An Econometric Approach to Optimizing Student Enrollment

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The concept of optimal enrollments has been studied extensively in enrollment management settings; however, most studies approach the problem qualitatively. Due to the complexity of the problem, a qualitative decision-making process can be overwhelming. An alternative is to use a quantitative approach. This paper will suggest a way of modeling the problem of optimal enrollments using mathematical optimization techniques.

INTRODUCTION

The concept of optimal enrollments has been studied extensively in enrollment management settings; however, most studies approach the problem qualitatively. Due to the complexity of the problem, a qualitative decision-making process can be overwhelming. An alternative is to use a quantitative approach. This paper will suggest a way of modeling the problem of optimal enrollments using mathematical optimization techniques. This approach has the benefit of being able to handle many variables and constraints at once and to uncover patterns that may be difficult to discern using qualitative approaches.

The model assumes that a decision must be made each year as to how many students of a given type to admit. Students may be classified according to academic qualifications, program of study, whether or not they would live on campus, whether they come from a primary or secondary market, or any other characteristic that is considered when making admissions decisions.

The problem formulation also assumes that students of different types act differently with respect to the institution; for example, if admitted, different types of students will enroll with different probabilities. Even among students who do enroll when admitted, different student types will progress at various rates through the system; for example, those students with stronger academic qualifications may have a higher chance of persisting and graduating. The problem with trying to mathematically describe such a system is that it is riddled with uncertainty. If a student is admitted, he may or may not enroll. If he enrolls for one year, he may or may not continue to his second year, and so on. When a student is admitted, it is not known whether or not that student will be successful; however, we may be able to estimate the student's chances of being successful based on the expected behavior of students of the same type.

This paper introduces a mathematical model that allows us to represent the uncertainty associated with the admittance, enrollment, and retention of students in an institution with common random variables. The model will also allow us to "optimize" our admissions decisions, while satisfying the constraints imposed. While it is unlikely that the model will be used directly by institutions to dictate admissions decisions, it may provide useful insight that could help admissions personnel make well-informed decisions and understand the implications of those decisions.

Description of Analysis

The model assumes that we can predict the behavior of the students who have applied based on the behavior of similar students who are or were already at the institution. That is, we need to be able to predict the future based on the past. In order to do this, we must somehow classify students into groups or types in order to track their behavior. To that end, we assume that students may be classified into types based on criteria that are used in admissions decisions. These criteria might include high school grade point average, standardized test scores, and whether or not they will live on campus or pay in-state or out-of-state tuition. The criteria used will vary among institutions.

Depending on the number of criteria used and the number of levels of classification within each (e.g. high GPA, medium GPA, or low GPA) there may be hundreds of different student types to consider, some of which may have only a few students. It is obviously much too complex to deal with hundreds of types; therefore, an effort can be made to reduce the number of student types by combining groups who act similarly with respect to the institution. By this we mean that the values of the yield, graduation and retention rates for the groups to be combined are not statistically different. If this is the case, then the combined group should act similarly to its member groups. If we do not combine groups who do not act similarly, then in the end we should be left with a small number of groups who are differentiated by their yield, retention, and graduation rates. This analysis can be done using Analysis of Variance (ANOVA) and should preferably be based on several cohorts to give as large a data set as possible.

Once we have reduced the student body to a small number of groups based on all cohort data available, we need to test the assumption that the behavior of these groups does not vary statistically from year to year. If this is the case, then it will be valid to predict the behavior of students of a given type who we would like to admit based on the behavior of past students of the same type. Therefore, for each student group or type, we can compare the yield, retention and graduation rates over several years to see if there are statistical differences over time. These tests (one for each student type and each rate) can be done by using a Chi Square test for a difference in proportions. If these tests hold, then the use of the model is validated.

Analysis of Institutional Data

The analysis described above was done on data for students enrolled in moderately sized mid western comprehensive university in the 1999-2005 cohorts. The criteria used to classify students were high school GPA, SAT (or ACT) scores, in-state vs. out-of-state, market (or yield level), and whether or not the student would live on campus. The last attribute could not be measured directly, but another variable representing whether or not the student lived within 50 miles of campus was used as a proxy. This is because the institution under analysis has a practice that students living further than 50 miles away must live in the residence halls.

When the initial classifications were done, there were 480 different student types present in the data. Groups were then combined using ANOVA in the following manner. The yield, retention (1-year through 7-year) and graduation rates (3-year through 7-year) of types that differed on only one variable, while all others remained constant, were compared. If there were no significant differences between the groups, then it was determined that that variable had no effect and the groups were combined. This analysis resulted in 25 distinct student types.

The 25 groups were then tested to determine whether the yield, retention and graduation rates varied from year to year. The results showed that, with the exception of a few outliers, that the rates remained very stable over the past several years. Thus, it was determined that the assumptions of the model held.

CONSTRAINTS INVOLVED IN ADMISSIONS DECISIONS

Since we found several student groups whose members act similarly to each other, we no longer need to look at each student individually in the admissions decision, but can now consider the number of students of each type to admit. These decisions must be made subject to certain constraints. There are two types of constraints that must be considered. Some of the constraints apply to the number of students we admit; for example, we cannot admit more students than we have applications. The others will be on the number of students who enroll, which is a random quantity based on the number of students we admit. We discuss each type of constraints below.

Constraints on the Decision Variables

Because we now only want to decide on the number of students of each type to admit, we will call these variables our "decision variables". There are several types of constraints on the decision variables that must be considered when admissions decisions are made. Some are real; for example, that we cannot admit more students that we have applications. Others are self-imposed; for instance, we may not want to admit too many remedial students who will require more resources. We call the latter constraints "capacity" constraints because they assume that the institution only has a certain capacity for certain types of students.

If we let x_i be the number of students of type i who we decide to admit, and let A_i be the number of applications from students of type i that we have, then for each student type, we will have the constraint:

$$x_i \leq A_i \quad \text{for all student types } i$$

Similarly, suppose there are certain upper bounds B_i on the number of students of type i that we wish to admit for some (not necessarily all) student types. Then our capacity constraints will be:

$$x_i \leq B_i \quad \text{for some student types } i$$

Capacity Constraints on the Outcome Variables

At the time when we admit a group of students, we do not know how many will enroll, and of those who do, how many will continue to enroll. These quantities are uncertain, and we just have to "wait and see" how they turn out. For this reason, we refer to the number of students who enroll as "outcome" variables.

There are several types of constraints that we might want to enforce on these outcome variables. One possible type of constraint might be capacity constraints similar to those we had on our decision variables. For example, you may want to limit the size of your freshman class. If you admitted too many of some types of students, for example those with high yield rates, you may end up exceeding that limit. Similarly, you may have a freshman dorm that houses all on campus freshmen. If too many on-campus freshmen were to enroll, then you would have overflow in your dorm.

The problem with trying to impose constraints on these outcome variables is that they are random quantities. We do not know with 100% certainty how many freshmen will enroll; thus, we can never ensure with 100% certainty that our capacity constraints will be met. However, based on our assumption that future students will behave similarly to past students of the same type, we can ensure that we will make a decision that will result in our constraints being met with a high degree of probability, say 95%.

We will discuss how this is done later in the discussion. However, at this point we can express these capacity constraints in mathematical terms. To that end, let X_i be the random number of students of type i who enroll as freshmen, and let U be the upper bound on the size of the freshman class. Then, if we sum the X_i over all types i, it will give us the total number of freshmen who enroll. Thus, the following constraint insists that we meet this bound on the size of the freshman class with a high degree of probability, represented by α_1:

$$\text{Prob}(\sum X_i \leq U) \geq \alpha_1$$

Similarly, we can impose capacity constraints for individual student types. If we let U_i be the upper bound on the number of students of type i who will enroll, then we have:

$$\text{Prob}(X_i \leq U_i) \geq \alpha_2$$

where α_2 is a probability close to 1. Note that α_2 does not need to be the same as α_1. Note also that we could create similar constraints to impose bounds on subsets of student types as well.

Quality Constraints on the Outcome Variables

When a student fails to persist, it affects institutional measures of success such as retention and graduation rates. Therefore, when we admit a group of students, we would like to be reasonably sure that the yield, retention and graduation rates that will result for that cohort will be acceptably high. These rates are based on the students who enroll, which is a random quantity, so we can never ensure with 100% accuracy that they will be met. However, using the past behavior of students to predict future behavior of students of the same type, we can ensure that these constraints will be met with a high degree of probability.

For example, suppose that you wish to satisfy, with a high degree of probability, the following constraints involving the outcome variables representing your yield, retention and graduation rates:

- that your yield rate, represented by Y, is sufficiently high, say at least p_y (where $0 \leq p_y \leq 1$).
- that your first year retention rate, R^1, is sufficiently high, say at least p_r (where $0 \leq p_r \leq 1$).
- that your four year graduation rate, G^4, is sufficiently high, say p_g (where $0 \leq p_g \leq 1$).

Then, your constraints would be expressed as:

$$\text{Prob}(Y \geq p_y) \geq \alpha_3$$
$$\text{Prob}(R^1 \geq p_r) \geq \alpha_4$$
$$\text{Prob}(G^4 \geq p_g) \geq \alpha_5$$

where α_3, α_4, and α_5 are (possibly different) probabilities close to 1. Note that we could also impose restrictions on these rates for specific student types instead of on the entire freshman class.

While the constraints on the decision variables are straightforward because you simply limit the number admitted to a certain level, the constraints on the outcome variables are more complex because they involve random variables. We will see later in the discussion how the assumptions made in the model will allow us to easily deal with these random constraints.

THE OBJECTIVE FUNCTION

The portions of the model that we have discussed so far allow us to ensure with a reasonable degree of confidence that the constraints we specify will be met. Subject to these constraints, we would like to admit the "optimal" number of students of each type. So, what is "optimal", or in other words, what is our objective?

Measures of Optimality

There are several possibilities for what would make up an optimal freshman class, depending on the institution and its mission. Note that each of these measures of optimality will be random quantities based on the outcomes variables because we are measuring the optimality of those who enroll and we do not know if the students will enter the institution and how long they will stay enrolled. Some possible ways to measure the optimality or desirability of a class are as follows:

1. Quality of the freshman class, as measured by pre-entry attributes such as high school GPA and standardized test scores, or by behaviors after enrolling, such as retention and graduation rates.
2. Revenues generated by the incoming class over the course of their tenure at the institution, which may be important to institutions with limited financial resources. Note that revenues also incorporate to some extent quality aspects such as retention because we are looking at revenues generated over the course of several years, not just in the first year. Thus, students who are likely to be retained will be more desirable because they will generate revenues over several years.
3. A Combination of Quality and Revenues which would involve a weighted average of the two measures.
4. How well the freshman class conforms to some ideal class whose mix is defined by the institution.
5. How well the freshman class meets certain institutional goals, such as increasing the number of high quality students or increasing recruitment in new markets.

Calculate some utility to the distance of each type to its desired level. Then, a total utility score is computed so that solutions can be compared. The quality measure is very valuable, but to some extent is dealt with in the constraints where we ensure certain levels of retention and graduation rates. Thus, here we will discuss the revenue measure, which has the added benefit of having a simple form.

Objective Function Involving Revenues Generated

In this type of model, we wish to express the measure of optimality in a form that will allow us to compare different decisions and decide which is best. In this context, such an expression is called an objective function. Here, we wish to express our objective function in terms of the outcome variables X_i.

In order to express the revenues generated over the students entire tenure at the institution, we need to distinguish the number of students who enroll in their first year from the number who enroll in their second, third, fourth, etc. Thus, let $X_i(n)$ be the number of students of type i who enroll in their n^{th} year, for n = 1, 2, 3, … (Since X_i, which we defined as the number of students of type i enrolling in their first year, can also be written as $X_i(1)$, we will henceforth refer to it in the longer notation to emphasize its relation to the other random variables $X_i(n)$.) Also, let $r_i(n)$ be the amount of revenues generated by students of type i in year n. Note that the quantity $r_i(n)$ is just a number. It is varies by student type because different students may pay different amounts for tuition (in-state vs. out-of-state, on-campus vs. commuter, etc.). The value of $r_i(n)$ is also allowed to vary from year to year to account for tuition hikes and other changes in fees.

In year n, each student of type i will generate $r_i(n)$ dollars in revenue; therefore, the total amount of revenue generated by students of type i in year n is $r_i(n)\bullet X_i(n)$. If we sum over all values of n, we will get the total revenues generated by students of type i over their entire tenure at the institution. Lastly, if we sum this quantity over all values of i, we will get the total revenues generated by all students we admit, which we denote by r. Thus, our objective function is:

$$r = \sum \sum r_i(n)\bullet X_i(n)$$

In order to find the optimal freshman class, we will want to maximize this quantity. As was true with the constraints on the outcome variables, this quantity is complex because it involves random variables representing the number of students who enroll, and as such is itself a random variable. We will not be able to maximize this quantity directly because we do not know the outcome of the $X_i(n)$. For that reason, we will want to maximize a function of this quantity that we can know in advance. Although there are many possibilities, the simplest approach would be to maximize the expected value, or mean, of this random variable. Although the expected value is no longer a random variable, it is still a complex quantity. Later in the discussion, we will show how the assumptions made in the model allow us to effectively deal with this aspect of the model.

THE MODEL

If we put together the various components we have discussed, we have the following mathematical model for the admissions decision:

$$\text{Maximize } E\left[\sum_i \sum_n r_i(n)\, X_i(n)\right]$$

Subject to:

$x_i \le A_i$ for all student types i

$x_i \le B_i$ for some student types i

$\text{Prob}(\sum X_i \le U) \ge \alpha_1$

$\text{Prob}(X_i \le U_i) \ge \alpha_2$

$\text{Prob}(Y \ge p_y) \ge \alpha_3$

$\text{Prob}(R^1 \ge p_r) \ge \alpha_4$

$\text{Prob}(G^4 \ge p_g) \ge \alpha_5$

where $x_i \ge 0$ for all student types i and
$0 \le \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, p_y, p_r, p_g \le 1$

The above problem, which consists of maximizing a quantity subject to constraints containing probabilities is a mathematical construct called a stochastic program. Although there are methods for solving such problems, they tend to be challenging problems to solve because of the uncertainty involved. The $X_i(n)$ are random quantities that depend on the number of students of that type who were admitted. Furthermore, the yield retention and graduation rates will all be calculated based on these $X_i(n)$. Thus, in order to make sense of this complex model, we need to get a handle on these random variables represented by the $X_i(n)$. We will discuss in the next section how these quantities can be expressed in terms of common distributions in order to greatly simplify the problem.

Probability Distributions of the Outcome Variables $X_i(n)$

Recall that the random variables $X_i(n)$ represent the number of students of type i who enroll in their n^{th} year. We know that the probability distribution of the random variables of $X_i(1)$ are related to the number of students of type i who we admit, namely x_i. If we assume that each of these x_i students acts independently of the others, and that each student of type i who is admitted will enroll with probability y_i, then $X_i(1)$ will be have a Binomial distribution with $n = x_i$ and $p = y_i$.

Furthermore, the value of $X_i(2)$ turns out to be the number of students of the x_i admitted who enroll the first year and the second year (assuming continuous enrollment). Thus, students must enroll in their first year, which they do with probability y_i, and then also enroll in their second year, which they do with probability equal to the first year retention rate denoted by R_i^1. Therefore, $X_i(2)$ also has a Binomial distribution with parameters $n = x_i$ and $p = y_i R_i^1$.

In general, a student who is admitted enrolls in period n with probability $y_i R_i^n$ where R_i^n is the n^{th} year retention rate of students of type i. Assuming each student acts independently, we will have for all n = 1,2,3,…

$$X_i(n) \sim \text{Binomial } (n = x_i,\ p = y_i R_i^n).$$

Furthermore, if the group sizes are sufficiently large, then this Binomial distribution can be approximated by a normal distribution. Generally, if the procedure described earlier results in a relatively small number of large groups, then this approximation will be valid. In this event, we have the following approximate probability distribution for $X_i(n)$:

$$X_i(n) \approx \text{Normal } (\mu = x_i y_i R_i^n,\ \sigma^2 = x_i y_i R_i^n (1 - y_i R_i^n))$$

Since the distribution of the random variables follows a normal distribution, and independent normal random variables can be added, it turns out that the constraints in the model that involve probabilities can be replaced with equivalent constraints that have no probabilities in them. In addition, we will be able to explicitly express the expected value in the objective function. Thus, by exploiting the properties of the normal distribution, we can reduce the above stochastic program to a much more manageable problem. We now explain the procedure that allows us to do this.

EXPLOITING PROPERTIES OF THE NORMAL DISTRIBUTION TO SIMPLIFY THE MODEL

The Objective Function

Recall that the objective function in the model involves an expected value of a sum of random variables. In general, this may not be an easy quantity to describe; however, when the variables added are independent and have normal distributions, the sum of the random variables will also be normally distributed. This is the case we have here. Thus, using the linearity of expected value, we have:

$$E\left[\sum \sum r_i(n)\, X_i(n) \right] = \sum \sum r_i(n)\, E[X_i(n)]$$

As we discovered above, the mean or expected value of each $X_i(n)$ is $x_i y_i R_i^n$. Therefore:

$$E\left[\sum_i \sum_n r_i(n)\, X_i(n) \right] = \sum_i \sum_n r_i(n)\, E[X_i(n)] = \sum_i \sum_n r_i(n)\, x_i y_i R_i^n = \sum_i x_i y_i \sum_n r_i(n)\, R_i^n$$

Since $r_i(n)$ and R_i^n are just numeric values that are determined by the physical system we are modeling, their sum over all values of n is just a function of i; therefore, let us express this sum as c_i. Similarly, y_i is just a numeric value as well; thus, the expected value in the objective function reduces to a simple linear function of the decision variables x_i, namely:

$$E\left[\sum \sum r_i(n)\, X_i(n) \right] = \sum_i y_i c_i x_i$$

where c_i and y_i are constants. As a result, we have removed all probabilistic quantities from the objective function.

Capacity Constraints on the Outcome Variables $X_i(1)$

In the previous section, we were able to remove all probabilistic attributes of the objective function by exploiting the properties of the normal distributions that govern the random variables $X_i(n)$. Using this same idea, we will also be able to deal with the probabilistic constraints involving $X_i(1)$, namely:

$$\text{Prob}(\sum X_i(1) \leq U) \geq \alpha_1 \quad (1)$$
$$\text{Prob}(X_i(1) \leq U_i) \geq \alpha_2 \quad (2)$$

Let us first deal with constraint (2). We know from our earlier discussion that $X_i(1) \approx \text{Normal}(\mu = x_i y_i R_i^1, \ \sigma^2 = x_i y_i R_i^1 (1 - y_i R_i^1))$. Therefore, by transforming this normal distribution into a standard normal, we have that (2) is equivalent to:

$$\text{Prob}([X_i(1) - \mu] / \sigma \leq [U_i - \mu] / \sigma) \geq \alpha_2 \text{ or } \text{Prob}(Z \leq [U_i - \mu] / \sigma) \geq \alpha_2$$

where μ and σ are linear functions of the decision variables x_i. Since we now just have a standard normal random variable, we can use any normal table to find the value of $[U_i - \mu] / \sigma$ such that the cumulative probability will be at least α_2. For example, if α_2 is 0.95, then $[U_i - \mu] / \sigma \geq 1.96$. Therefore, the probabilistic constraint (2) is equivalent to the following non-probabilistic constraint:

$$[U_i - \mu] / \sigma \geq F^{-1}(\alpha_2)$$

where F^{-1} is the inverse standard normal distribution. Rearranging the above inequality, we have that:

$$U_i - \mu \geq \sigma F^{-1}(\alpha_2)$$

which is equivalent to:

$$[U_i - \mu]^2 \geq \sigma^2 [F^{-1}(\alpha_2)]^2$$

if $U_i \geq \mu = x_i y_i R_i^1$. Thus, we are left with the following two constraints that are equivalent to the original probabilistic constraint (2):

$$[U_i - x_i y_i R_i^1]^2 \geq (x_i y_i R_i^1 (1 - y_i R_i^1)) [F^{-1}(\alpha_2)]^2$$

$$U_i \geq x_i y_i R_i^1.$$

These constraints have no probabilistic elements, and the first is quadratic in the decision variables x_i, while the second is linear.

A similar method can be used for constraint (1) to reduce it to two non-probabilistic constraints in the decision variables. The only difference is that:

$$\sum X_i(1) \approx \sum \text{Normal}(\mu = x_i y_i R_i^n, \ \sigma^2 = x_i y_i R_i^n (1 - y_i R_i^n))$$
$$\approx \text{Normal}(\sum \mu = \sum x_i y_i R_i^n, \sum\sigma^2 = \sum x_i y_i R_i^n (1 - y_i R_i^n))$$

and so the parameters μ and σ used in the above method would take on different values.

Constraints on the Quality Measures Y, R^1 and G^4

The probabilistic constraints involving the yield, retention and graduation rates can also be replaced with non-probabilistic constraints using the properties of the normal distribution. Recall that the yield rate is the percentage of those admitted who enroll in their first year, so $Y = \sum X_i(1) / \sum x_i$. Therefore, $Y \geq p_y$ is equivalent to $\sum X_i(1) \geq p_y \sum x_i$ and the constraint:

$$\text{Prob}(Y \geq p_y) \geq \alpha_3$$

is equivalent to:

$$\text{Prob}(\sum X_i(1) \geq p_y \sum x_i) \geq \alpha_3.$$

Again, since $\sum X_i(1)$ has a normal distribution, we can transform it to a standard normal and use the method described above to find two equivalent constraints that are quadratic or linear functions of the decision variables x_i and have no probabilistic elements. These constraints are:

$$[p_y\sum x_i - \mu]^2 \leq \sigma^2 [F^{-1}(1-\alpha_3)]^2$$

$$p_y\sum x_i \geq \mu$$

where $\mu = \sum x_i y_i R_i^n$ and $\sigma^2 = \sum x_i y_i R_i^n (1 - y_i R_i^n)$.

The constraints involving R^1 and G^4 are a bit more complex. Note that R^1, the first year retention rate, is equal to the percentage of students who enrolled in their first year who also return for their second year. Thus $R^1 = \sum X_i(2) / \sum X_i(1)$. A similar function can be written for G^4. Thus, these two quantities are quotients of random variables. A procedure similar to the one described above can also be used to replace these constraints with non-probabilistic functions of the decision variables; however, we do not discuss this process here as it is a complex procedure involving bivariate normal distributions.

APPROACHES TO SOLVING THE SOLUTION

In the previous section, we showed that the stochastic program described above can be replaced with an equivalent problem in which we wish to maximize a linear function of the decision variables subject to constraints that are either linear or quadratic in those decision variables. This type of problem is called a nonlinear program, and since it has no probabilistic aspects, is much easier to solve than the original problem. There are established methods for solving such a problem in the field of Operations Research (Bazarra, Sherali, & Shetty, 1993).

The solution to this nonlinear program will give us the "optimal" solution to the problem. This optimal solution informs the decision-maker how many of each student type to admit (the values of the decision variables x_i). The solution is "optimal" in the sense that it maximizes expected revenues while maintaining acceptable retention and graduation rates and ensuring that any limits on the number of students are met with a high degree of probability.

The same approach can be used to discover optimal solutions for the alternative objective functions discussed earlier. The solutions of the nonlinear program in those cases will then be optimal in the sense of the objective function specified.

Once we have such a solution to the model, there are several possibilities for utilizing it. Chances are that admissions offices are not going to use the optimal solution to dictate their decisions; however, there may be other information that could be used to supplement the decisions made in more traditional ways. For example, an admissions office could use the decision to judge how close their practices are to optimal. While this may not mean an overhaul of their system, it may point out areas that could be targeted for improvement.

Although the solution of such a system has not yet been fully implemented, it is conjectured that other useful information that such optimal solutions may provide may include certain qualitative properties that such solutions exhibit. For example, there may be cases where it is always optimal to admit all students of a certain type that apply, which would certainly be information that could prove useful to admissions offices. Additionally, such an optimal solution may provide a sort of ordering among the student types, so that it is always better to admit a student of one type rather than a student of the other. Still other types of information may be obtained from optimal solutions, depending on the real system on which the model is based. The next step would be to implement such a model for a real system and determine which of these types of information is available.

EXTENSIONS OF THE MODEL

The model described assumes that we make an admissions decision once and are done with it, but in reality, this same decision is made year after year. In addition, the decision we make in one year may affect our decision in a subsequent year. For example, if too many of one student type enroll in one year, we may need to curb the number of those students who are admitted the next.

Furthermore, restrictions on certain student types may be more complex than we stated earlier. For example, it may be that in any given year there can be no more than a specified number of students, both new and returning, who will live on campus. In addition to the uncertainty created by the fact that an admitted student may not enroll, there is uncertainty as to whether students who lived on campus last year will enroll and live on campus again this year. Thus, the decisions we made in the last several years affect the decision we will make this year.

This type of problem is a generalization of the stochastic program that was stated earlier. This more general problem involves maximizing the expected value of a random quantity based on random constraints, such that several decisions are made over many periods and the decisions we make in one year may influence our decision in subsequent years. This type of mathematical construct is called a stochastic dynamic program.

Solving a stochastic dynamic program is generally extremely difficult. This is because at each step, we need to look into the future and determine what the possible outcomes might be, and then ensure that we are doing the best we can do today, no matter which of those outcomes occurs. For more information on stochastic dynamic programming (Ross, 1983).

In this dynamic problem, or multi-stage problem as it is sometimes called, we may still replace the probabilistic constraints with non-probabilistic ones as discussed earlier; however, there is no way to totally remove the probabilistic component from the dynamic model. This is because of the constraint that states that we cannot admit more students of a given type than we have applications from students of that type. While we know this year how many applications we received this year, we do not know how many we will receive in future years, and we need this information to make our decision today. Thus, the number of applications in future years introduces a random aspect that cannot be removed from the model.

One approach to solving this dynamic model with a random application process is to try to describe the probability distribution of the application process. If this is successful, we may be able to find the optimal solution of such a model. If this approach is not successful, heuristics may be employed to approximate the optimal solution.

IMPLICATIONS OF THE MODEL

Eventual goals of the model include the development of a tool that allows enrollment managers to calculate admissions levels for each student type that will best accomplish institutional goals. The tool would also allow users to evaluate how close current admissions policies come to those optimal levels and to identify areas for improvement. Additionally, such a tool would inform admissions personnel of any qualitative properties of optimal solutions that might assist in their admissions decisions. The tool would also have the capability to allow the enrollment manager to perform “what if” analyses that are able to quickly predict how a change in policy or recruitment will affect revenues, institutional quality and yield, retention and graduation rates.

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