D} f(x) \\ & \text{subject to } g_i(x) \leq b_i, \text{ where } x = (x_1, x_2, \dots, x_n), x \subseteq R^n, i = 1, \dots, m, b_i \in R \end{aligned} \quad (8)$$

The method of Lagrange multipliers defines a new function:

$$h(x) = f(x) + \lambda_1(g_1(x) - b_1) + \lambda_2(g_2(x) - b_2) + \dots + \lambda_m(g_m(x) - b_m) \quad (9)$$

Then, use partial derivatives to find critical values of Equation (9), which also satisfies all given constraints, such that:

$$\begin{aligned} \frac{\partial h}{\partial x_i} &= \frac{\partial f}{\partial x_i} + \lambda_1 \frac{\partial g_1}{\partial x_i} + \lambda_2 \frac{\partial g_2}{\partial x_i} + \dots + \lambda_m \frac{\partial g_m}{\partial x_i}, i = 1, \dots, n \\ \lambda_j(g_j(x) - b_j) &= 0, i = j, \dots, m \end{aligned} \quad (10)$$

Solve the system of equations in Equation (10) to determine critical values, which are used to find maximizers of Equation (8) by physical or intuitive arguments (Stewart, 2013).

REGRESSION ANALYSIS IN PRICING, LEARNING CURVE AND COSTS

To find relationships among variables in quantitative models and employ them, management or cost accountants often use regression analysis (Draper, 1998). A scatterplot might be helpful to illustrate the possible correlation for two variables, but a regression analysis can further provide a specific function model. Linear regression seems to be more popular for its simplicity. This paper focuses on scenarios in which nonlinear relationships are observed. Although the sacrifice of simplicity may bring challenges in modeling and simulations, cost and profit analysis might be more accurate. Meanwhile, with the development of computer software packages, such as SPSS, Excel, and MATLAB, it becomes convenient and handy for cost accountants and management to implement advanced models and computer technology in data analysis without much background in advanced mathematics and statistics fields.

Throughout this paper, nonlinear regression models will be applied to collected data sets. When nonlinear models become available from regression analysis, a constrained optimization problem can be set up and solved by the method of Lagrange multipliers. More importantly, computer technology methods also will be introduced in order to demonstrate how to find optimal solutions to nonlinear constrained models.

CONSTRAINED OPTIMIZATION AND NONLINEAR MODELS IN EXAMPLES

SuperLive Company Background

SuperLive Company manufactures and sells stainless-steel pressure cookers and 5 QT stainless-steel bowls. Every year, SuperLive receives large orders from retailers all over the world, and its products are popular. The company has an annual manufacturing capacity of 1 million pressure cookers and 3 million 5 QT bowls. SuperLive uses stainless-steel materials purchased from its suppliers in manufacturing pressure cookers and 5 QT bowls. The company recently discovered that production resources may not be

well allocated and used; hence, its CEO believes that the company may need to investigate how to best utilize resources to improve the company’s overall contribution margin. In order to understand how exactly the company should allocate its resources to production and sales, the CEO has appointed Steve Smith, a cost accountant, to collect market data, SuperLive budget information, and previous operating results so that Steve can conduct a thorough research. The CEO expects to receive the report from Steve, outlining recommendations of allocations of resources, marketing strategies, and budgeted sales for the purposes of maximizing the company’s contribution margin.

Price and Sales Volume Modeling

Steve has noticed that, in previous sales records, sales of pressure cookers and bowls in units change as the sales price of each product moves up or down. Therefore, Steve wants to include this analysis in the research. Steve has requested more detailed sales data from the accounting and marketing departments over the last several years. Initial scatterplots reveal that sales volume and price do not seem to follow a linear correlation for both products, as illustrated in Table 1 and Figure 1. Therefore, Steve wants to use nonlinear regression models to simulate the relationships between sales volume and selling price.

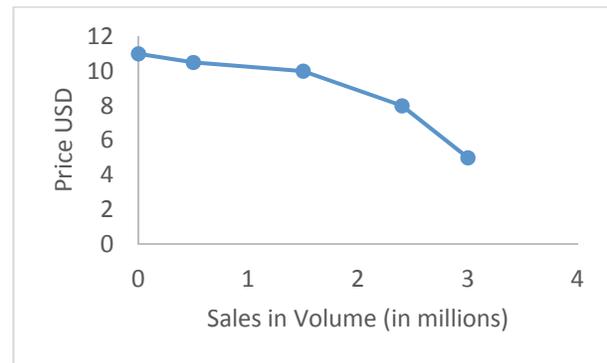
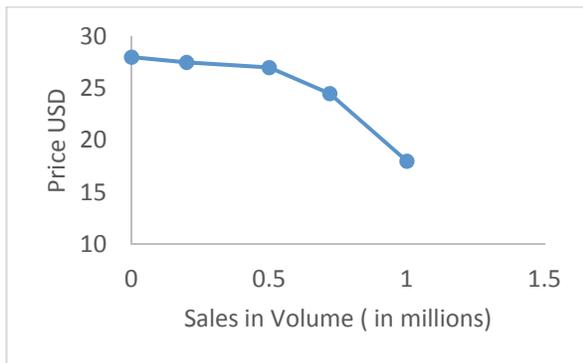
**TABLE 1
SALES VOLUME AND SELLING PRICE**

Pressure Cooker		5 QT Bowl	
Sales Volume (in millions)	Unit Price (USD)	Sales Volume (in millions)	Unit Price (USD)
1	17.99	3	4.99
0.72	24.49	2.4	7.99
0.5	26.99	1.5	9.99
0.2	27.49	0.5	10.49
0	27.99	0	10.99

**FIGURE 1
SCATTER PLOTS OF SALES VOLUME AND SELLING PRICE**

(a) Pressure Cooker

(b) 5 QT Bowl



To convince himself to use nonlinear regression models, Steve tries both linear and nonlinear models. R-squared values demonstrate that, for these two products, it might be better to use nonlinear models for studying the relationship between price and sales volume, as seen in Table 2 and Figure 2, because nonlinear models have better fit of data in terms of R-squared values.

Learning Curves in Direct Labor Hours

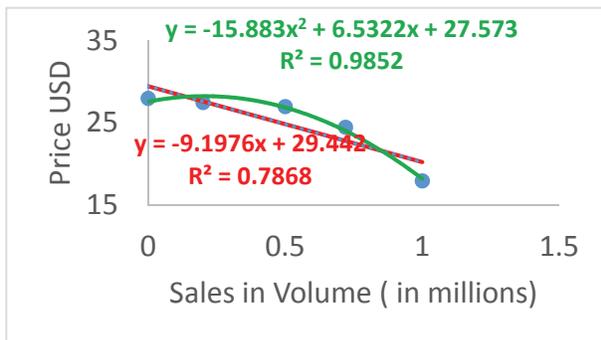
Steve has visited SuperLive production department and interviewed the manager and workers. Steve has seen that some of procedures in the manufacturing process must be done manually by workers. It is also found that labor-hours per unit decline as units of production increase, which shows that workers become more efficient and productive in some production procedures. In the literature, the learning curve also may be called the “experience curve,” which has been well adopted by many companies, including Dell Computer and Walmart. Steve would like to adopt the learning curves in analysis. Steve also feels that the cumulative average-time learning model should be applied to manufacturing the pressure cookers and 5 QT bowls. In most literature, nonlinear or power functions are applied. Steve does scatterplots for data obtained from the production manager and applies power regressions to model learning curves, as illustrated in Table 3 and Figure 3. Both power models have a good fit of data; however, linear regression and polynomial regression models are not as good as power models.

TABLE 2
REGRESSION MODELS WITH CORRELATION COEFFICIENTS

	Pressure Cooker		5 QT Bowl	
	Nonlinear Regression	Linear Regression	Nonlinear Regression	Linear Regression
Model	$y = -15.883x^2 + 6.5322x + 27.573$	$y = -9.1976x + 29.442$	$y = -0.902x^2 + 0.8766x + 10.706$	$y = -1.8088x + 11.567$
R-Squared	0.9852	0.7868	0.9811	0.8528
Recommendation	Yes	No	Yes	No

FIGURE 2
REGRESSION MODELS OF SALES VOLUME AND SELLING PRICE

(a) Pressure Cooker



(b) 5 QT Bowl

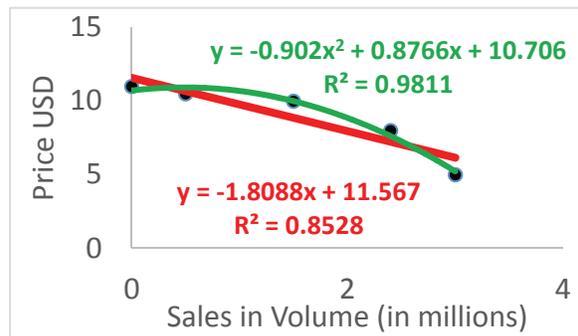
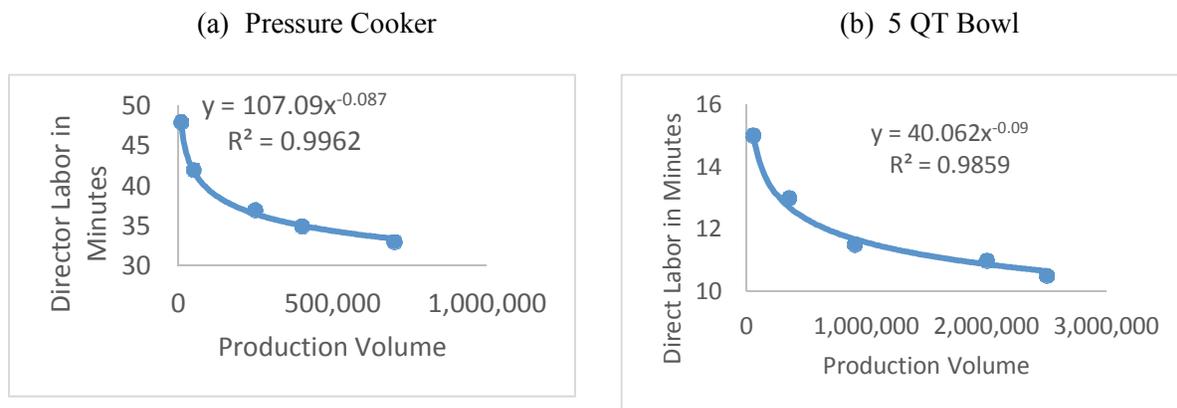


TABLE 3
LEARNING CURVE: UNITS PRODUCED AND DIRECT LABOR HOURS

Pressure Cooker		5 QT Bowl	
Production Volume	Direct Labor Hours /unit (in minutes)	Production Volume	Direct Labor Hours /unit (in minutes)
700,000	34	2,500,000	10.5
400,000	35	2,000,000	11
250,000	37	900,000	11.5
50,000	42	350,000	13
10,000	48	50,000	15

FIGURE 3
LEARNING CURVE: UNITS PRODUCED AND DIRECT LABOR HOURS



Variable Costs and Production Volume

Contribution margin calculation relies on the information of revenue and variable costs. It is a critical important indicator of profitability and operations effectiveness. Although sometimes variable costs per unit are considered constant, Steve notes that it is not the case for SuperLive. After Steve speaks with the purchase department and production department, he understands that several factors make variable costs per unit change as production volume changes. Purchase department manager confirms that, with a larger order of raw materials such as stainless steel, suppliers are willing to offer greater discounts. Input of direct labor hours per each product uses a learning curve and hence behaves nonlinearly. Therefore, Steve believes that variable costs per unit should be expressed in terms of production volume. Using historical data and statistical analysis, Steve is convinced and applies a nonlinear regression model to study the relationship between variable costs per unit and production volume, as shown in Table 4 and Figure 4.

TABLE 4
UNITS PRODUCED AND VARIABLE COSTS/UNIT

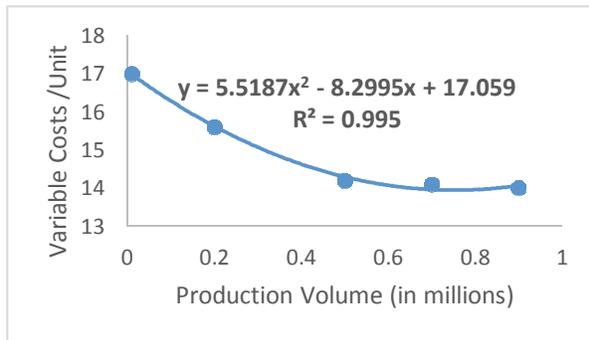
Pressure Cooker		5 QT Bowl	
Production Volume (in millions)	Variable Costs/Unit (USD)	Production Volume (in millions)	Variable Costs/Unit (USD)
0.9	14	2.7	3
0.7	14.1	2.1	3.5
0.5	14.2	0.9	6
0.2	15.6	0.1	8.5
0.01	17	0.05	9

Resource Constraints and Budgeted Production Plans

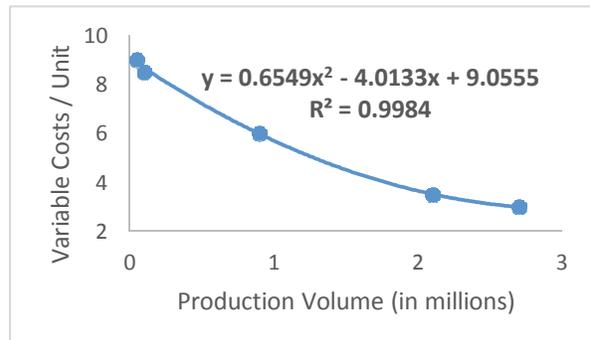
Steve knows that SuperLive has limited resources in production and business. After visiting the production, accounting, and human resources departments, Steve finds the budgeted resources available for current production. First, given the number of machines and workers in the production lines, SuperLive has a total of 700,000 direct labor hours that can be used to manufacture these products; second, the pressure cookers and 5 QT bowls use stainless-steel materials. It takes 2 kg of stainless-steel materials to make a pressure cookers and 1 kg of stainless-steel materials to make a 5 QT bowl, according to standard operating procedures (SOP). SuperLive may be able to purchase 3 million kg of stainless-steel materials for its production annually due to limited capital resources. SuperLive has no beginning and ending inventories for finished products and raw materials due to the high costs of inventory storage and its strategy of lean production.

FIGURE 4
UNITS PRODUCED AND VARIABLE COSTS/UNIT

(a) Pressure Cooker



(b) 5 QT Bowl



Maximization Problem Modeled in the Case of SuperLive

Steve collects all the available information and data and feels ready to investigate how SuperLive should plan its production and sales so that resources are effectively used to maximize its contribution margin.

First, Steve sets up the objective function to maximize:

- 1) for pressure cookers, let n_1 , p_1 and vc_1 be the number of units manufactured and sold, the selling price per unit and the variable costs per unit; from Figures 2 and 4, the contribution margin is

$$\begin{aligned}
& n_1(p_1 - vc_1) \\
& = n_1(-15.883n_1^2 + 6.5322n_1 + 27.573 - 5.5187n_1^2 + 8.2995n_1 - 17.059) \quad (11) \\
& = -21.4017n_1^3 + 14.8317n_1^2 + 10.514n_1
\end{aligned}$$

- 2) for 5 QT bowls, let n_2 , p_2 and vc_2 be the numbers of units manufactured and sold, the selling price per unit and the variable costs per unit; from Figures 2 and 4, the contribution margin is
- $$\begin{aligned}
& n_2(p_2 - vc_2) \\
& = n_2(-0.902n_2^2 + 0.8766n_2 + 10.706 - 0.6549n_2^2 + 4.0133n_2 - 9.0555) \quad (12) \\
& = -1.5569n_2^3 + 4.8899n_2^2 + 1.6505n_2
\end{aligned}$$

Hence, the total contribution margin is the result of adding Equations (11) and (12):

$$CM(n_1, n_2) = -21.4017n_1^3 + 14.8317n_1^2 + 10.514n_1 - 1.5569n_2^3 + 4.8899n_2^2 + 1.6505n_2 \quad (13)$$

Steve also includes a few constraints expressed in inequalities:

- 1) Relevant range, for pressure cookers, $0 \leq n_1 \leq 1$ million ; for 5 QT bowls, $0 \leq n_2 \leq 3$ million
- 2) Direct labor hours,
 $n_1 \cdot 107.09(1000000n_1)^{-0.087} + n_2 \cdot 40.062(1000000n_2)^{-0.09} \leq 42$ million minutes
- 3) Stainless-steel materials, $2n_1 + n_2 \leq 3$ million kgs

Steve, hence, has set up the optimization problem as

$$\begin{aligned}
& \max CM(n_1, n_2) = -21.4017n_1^3 + 14.8317n_1^2 + 10.514n_1 \\
& \quad - 1.5569n_2^3 + 4.8899n_2^2 + 1.6505n_2 \\
& \text{subject to :} \\
& 0 \leq n_1 \leq 1, 0 \leq n_2 \leq 3 \\
& n_1 \cdot 107.09(1000000n_1)^{-0.087} + n_2 \cdot 40.062(1000000n_2)^{-0.09} \leq 42 \\
& 2n_1 + n_2 \leq 3
\end{aligned} \quad (14)$$

The method of Lagrange multipliers, as previously illustrated, can be applied to solve Equation (14) but might be challenging to most accountants. For instance, the optimization problem can be formulated as the following:

$$\begin{aligned}
& h(n_1, n_2) = -21.4017n_1^3 + 14.8317n_1^2 + 10.514n_1 - 1.5569n_2^3 + 4.8899n_2^2 + 1.6505n_2 \\
& \quad + \lambda_1(n_1 \cdot 107.09(1000000n_1)^{-0.087} + n_2 \cdot 40.062(1000000n_2)^{-0.09} - 42) + \lambda_2(2n_1 + n_2 - 3)
\end{aligned} \quad (15)$$

Then, a system of equations can be set up from Equation (15):

$$\frac{\partial h}{\partial n_1} = \frac{\partial h}{\partial n_2} = 0$$

$$\lambda_1(n_1 \cdot 107.09(1000000n_1)^{-0.087} + n_2 \cdot 40.062(1000000n_2)^{-0.09} - 42) = 0 \quad (16)$$

$$\lambda_2(2n_1 + n_2 - 3) = 0$$

By solving the system of equations in Equation (16), a set of critical values is available; thus, the solution to Equation (14) yields the maximum contribution margin. Solving Equation (16) might be beyond the scope of this paper; normally, it can be difficult for accountants to find its analytical solutions.

Most problems such as Equation (14) can be solved by using Excel Solver toolboxes or even advanced computer technology, such as MATLAB and SAS. Steve then uses Excel Solver toolboxes to find the optimal solutions of production activities of pressure cookers and 5 QT bowls to maximize the SuperLive contribution margin. The maximum contribution margin is \$16,821,323 USD when 559,100 pressure cookers and 1,881,800 5 QT bowls are manufactured with a set of resource constraints, as shown in Table 5. The detailed application of computer technology is shown in the next section.

TABLE 5
OPTIMAL SOLUTIONS AND MAXIMIZED CONTRIBUTION MARGIN

Optimal solutions	Pressure Cooker	5 QT Bowl
Sales in volume	559,100	1,881,800
Unit price	\$26.26	\$9.16
Direct labor hours	33.86 minutes	10.92 minutes
Variable costs per unit	\$14.14	\$3.82
Total maximum contribution margin (USD)	\$ 16,821,323	

EXCEL TOOLBOXES TO MODEL SUPERLIVE PRODUCTION

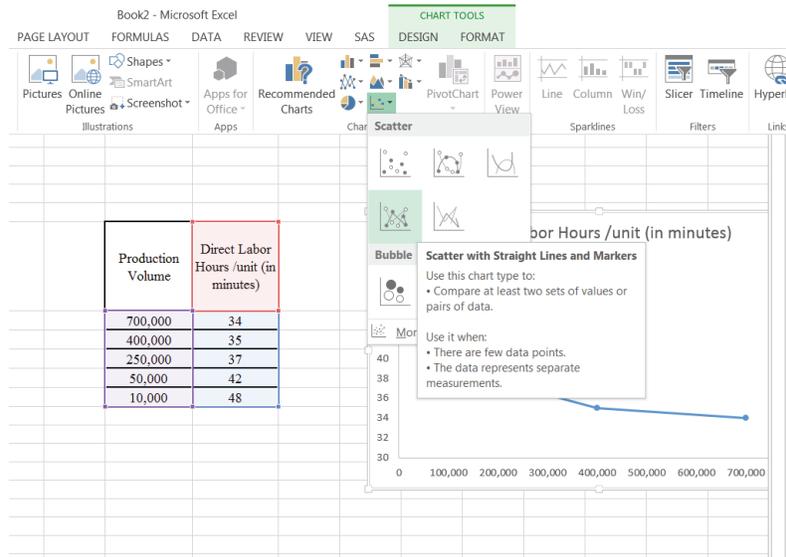
Whenever nonlinear models are applied and have multiple variables, it is always challenging to model and find optimal solutions. Excel toolboxes are always handy and useful to management and accountants when they are dealing with sophisticated models. Therefore, management and accountants may not need to know advanced mathematics, such as calculus, numerical optimization, and gradient methods. After management and cost accountants define variables, set up objective functions to optimize and develop constraints based on company business operations; they can rely on computer technology to find the solutions to the objective functions.

Regression Analysis

Many functions shown previously are modeled using Excel after collecting the necessary data sets. Data will be loaded into Excel and selected for a regression analysis.

As shown in Figure 5, click INSERT =>SCATTER=>SCATTER; then users will see a scatterplot of selected data. From the scatterplot, users might be able to determine which models are more applicable (options include linear, polynomial, power, exponential, and logarithmic models). Users can try a few different models in the same figure. For instance, click on the + => CHART ELEMENTS => TRENDLINE =>MORE OPTIONS. For every model that users try, the boxes of “Display Equation on Chart” and “Display R-Squared Value on Chart” should also be checked. R-squared values indicate the fitness of model to the data, and corresponding equations can be further studied and applied. Data of pressure cookers in Table 3 are used to show detailed analysis. Linear, power, and polynomial models are applied, as shown in Figure 6.

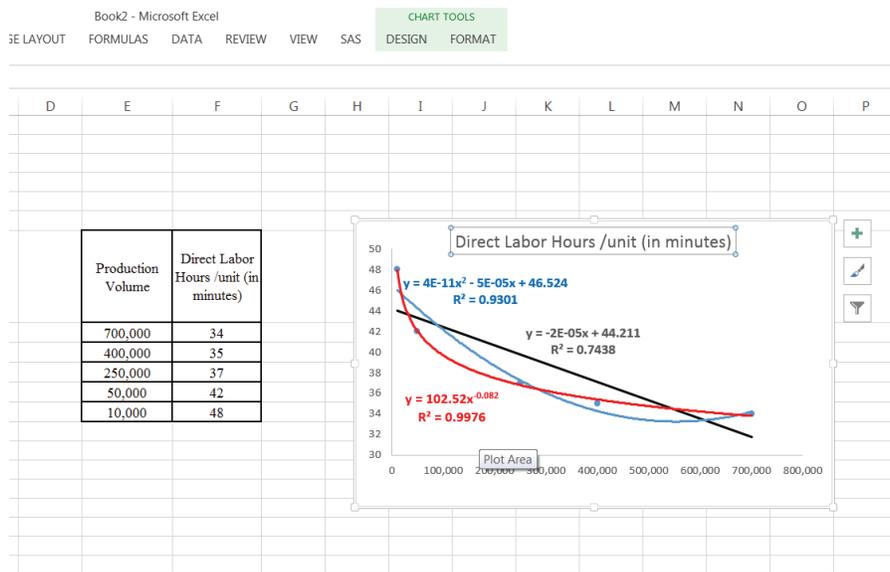
**FIGURE 5
INPUT COLLECTED DATA AND SCATTER PLOT**



Excel Optimization Solver

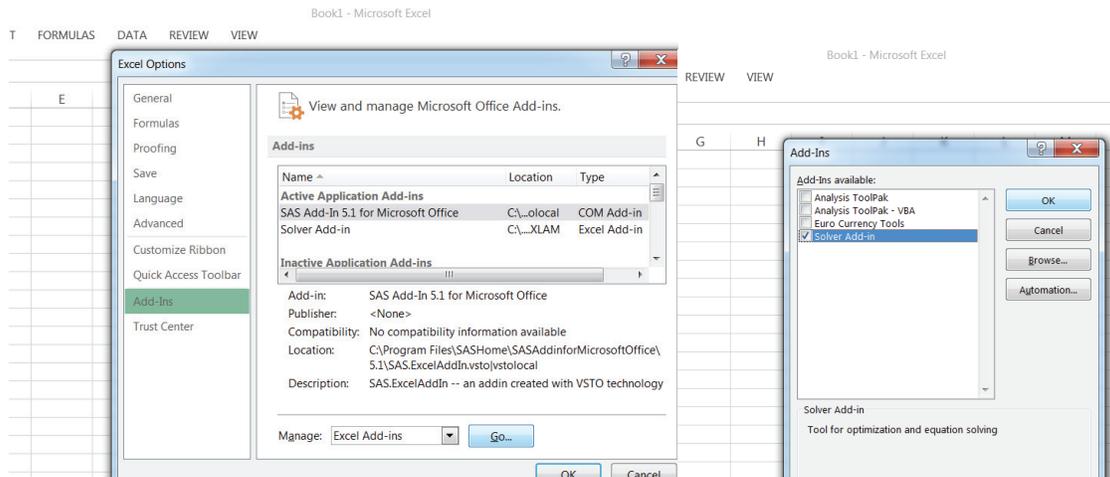
After management and cost accountants well study business operations with determined models for price, variable costs, constraints, and objective functions, they will need to know how to effectively use resources to maximize contribution margins or profit or to minimize financing costs. Excel Solver can assist business users to skip sophisticated computational steps without much background in quantitative fields, efficiently find optimal solutions to models, and make recommendations of business operations to the company.

**FIGURE 6
REGRESSION ANALYSIS WITH SEVERAL MODELS**



Excel Solver may need to be initiated in the beginning for the first-time user. Users can click FILE => OPTIONS => ADD-INS => MANAGE => EXCEL ADD-INS => GO. Then users will see a pop-up window and need to click SOLVER ADD-IN => OK to add Excel Solver. Figure 7 shows steps for adding Excel Solver.

**FIGURE 7
EXCEL SOLVER**



Once Excel Solver is set up, users can solve the model. To demonstrate how Steve is able to find the optimal solution and make recommendations to CEOs about business operations, Figures 8–11 include detailed steps. For instance, Steve gets the model as in Equation (14):

$$\begin{aligned}
 \max CM(n_1, n_2) &= -21.4017n_1^3 + 14.8317n_1^2 + 10.514n_1 \\
 &\quad - 1.5569n_2^3 + 4.8899n_2^2 + 1.6505n_2 \\
 \text{subject to :} & \\
 0 \leq n_1 \leq 1, 0 \leq n_2 \leq 3 & \\
 n_1 \cdot 107.09(1000000n_1)^{-0.087} + n_2 \cdot 40.062(1000000n_2)^{-0.09} \leq 42 & \\
 2n_1 + n_2 \leq 3 &
 \end{aligned}
 \tag{14}$$

**FIGURE 8
VARIABLES, CONSTRAINTS, AND OBJECTIVE FUNCTION**

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I
1	Variables		Production						
2	Pressure Cookers		0.1						
3	5 qt bowls		0.2						
4									
5	Constrains								
6	Relevant Range 1		0	0.1	1				
7	Relevant Range 2		0	0.2	3				
8	Direct Labor Hours			6.604202	42				
9	Direct Materials			0.4	3				
10									
11									
12									
13									

The formula bar for cell D8 shows: $=C2*107.09*(1000000*C2)^{-0.087}+C3*40.062*(1000000*C3)^{-0.09}$

**FIGURE 9
OBJECTIVE FUNCTION IN EXCEL**

The screenshot shows an Excel spreadsheet with the following data:

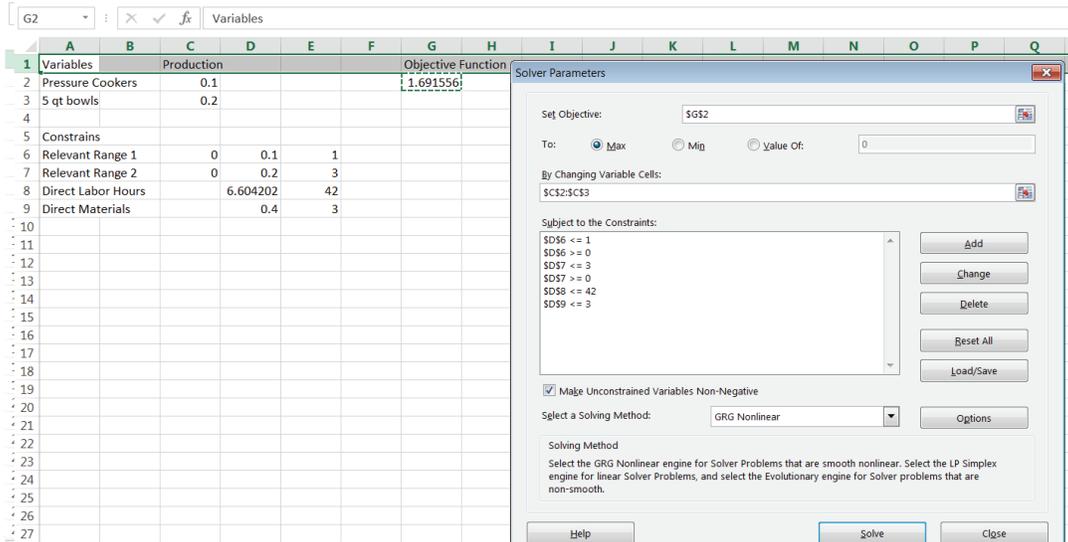
	A	B	C	D	E	F	G	H	I	J	K
1	Variables		Production				Objective Function				
2	Pressure Cookers		0.1				1.691556				
3	5 qt bowls		0.2								
4											
5	Constrains										
6	Relevant Range 1		0	0.1	1						
7	Relevant Range 2		0	0.2	3						
8	Direct Labor Hours			6.604202	42						
9	Direct Materials			0.4	3						
10											
11											
12											
13											

The formula bar for cell G2 shows: $=-21.4017*C2^3+14.8317*C2^2+10.514*C2-1.5569*C3^3+4.8899*C3^2+1.6505*C3$

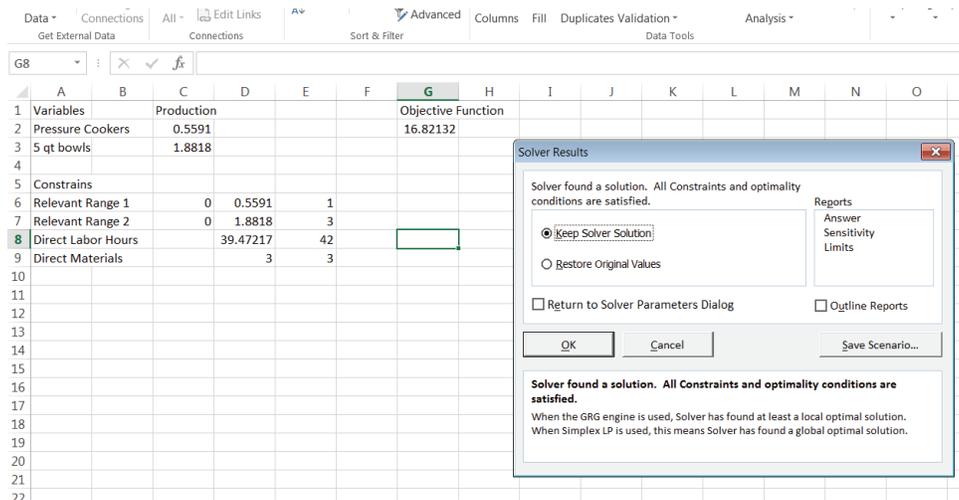
Steve inputs variables, constraints, and objective function into an Excel file, as in Equation (14). Figure 8 shows that Steve first defines n_1 and n_2 to be cells C2 and C3 and starts with some random number 0.1 and 0.2 in millions; then Steve types in all constraints in inequalities; finally, Steve inputs the objective function, as in Figure 9.

After Steve finishes all the above steps, he can use Excel Solver to find the optimal solution to the model in Equation (14). As shown in Figure 10, Steve starts Excel Solver and inputs all parameters, including the objective function in G2; variables C2 and C3; and other constraints in D6, D7, D8, and D9. Users can finish it by clicking DATA => SOLVER to start Excel Solver and then input “Set Objective,” “By Changing Variable Cells,” and “Subject to the Constraints.” Finally, click “Solve” to find the optimal solution.

**FIGURE 10
EXCEL SOLVER**



**FIGURE 11
OPTIMAL SOLUTIONS**



As the results show in Figure 11, the SuperLive contribution margin is maximized at $n_1 = 559,100$ and $n_2 = 1,881,800$ with the maximum value of \$16,821,323 USD. Steve can then finish his report and make recommendations to the CEO about SuperLive business operations, especially how to effectively use resources. Table 5 includes all the information regarding the maximum contribution margin and related operating plans.

Steve also might conclude that SuperLive just manufactures 56% of capacity of pressure cookers and 60% of capacity of 5 QT bowls. For any remaining idle capacities in production, SuperLive may want to consider how to effectively manage its fixed costs and find a way to deal with idle capacities. Steve has demonstrated how to use Excel toolboxes to study sophisticated business models, especially dealing with nonlinear models, including polynomial and power functions. Although the foundation is

advanced mathematics, Steve can skip all these challenging quantitative steps and is still able to find optimal solutions to the objective functions. In fact, Excel Solver uses the method of Lagrange multipliers and gradient methods to search for optimal solutions.

SUMMARY AND CONCLUSIONS

Business operations are becoming increasingly sophisticated; hence, business professionals need to be more open to all possible quantitative models, linear or nonlinear. Big data now plays an important role in the business world, and, with the overwhelming collection of business data, applying computer technology can facilitate using these collected data to make effective business decisions and to effectively and efficiently manage business operations. This paper tries to broaden the background of cost accountants so that they are able to address important business issues. This paper studies related examples and aims at introducing nonlinear constrained models in the scope of regression analysis and optimization problems.

Price and sales volume in units, learning curves, variable costs per unit and sales volume in units, and multiple product sales—all these topics are often seen in many business operations. However, nonlinear models can make these topics difficult to study and be frustrating to accountants or management. SuperLive is a typical company that has to deal with all these issues. Advanced quantitative models definitely can be useful to data analysis and optimal decision-making. The example also considers nonlinear constraints in resources, which is also typical in the business world, such as raw materials constraints, director labor constraints, and capital constraints. Therefore, accountants need to acquire skills to deal with constrained optimization problems. Excel Regression and Solver toolboxes demonstrate the power and convenience in studying sophisticated data sets, modeling business operations, and finding optimal solutions, such as in the case of SuperLive.

In summary, this paper demonstrates that accounting students and business professionals can learn how to use computer technology to study sophisticated business operations. When facing sophisticated data sets and patterns, researchers should not choose to oversimplify the model or data patterns; instead, they need to find appropriate computer technology tools to study these issues for the purposes of making accurate recommendations or predictions for business operations. In order to better prepare business professionals to deal with these important issues, as discussed in this paper, in today's business world, advanced modeling topics particularly in the field of cost accounting should be incorporated into accounting education in the undergraduate and graduate levels. This paper can be used as a case study or project in an accounting or business course.

REFERENCES

- Bertsekas, D. P. (1996). *Constrained optimization and Lagrange multiplier methods*. Athena Scientific.
- Chvatal, V. (1983). *Linear programming*. W H Freeman & Co (Sd).
- Draper, N. R. & Smith, H. (1998). *Applied regression analysis*, 3rd ed. Wiley.
- Drury, C. (2007). *Management and cost accounting*. Cengage Learning Business Press.
- Horngren, C. T., Foster, G., & Datar, S. M. (2015). *Cost accounting*, 15th ed. Pearson.
- Landsburg, S. (2013). *Price theory and applications*. South-Western College Pub.
- Nocedal, J. & Wright, S. (2006). *Numerical optimization*. Springer.
- Stewart J. (2013). *Multivariable calculus*, 7th ed. Cengage Learning.
- Togo, D. F. (2008). Amaya Company: financial considerations for product-mix LP models. *Journal of Business Case Studies*, 4(11).
- Wade, W. R. (2010). *An introduction to analysis*, 4th ed., Upper Saddle River, NJ: Pearson Prentice Hall.
- Warren Jr., J. D., Moffitt, K. C., & Byrnes, P. (2015). How big data will change accounting. *Accounting Horizons*, 29(2), 397–407.
- Wu, D, Li, J. (2016). *Applications of nonlinear models in contribution margin and profit analysis* (submitted manuscript).