Eliciting Student Thinking in Support of Adaptive Expertise in Teacher Candidates

Diana Sherman
Saint Anselm College

Having skilled mathematics instruction at the elementary level is entirely dependent on teacher preparation. Understanding how teachers develop the adaptive expertise necessary to skillfully instruct elementary mathematics students is important for teacher education programs. Qualitative data analyses were applied to 31 elementary teacher candidates' teaching. Split into two cohorts, one cohort experienced sustained instruction in developing their practice of eliciting student thinking. Findings suggest that teachers do not go from lecturing to skilled eliciting. Instead, there is a developmental process of moving from making space for students' voices in the classroom to using student contributions to focus on mathematical content.

Keywords: adaptive expertise, preservice teachers, eliciting student thinking, mathematics education

INTRODUCTION

As students in the United States fall farther and farther behind their international peers in mathematics and science achievement (Ginsburg et al., 2009), teachers, parents, policy makers and other stakeholders in education have increasing cause for concern. Low student achievement implicates poor teaching and teachers. Poor teaching results from a lack of content knowledge, the assumption that talking at students is the same as teaching, and that listening is synonymous with learning (Bransford, Brown, & Cocking, 2000). As it turns out, standing at the white board and repeating algorithms does not help children understand mathematics deeply and conceptually. Poor teaching is further complicated by the problem of math anxiety. For elementary teachers in particular, anxiety around learning and doing mathematics is found to be a commonplace phenomenon (Bursal & Pagnozas, 2006; Gresham, 2007). As one can imagine, such anxieties impact teachers' abilities to plan and carry out successful mathematics lessons.

We need good math teachers who are capable of pairing content knowledge with conceptual knowledge, teachers who know how people learn and can access the prior knowledge and the misconceptions of their learners (Ma, 2010). In order for a teacher to truly identify what students know, they must elicit students' thinking. Lobato, Clarke & Ellis (2005) explain that "eliciting" student thinking occurs when the teacher's actions serve as a way to draw out students' "images, ideas, strategies, conjectures, conceptions, and ways of viewing mathematical situations" (p. 111). Teachers then use this knowledge of students' thinking to adjust content and methods during teaching to meet the needs of diverse learners. The work of adjusting one's own knowledge and practice to meet the needs of students, contexts, and content is known as adaptive expertise (Baroody & Dowker, 2013). Research findings provide insight on how to impart adaptive expertise on teacher candidates and asses this capacity in their teaching practice (e.g., Anthony, Hunter & Hunter, 2015; Crawford et al., 2005; Soslau, 2012). Yet little research focuses on how adaptive expertise might begin to be developed in preservice teachers.

This study responds to a need to better understand whether and how preservice teachers may develop the practice of adaptive expertise. Although adaptive expertise is most often tied to in-service and expert teacher practice (e.g., Hayden, Rundell, & Smyntek-Gworek, 2013), aspects of this practice, namely knowledge of student thinking as displayed by the practice of eliciting student thinking, are fertile ground for this research. In light of the need for high quality elementary mathematics teachers, this study examines how eliciting student thinking may be learned by preservice teachers and asserts that the enactment of such a practice allows novice teachers to develop adaptive expertise. This study addresses the following overarching research question and sub-questions: *What evidence of eliciting can be found in teacher candidates' teaching?*

- How does exposure to the practice of eliciting student thinking through course session and materials impact candidate's enactment of eliciting student thinking during teaching?
- What is the quality of candidate's eliciting during teaching?

Current research from the learning sciences describes how people learn (e.g., Donovan & Bransford, 2005; National Research Council, 2000). The study presented in this paper connects the work of adaptive expertise and how people learn to the specific teaching practice of eliciting student thinking. Given the lack of research on the developmental progressions of specific high-leverage (or core) practices that contribute to teaching expertise and student learning (e.g., Ball, et al., 2009; Grossman, 2018), this close look at pre-service teachers' eliciting practice describes and informs the early development of one such high-leverage practice in the larger progression of a teacher in developing adaptive expertise. After a review of the literature, the context and methods for this study are described. The findings are presented by first comparing the eliciting practice of two cohorts of teacher candidates and then by describing the treatment cohort's learning to eliciting student thinking as it supports development toward adaptive expertise. The discussion connects the study's findings back to the current research landscape of teacher education and mathematics teacher preparation and offers implications for the field.

REVIEW OF LITERATURE

This research draws three areas of literature to conceptualize the work of learning the practice of eliciting student thinking in support of developing adaptive expertise. First, theory on how people learn, specifically the conditions that facilitate learning in classrooms, are described. Then, characterizations of high-quality teaching and the practices that support skilled teaching are presented. Last, the underlying assumption that teaching can be learned is theorized.

How People Learn in Classrooms: The Need for a Skilled Teacher

Learning is not just a transfer of information. We often hear that people learn through experience (Dewey, 1923), rather than through solely passive or receptive modes (e.g., listening or reading). In fact, research on learning explains that learners take in new information in more "effortless" ways when they can connect it to prior knowledge (Bransford et al., 2005) and integrate what is learned across subject matter, and over time (Peterson, Clark, & Dickson, 1990). Activating prior knowledge and experiences allows humans to organize and integrate new information into existing conceptions (Bruner, 1960; Piaget, 1964). Further, research on learning and human development tells us that learning is social (Vygotsky, 1978). The brain is not fixed, but rather it, "develops as a function of the social activities in which people are engaged" (Bransford et al., 2005, p. 65). Thus, classrooms serve as a kind of social laboratory where ideas are shared, developed, and defined. Students must have opportunities to voice their ideas and use the classroom airtime (or talk time) to explain and reason through content. In order for such a classroom space to exist, it must be developed, lead, and maintained by a knowledgeable teacher (Cobb, Yackel & Wood, 1992). Therefore, a knowledgeable, or skilled, teacher must have an understanding of both the students, as well as the content, they teach.

Skilled teachers must not only understand the complexities of students' experiences (Boaler, 1998), including their background and life outside of schooling (Hollins, 2011), but also make sense of the differing perspectives and experiences learners bring to the classroom in light of the content being taught. Teachers must then determine how both of these conditions, learner contexts and curricular content, will manifest themselves in the social interactions of the classroom (Peterson, Clark, & Dickson, 1990), and decide how to structure classroom activities in response to these conditions. In other words, rather than seeing learning as a routine of transferring information from teacher to student, teachers must develop adaptive expertise (Bransford et al., 2005).

Connecting Adaptive Expertise to Eliciting Student Thinking

A characteristic of high-quality teaching, adaptive expertise is the ability to flexibly apply knowledge from a known context to a new context, through recognition of contextual similarities (Verschaffel, Torbeyns & Van Dooren, 2009). One central context of teaching is the students in the classroom (Hollins, 2011). In order for teachers to act as adaptive experts, they must have knowledge of their students and their students' thinking (e.g., Fanke & Kazemi, 2001; Nuthall, 1997). This work often manifests itself in teachers' practice as asking question to elicit student thinking (NCTM, 2014; TeachingWorks, 2020). Termed "high leverage," eliciting student thinking requires teachers to focus on student ideas, and flexibly respond to students' verbal and written contributions in light of the learning goals and context. High-leverage practices, in the field of education, describe fundamental actions teachers take on a daily basis that comprise the work of teaching. TeachingWorks.org explains that these practices are "high-leverage" not only because "they matter to student learning but because they are the basis for advancing skills in teaching" (TeachingWorks, 2020). When enacted consistently and competently, these practices result in student achievement. The high-leverage practice of eliciting student thinking allows teachers to *adapt* their teaching practice based on the context of the classroom; namely students and content. Eliciting practice also provides a basis for knowledge about the intersection of students and content. How students understand, or misunderstand, content is of importance to teachers because the identification of student misconceptions, specifically in mathematics and science classrooms, often form the basis for teachers' elicitations of student thinking (Borasi 1994; Fouche′, 2015; Kazemi and Stipek 2001; Thompson & Logue, 2006). Further, understand of student thinking, both procedural and conceptual, supports teachers' application of this knowledge toward student achievement (Sherman, 2016). Eliciting student thinking, where the teacher draws out students' verbal and written contributions to highlight thinking and understanding, can be done in two specific ways: posing tasks and asking questions

Connecting Knowledge of Student With Knowledge of Content

When enacted skillfully, the practice of eliciting student thinking allows teachers to gain information about student experiences while simultaneously maintaining the centrality of the content being taught. Part of the work of skilful eliciting requires teachers to pose tasks and ask questions that are cognitively meaningful for students (Boaler, & Brodie, 2004) and conceptually rigorous (Kazemi, & Stipek, 2009). Tasks and questions that require students to engage with content and reasoning are especially useful for both teachers and students. For students, a question about their method or reasoning requires greater cognitive demand, and therefore deeper, more critical, thinking. For teachers, the answers that students provide to these types of questions, both gives the teacher important information about learners, and supports their development of adaptive expertise in teaching. This work requires skilled eliciting on the part of the teacher.

One feature of skilled elicitations is posing tasks and asking questions that provide space (e.g., Cazden, 1988; Chapin et al., 2009) or talk time for students to contribute in meaningful or cognitively rigorous ways (Bransford et al., 2005). Asking a question that can be answered with "yes" or "no" response does not require a student to think in complex ways, especially when compared with a question that is open-ended and probes for a student to explain their meaning or method of solution. Instead of imposing their own ideas or understanding on the students by telling them what to think and simply transferring knowledge, skilled teachers must ask questions in order to understand what the students are thinking, and use the information gained in connecting to curricular content to plan and enact student-centered instruction (e.g., Rogers, 1979; Rogers, Lyon & Tausch, 2013).

A second feature of skilled eliciting is the ability to infuse curricular content into tasks and questions posed during instruction. The primary way teachers do this work in by including academic language (Cadwell & Leslie, 2012) in the elicitations they pose. Multiple teacher performance assessments (e.g., PACT, edTPA and NH-TCAP) include academic language in their written prompts and rubrics for assessment. The New Hampshire Teacher Candidate Assessment of Performance, or NH-TCAP, defines academic language as "language needed by student to understand and communicate in the academic disciplines. Academic language includes such things as specialized vocabulary… and other language-related activities typical of classrooms" (New Hampshire Institutes of Higher Education Network, 2013, p. 25). By including academic language in oral directions, prompts, and questions, teachers support student comprehension and use of discipline-specific language. Eliciting a student's method or reasoning occurs when a student is asked to further elaborate and describes (1) the method, or procedure, they use to solve a task, and/or (2) the reason(s) for their answer or for their choice in applying a particular method of solution to a given problem (Sleep & Boerst, 2012).

Using student thinking together with knowledge of content and practice (Ball, 1988; Ball, Thames, & Phelps, 2008) is one way teachers demonstrate adaptive expertise. Developing adaptive expertise is not a simple task (Darling-Hammond & Bransford 2007; Baroody & Dowker, 2013). The knowledge and skill required for such teaching expertise are not innate, but learned (Ball & Forzani, 2009). In fact, teacher preparation programs operate from the assumption that teaching can be taught, and relies on a set of common teaching techniques that can be learned by teacher candidates through experiences with planning, instructing, and reflecting (Grossman, & McDonald, 2008; Hatch & Grossman, 2009; Lampert, 2009; Shaughnessy & Boerst, 2017; Sleep, Boerst, & Ball, 2007). In order for teachers to skillfully adapt their knowledge of content for student learning, they must first know what students know. Although there has been research on the implications for adaptive expertise, specific connections to mathematics teaching and learning to teach are less common. In particular, looking at methods courses as a site for the development of core practices that lead to the development of adaptive expertise, is understudied. Further, methods courses, in contrast to content courses, are often a site where preservice teachers learn to integrate knowledge of students with knowledge of content (e.g., Clift, & Brady, 2005; Quinn, 1997). This makes methods courses an ideal site to investigate how preservice teachers learn specific teaching practices, such as eliciting student thinking, in their development toward adaptive expertise.

METHODS

Given the purpose of this research is to better understand how the practice of eliciting student thinking supports preservice teachers' development of adaptive expertise in teaching mathematics, two cohorts of preservice elementary mathematics teachers are compared. This comparison allows for analysis and evaluation of the impact a methods course (treatment), focused on eliciting and utilizing student thinking during instruction, had on preservice teachers' actual performance of eliciting student thinking in elementary mathematics classrooms.

Participants in this study include a convenience sample (Weiss, 1994) of 31 preservice teachers from two different cohorts of elementary education majors at a small, Catholic liberal arts college in the Northeast. Cohort 1 is comprised of 13 teacher candidates, including 12 female, 1 male, and 1 individual who identifies as Asian. Cohort 2 is comprised of 18 teacher candidates, including 16 female, 2 male, and 1 individual who identifies as Latina (see Table 1). Although these cohorts are overwhelmingly white and female, their gender and racial distribution are typical of the teaching profession (Zumwalt & Craig, 2005). Both cohorts completed the statewide New Hampshire Teacher Candidate Assessment of Performance (NH TCAP) in their last year of college studies.

TABLE 1
PARTICIPANT INFORMATION

	Cohort 1	Cohort 2
Participants	13	18
Gender	12 Female; 1 Male	16 Female; 2 Male
Race/Ethnicity	12 White; 1 Asian	17 White; 1 Latina
Math Methods	Spring 2016	Spring 2017
TCAP	Fall 2016	Fall 2017
Graduation with Certification	May 2017	May 2018

Data Sources

The NH TCAP was adapted from the Performance Assessment for California Teachers (PACT) for a New Hampshire context (IHE Network, 2013; Reagan et al., 2016) and assesses preservice teachers' practice across six strands: (1) Contextualizing, (2) Planning and Preparing, (3) Instructing, (4) Assessing, (5) Reflecting, and (6) Academic Language. Strands 2-6 are aligned to 12 rubrics, with strand 1 woven into the prompts for all other strands. In the Instructing strand, preservice teachers are instructed to, "Provide one or two video clips of no more than fifteen minutes total." This section further directs preservice teachers to, select clip(s) that (a) "demonstrate how you engage students in understanding mathematical concepts and participating in mathematical discourse" and (b) "include interactions among you and your students and your responses to student comments, questions, and needs" (New Hampshire Institutes of Higher Education Network, 2015, p. 15). Knowing that eliciting student thinking is a practice that most often occurs during classroom discussions and small group work, these TCAP directions provide sufficient cues to preservice teachers to focus on student thinking as a means for "engaging students in understanding" and responding to student "comments, questions, and needs."

As described above, this study compares two cohorts of preservice elementary mathematics teachers in order to investigate their development of the practice of eliciting student thinking as a signal of developing adaptive expertise. Cohort 1 serves as a control group, as they completed their elementary mathematics methods course without any specific instruction about eliciting student thinking as a means to develop adaptive expertise. The second cohort serves as the treatment group. This group experienced sustained instruction in developing their practice of eliciting students thinking in two concrete ways. First, Cohort 2 was exposed to the practice of eliciting student thinking during their elementary mathematics methods course. This exposure included (a) course readings that both describe eliciting and its benefits for student learning (e.g., National Council of Teachers of Mathematics (NCTM), 2014; Stein & Smith, 2011) and provide examples of teachers engaging in the practice of eliciting student thinking (e.g., Bastable, & Russell, 1999; Carpenter, Franke, & Levi, 2003), (b) group viewing and analysis of video records of teachers engage in the practice of eliciting student thinking, (c) teaching rehearsals where preservice teachers planned for and enacted eliciting moves during their instruction, as well as (d) feedback given on lesson plans and assignments that focused preservice teachers' attention to the work of eliciting student thinking. Second, Cohort 2 was required to use a lesson planning template that specifies the identification of "academic language" and "student misconceptions." Student misconceptions often form the basis for teachers' elicitations of student thinking, and academic language use, specifically when eliciting student thinking, increases the quality of teacher elicitations. Not only was this lesson planning template required for all plans constructed during the elementary mathematics methods course, it was also required for all lesson plans submitted as part of the TCAP. These requirements allowed for consistency, and hopefully a greater likelihood of transference of skills, across these two experiences of mathematics teaching and learning for preservice teachers in Cohort 2.

For this study video records of teaching from the preservice teachers' *Instructing Strand* of the TCAP were transcribed and analysed. Video records were collected for both cohorts through an archival, password-protected software program called TaskStream. During data processing and preparation (Miles & Huberman, 2014), videos were transcribed and then all textual data were scrubbed of identifiers (e.g., names and locations) to protect the anonymity of both candidates and their elementary-aged students. All names presented in this research are pseudonyms

Analysis Procedures

Given the purpose and data sources of this study, qualitative analysis was used to create descriptive cases (Yin 2009). Case study analysis is particularly useful when investigating and describing an intervention or treatment (Yin, 2009). Each preservice teacher was viewed as an individual "case." In the first phase of data analysis and a set of a priori codes informed by past research on teacher eliciting (e.g., Boaler, 1998; Sleep & Boerst, 2012) were applied to a set of six cases, three from each cohort (Goetz & LeCompte, 1984). During this initial coding process two organizational categories (McMillian & Schumacher, 2001) emerged: (1) Procedural Elicitations and (2) Reasoning Elicitations. Aligned with Sleep & Boerst (2012), the majority of the coded excerpts within the *Procedural Elicitations* category included the teacher giving directions or leading student through a method or procedure. Coded excerpts within the *Reasoning Elicitations* category were more diverse (also aligned with Sleep & Boerst, 2012), and included a wider array of teaching moves including asking for agreement, linking mathematical ideas, asking for meaning (e.g., "Why?" and "How do you know that?") and revoicing central mathematical ideas as described by a student.

In the second phase of analysis the remaining 25 cases were examined using the same process as employed in phase one. In keeping with Yin's (2006) case study methods, and as a way to triangulate the data (Cresswell, 2002), prepositions from the literature and the TCAP document were reviewed. This review resulted in two questions about the data and the nature of elicitations: (1) Did an increased number of elicitations increase the amount of teacher or student talk within the 15-minute teaching clip? and (2) Were preservice teachers' using mathematical academic language, as prescribed by the TCAP in their elicitations? Upon asking these questions, all coded excerpts, in both the *Procedural* and *Reasoning* categories, were tagged as longer (more than three lines of text) or shorter (fewer than three lines of text), and a new code of "academic language" was applied. This coding allowed for purchase on the second research sub-question, "What is the quality of candidate's eliciting during teaching?"

In the third phase of analysis questions of validity were addressed. Although triangulation was performed in the second phase of data analysis, this third phase included a deeper dive into the literature on eliciting student thinking, adaptive expertise, and case study methodology (Creswell, 2002), as well as re-reading coded quotes from the original teaching transcripts and comparing each code within individual cases, and across cases (Miles & Huberman, 1994). From this work, more refined code definitions were achieved, and the revised codes were applied across all 31 cases (Anfara, 2002). Recognizing the limitations of cases from one preparation program, at one institution in one state, this study does not provide a representative understanding of how preservice teachers learn and enact the practice of eliciting student thinking towards the development of adaptive expertise. Similar to other qualitative case studies, the analysis provided here isn't generalizable, however this is not the goal (Stake, 2010; Yin, 2009). Instead, these cases provide new insight into the ways in which teacher preparation programs may emphasize the practice of eliciting student teaching in order to foster meaningful enactment of the practice. Further, the current field of teacher education has not yet conceptualized or described the developmental trajectory of the practice of eliciting student thinking, nor its' role in supporting novice teachers in their acquisition of adaptive expertise. These cases begin to investigate connections between central practices (e.g., eliciting student thinking) in the teaching discipline and adaptive expertise.

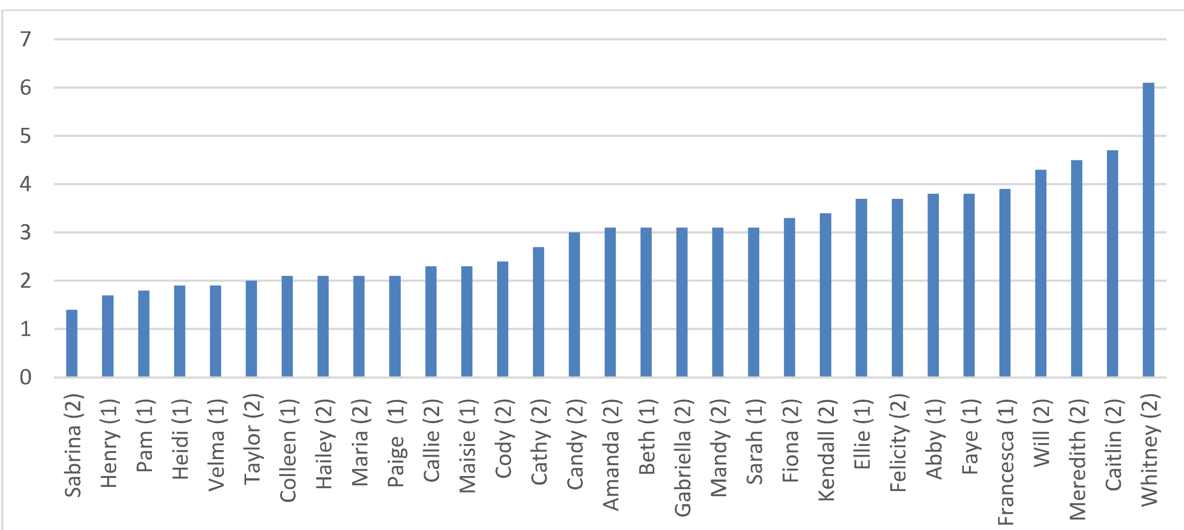
FINDINGS

Based on transcriptions of the teacher candidates' teaching, Cohort 2 (treatment group) not only posed more elicitations per minute, providing more opportunities for student talk than their Cohort 1 (control group) counterparts, but Cohort 2 relied less on lecturing than their Cohort 1 peers. Cohort 1 candidates more often used lecture or telling (e.g., longer stretches of teacher talk with no student contributions) during their teaching episodes. When teacher candidates talked for longer stretches of the class session, they dominated the airtime, leaving little room for student voices and ideas to be heard or developed. Although candidates in Cohort 2 generally elicited student thinking many more time per minute of instruction than their Cohort 1 peers, this increase in frequency of elicitations did not correspond to the quality of elicitations, where candidates were eliciting student for their method or reasoning, or including mathematics specific academic language in their elicitations. In terms of academic language use, candidates across both cohorts utilized mathematical terms, such as naming a concept (e.g., distributive property) or a mathematical representation (e.g., tens frame). Overall, the most common elicitations candidates posed that included academic language elicited student thinking about place value concepts, where the language of the "place" of a number (e.g., ones place, tens place) were most frequent.

Eliciting Student Thinking: Posing Tasks & Asking Questions

Teacher candidates from both Cohorts did indeed elicit student thinking during their instruction. Overall, the teacher candidates in Cohort 2 posed more tasks and asked more questions per minute of instruction when compared to their Cohort 1 peers. In fact, the average rate of elicitations per minute for Cohort 2 was 3.18, whereas the average rate of Cohort 1 was 2.51. To get a broader picture of this data, Figure 1 models the elicitations per minute for all 31 teacher candidates.

FIGURE 1
ELICITATION PER MINUTE



Each teacher candidate's cohort number (1 or 2) is indicated in parenthesis after their name on the horizontal axis of the graph. As shown in Figure 1, all candidates who had higher rates of elicitation (4.0 elicitations or more per minute) were in Cohort 2. Of the candidates who had lower rates of elicitations (2.0 elicitations or fewer per minute), four out of five of these candidates were in Cohort 1. This means that candidates in Cohort 2 had higher rates of elicitations per minute and were, therefore, making more space for student talk, or student *airtime,* during their instruction.

Airtime: Student vs. Teacher Talk

When thinking of a segment of instruction, one might consider a segment of a radio or television program, where "airtime" describes the time, or any part of the time, when the program is "on the air" for viewers to hear. Teaching segments, similarly, may be thought of as programs where teachers are "on the air" and students are tuning in to listen to the lesson. For this study, *airtime* refers to the instructional time when teachers and students are engaged in verbal exchanges. Airtime is not only when the teacher is speaking, but also when students are speaking, as research supports that students-to-student learning is an essential component of the social way in which people learn.

In particular, teacher candidates in Cohort 1 took up more airtime, talking for longer stretches, when they were describing a procedure or giving directions. For example, Heidi (Cohort 1), when teaching the traditional subtraction algorithm for the problem 347-58, used a long stretch of talk, posing no questions until the end a ten-sentence lecture.

Heidi: There you go, 17. We're taking a set of 10 from the tens place, so seven plus 10 is 17. We did a lot of big steps. Good job Liam! Liam looked ahead, seven minus eight, we couldn't do. Four minus five we can't do, so we borrowed from the hundreds place. This three [in the hundreds place], we borrowed 100, so it's almost like thinking 300 minus 100, we're getting 200. That's why we have the two [in the hundreds place], then we put the one [hundred] by the four [in the tens place] to make it 14 [tens]. We borrowed 100, so now it's 14 here [in the tens place]. Then we borrowed again from—because we can't do seven minus eight. What's 17 minus eight?

Of the 128 words that Heidi utters, only 4 were used to ask the question: "What's 17 minus eight?" By talking for such a lengthy stretch of time without posing a question, Heidi used the majority of the airtime, or talk time during this instructional segment, for teacher talk. In doing so, she eliminated airtime for student talk.

Similarly, Pam (Cohort 1) took up a large chunk of airtime, about four minutes, to give directions for setting up a problem on an open number line. Pam has her students use a numberline to represent a world problem that asks which rest area is closest to Exit 33. This means students must determine which number, 30 or 40, is closest to 33.

Pam: Yep. Do it right now. Everyone should be picking up their pencils. We're gonna write this in our math notebook. We're gonna write it just as a number line, okay? [teacher talk continues]...We wanna figure out, is it more efficient for us to drive to exit 40 to go to the rest stop or go back to exit 30 to use the rest stop? Jamie?

From this transcript excerpt it is apparent that Pam's uses a long stretch of talk where she does not pose any questions. In many cases, teacher candidates from Cohort 1, like Pam, appear to ask a question when they say, "Okay?" But given that an opportunity for students to verbally contribute is not provided, the question of "Okay?" becomes rhetorical, and is used to produce an effect or to make a statement, rather than to elicit students' contributions. It is not until the end of the direction-giving stretch that Pam specifically posed a question for a student (Jamie) to answer.

Conversely, Meredith (Cohort 2) elicited student contributions throughout the segment, where she takes the class through the mathematical procedure for adding 105 + 537.

Meredith: Who can read me the problem for #3? Sarah, what's the problem?
Student: One hundred five plus five hundred thirty-seven.
Meredith: I'm gonna draw our columns so it's a little bit easier. What's Step 1? Ibram, what's Step 1? Where do I start?
Student: In the ones [place], five plus seven.
Meredith: Where do I write it?
Student: Right at the side of where the first equal should be.

Meredith: Over on the side here, right? Five plus seven. What is five plus seven?
Student: Twelve.
Meredith: What's Step 2? Sofia, what's Step 2?
Student: Add 90 plus 30.
Meredith: Add 90 plus 30. Is anyone confused where Sofia got 90 plus 30? She didn't do nine plus three. She did 90 plus 30. Is anyone confused where she got those numbers? Where'd she get 'em? Someone tell me. Raise your hands. Where'd she get 'em, Asia?

Both Meredith (Cohort 2) and Pam (Cohort 1) spend the same amount of airtime (four minutes) in the examples presented. However, Meredith elicited her students' thinking 20 times in the four minutes of instruction, whereas Pam elicited students' thinking only six times in the same four-minute span.

Quality of Elicitations

Number of elicitations is not synonymous with the quality of elicitations. Although Pam poses fewer elicitations in her teaching segment than Meredith, these six elicitations focus on student's conceptual knowledge of the mathematics. For example, when Pam asks, "Why is it 30?" she is probing the student's correct answer, requiring the student to explain their reasoning for the answer 30. Meredith, who poses many more elicitations than Pam during her instruction, asks questions that are mainly procedural (e.g., "Where do I write it? Or "What's Step 2?"). In Meredith's case, more elicitations do not equate with higher-quality elicitations, or elicitations that require students to really think conceptually about mathematical content.

The quality of candidates' elicitations ranged from low to high quality elicitations across both cohorts. High quality elicitations fell into one of two categories: (1) elicitations that included academic language, or (2) elicitations for a students' method or reasoning. In terms of the first category of elicitation, candidates in both cohorts included academic language in their elicitations to highlight the academic, or mathematical, language that is necessary for students to demonstrate competency. Including academic language in an elicitation means that mathematics specific language, or language that has specific meaning when applied to doing mathematics, appeared in the elicitation. For example, when Francesca (Cohort 2) asked her students, *"Is this the part or the whole?"* she is eliciting her students understanding about the part-part-whole method for adding numbers. In this case, the numbers 4 and 6 are the parts and whole is 10. When used in everyday life, the words "part" and "whole" may take on an entirely different meaning. The word "part" could be used to describe a role in the school play. In Francesca's classroom, she used the word "part" to describe one of the two addends that make up the sum of 10. In terms of the second category, overall, candidates from Cohort 2 more often posed questions that elicited students' method or reasoning. For example, Maria (Cohort 2) elicits her student's method, by asking, *"Can you explain what you did there?"* Her use of the word "explain" signals the need for the student to describe their method for solution. In another example, Pam (Cohort 1) asks, *"Thirty? Why is it 30?* to elicit a student's reasoning. Pam repeats a student's answer and asks the student to elaborate on her reasoning for the answer 30.

Academic Language

Candidates from Cohort 2 (treatment) more frequently included academic language in their elicitations than Cohort 1. In fact, 77% (11/13) of candidates in Cohort 1 included academic language in their elicitations, whereas 100% (18/18), or all, candidates in Cohort 2 included academic language in their elicitations. Elicitations that included academic language took three different forms in candidate's teaching: (1) *mention*, (2) *focus*, and (3) *support* of academic language.

Elicitations That Mention Academic Language Use. The first form of academic language elicitations describes a specific mention of an academic language, such as concepts (e.g., distributive property; place value; equality) or representations (e.g., tens frame; area model; numberline), within the elicitation. Multiple candidates used academic language about place value concepts in their elicitations. Knowing the central role place value concepts plays in elementary mathematics content (Baroody, Laie & Mix, 2006; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000), these inclusions are important as they further the conceptual learning for students. Examples of place value academic language from the transcripts include specific talk about the place value (e.g., column or place) of a digit or number. For example, Meredith (Cohort 2) elicited, *"What am I gonna put in this tens column?"* to ask students which digit goes in the tens place of the answer position. Likewise, Cathy (Cohort 2) elicited the digit in the "tens place" from her students who are multiplying a pair of two-digit numbers. In these cases, the words "place" and "column" are used to focus students on the value of the digit that is worth ten times its value. Additionally, Kendall (Cohort 2), focused her kindergarten students on a central place value concept of making 10 when she asked, *"You started with two and how many more did you need to get 10?"* In this example, Kendall is supporting her students in verbalizing their physical work of moving cubes into groups of ten. Her inclusion of *"how many more"* signals the addition that students are physically acting out as they move eight cubes to join the starting two cubes to make a total of ten cubes. This is significant because Kendal is supporting very young students, kindergarteners, in talking about the mathematical manipulations they are performing with physical objects. In narrating ("you started with two") and naming ("to get to 10") the work, Kendall supported her students in both comprehending and using academic language. Use of academic language in elicitations for upper elementary grade students is also important, as it supports their comprehension and use of these math specific terms. Sarah (Cohort 1) used the math term "distributive property" in her elicitation of students, when she asked, *"Do you remember what the distributive property is?"* In this example, Sarah is eliciting students' prior knowledge from a previous lesson, using the academic language, "distributive property" to focus her fourth-grade students on central place value concept.

In her work with first graders, Caitlin (Cohort 2) used academic language to describe a mathematical representation called a tens-frame. She used the vocabulary word "tens-frames" to elicit her students' reasoning about the purpose of this mathematical representation: "What do tens-frames help us do?" In a similar way, Will (Cohort 2), supports his fifth-grade students in explaining the academic language "area model" by asking, "What is an area model?" Both Caitlin and Will are drawing their students' attention to important academic language. They each use mathematical vocabulary to communicate central mathematics ideas to students and to support students in using the same specialized vocabulary. In this way, these candidates are not just eliciting, but eliciting with a focus on larger mathematical ideas and concepts.

Elicitations That Focus on Academic Language. The second form of academic language elicitations describe elicitations that were used to draw students' attention to a specific example of academic language, by modeling its use in talk or writing. In the example below, Meredith (Cohort 2) chooses to focus her students on the academic language "partial sums." She has used an elicitation to follow up on the student's verbal contribution, eliciting the academic language for what he is describing as "them."

Sean: You have to add them all up.
Meredith: What are these called? What is "them?"
Sean: The partial sums.

Meredith's elicitation presses the student to use academic language, rather than a common noun (them) to be more accurate and specific in their mathematical talk. In this way, she is supporting her students' academic language used and comprehension. In a similar way, Hailey (Cohort 2) elicited the meaning of the academic language word "equal," when she asked, *"Can someone tell me what equal means?"* Again, the candidate is focused on a concept, equality, that is central to the learning of mathematics. In order to support conceptual understanding of mathematics in elementary students, teachers must use correct, mathematical, academic language and elicit students' thinking of this language. In doing so, candidates in this study demonstrate a higher-quality elicitation, because it included academic language and is focused on central mathematical content, necessary for current grade level and future grade level learning.

Elicitations That Support Academic Language. The third form of academic language elicitations describe elicitations that referenced resources that support academic language used for students. For example, Amanda (Cohort 2) begins instruction focusing students on "math vocab words" or academic language, saying, "*Today we have two new math vocab words. Do you remember how last week we put them right up there on the math vocab wall?* In this example, Amanda is referencing the word wall in the classroom that serves as a resource for students to remember, comprehend, and use math academic language. Other examples of this kind of academic language elicitation reference posters on classroom walls, charts, or other visual aids that are accessible to students within the classroom.

Eliciting Student Method or Reasoning

In the work of eliciting a student's method or reasoning, candidates across cohorts used both general and specific elicitations to uncover student thinking. In multiple cases, candidates employed a more general elicitation, asking, *"How do you know that?"* Although not explicitly specified, in these cases "that" refers to the answer the student produces or the procedure the student employed to solve the problem. For example, Francesca (Cohort 1) used *"How do you know that?"* as a general elicitation to get a student to describe her solution of 10 to the problem 5+5. Following Francesca's elicitation, *"How do you know that?"* the student could have described her procedure for solving 5+5, or she could have explained how she knew that 5+5=10. In either case, the student would need to reason about the mathematics, and explain her reasoning or the reasoning behind her method of solution.

Similarly, Mandy (Cohort 2) responded to a student's correct answer to a division problem by asking, *"How'd you know that?"* By directly asking about knowledge (using the word "know" in the elicitation), Francesca and Mandy accessed their students' conceptual knowledge of mathematics. In doing so, each candidate provided an opportunity for their students to both share their mathematical thinking with their classmates, while simultaneously proving an opportunity for the candidate to understand and assess the knowledge of the student who is speaking.

The majority of the elicitations that were coded as method or reasoning began with the question words "How" or "Why." As presented in previous examples, versions of, *"How do you know that,"* which includes the question word "How," were regularly used by candidates, to elicit a student's reasoning. Other examples of the question word "How" being used to elicit a student's method include Callie (Cohort 2), who used the question, *"How would we write that?"* to elicit her student's thinking about representing two-digit numbers. Similarly, Maria (Cohort 2), who use "How" to elicit her students' method for solving a multi-step word problem involving multiple costs, when she asked, *"How did you add all of that together?"*

In multiple cases, candidates in cohort 2 used the word "How" in the phrase "How many" to elicit students' procedure for solving a given math problem. For example, Cathy (Cohort 2) asked, *"How many do we put in the ones[place]?"* to elicit students' thinking of place value concepts when multiplying a pair of two-digit numbers. Similarly, Felicity (Cohort 2) used *"How many hops did Kailey take to get to 24?"* in her elicitation of a student's use of an open numberline to solve the subtraction equation 24 - 16. It is important to note that Felicity's elicitation is more complex, and of higher quality that Cathy's, because it asks students to attend to the thinking, or method, of their classmate. Instead of eliciting a student's *own* thinking about their *own* method or procedure, Felicity chose to highlight the method of a peer, and in doing so, she asked students to put their own method of solution aside and analyze Kailey's method. From the data, only a handful of candidates in Cohort 2 used this kind of high-quality elicitation during their teaching, and no candidates from Cohort 1 employed this kind of elicitation where one student's ideas are made focal for whole class analysis.

Across both cohorts, candidates used "why" questions for both general and specific elicitations. In fewer than 10 cases, candidates in Cohort 2 used a general elicitation followed by a more specific eliciting probe to get students to describe their method or reasoning. This eliciting work, like the example of Felicity, is evidence of higher-quality elicitations. When a teacher is able to string together multiple, meaningful elicitations, student thinking and the reasoning behind their thinking is highlighted. In doing so, teachers promoted the thinking behind the solution method not just the correctness, or completeness, of the method. In the case of Whitney (Cohort 2), she is able to elicit both the correct answer to a sequencing problem, as well as the reasoning behind the correct answer. When a student correctly responds to her initial elicitation, she probes the student's thinking further, eliciting the student's reasoning:

Whitney: What do we think the next number in our pattern is going to be? We went 10, 20, 30.
Student: Forty?

Whitney: Forty? Why do you say 40?

In this example, Whitney is not just eliciting for the correct answer, she is eliciting the student's reasoning to support him, and his classmates, in recognizing the important mathematical pattern of adding ten that is occurring in the sequence.

Interestingly, no examples where candidates probed an incorrect answer with an elicitation for reasoning were found in the data. This is likely due to the complex nature of responding to student's incorrect, or unexpected answers during mathematics instruction (Rougée, 2017). A large body of research details the challenges both novice and experienced teachers have when responding to incorrect answers (e.g., Jacobs, Lamb, & Philipp, 2010; Steuer, Rosentritt-Brunn, & Dresel, 2013). Yet evidence of higher-quality elicitations, including those made by Felicity and Whitney, support the notion that learning to elicit student thinking is not a consistent trajectory, and that learners develop the practice at different rates. In fact, different aspects of the practice including indicators of quality, may develop in staggered or inconsistent ways.

DISCUSSION

The research presented in this paper investigates the role the practice of eliciting student thinking plays in developing teacher expertise in novice teachers. This project impacts education research and teaching instruction in two main arenas. First, the research and findings from this set of cases, and this study, informs the landscape of teacher education and provides insight into improving the quality of elementary mathematics teacher preparation, and by extent K-6 student learning and achievement in mathematics. Second, the findings signal the need for a more complex description of the developmental trajectory of teacher candidates as they learn a mathematics teaching practice and develop adaptive expertise. Specific insight into how individual teaching practices support developing adaptive expertise is needed, and this research serves as the basis for such work.

Insight for Improving Teacher Education

Elicitation Quantity vs. Quality

One main finding from this research is that more frequent elicitations during instruction is not synonymous with the quality of such elicitations. This finding illuminates the "bumpy" nature of learning (Schraw, Olafson & Lunn Brownlee, 2017), especially, learning to teach. Teacher candidates in this study did not go from solely lecture to skilled eliciting. Some candidates, like Meredith (Cohort 2), infused more student airtime into their practice, with the quality of the opportunities for student talk being lower. Other candidates, like Pam (Cohort 1), continue to use long stretches of teacher talk, which *do not* allow for students to contribute their thinking, while incorporating higher quality elicitations that *do* allow students to share their thinking. Given these mixed results in consistency of eliciting practice, it becomes important for teacher educators to review both the quantity (airtime) and quality (focus on students' conceptual thinking) of teacher candidate elicitations.

The inclination to ask more question, or pose more elicitations, during instruction, does signal an orientation toward student thinking. A teacher cannot learn about student thinking unless it is elicited from students (verbally or in writing). An orientation toward student thinking, or a desire to know and use student ideas during instruction, positions teacher candidates well to develop adaptive expertise, where knowledge of students and related contexts is primary.

Developing Adaptive Expertise

Adaptive experts possess a specific knowledge blend of innovation and efficiency (Bransford et al., 2005). In teaching, adaptive experts must possess what Schulman (1987) terms "pedagogical content knowledge," or (PCK): knowledge of pedagogy (students and learning) and knowledge of content (subject matter and application). But unlike more generalized knowledge, or expertise in a given field, PCK includes understanding how content is learned, as in schools by young pupils. Knowing the likely misconceptions of students is not common knowledge for experts or for learners. Humans are complex as learners, and well-trained over twelve plus years of comprehensive schooling to best understand their *own* learning (Lortie, 1975). Yet, this work as a learner does not well prepare an individual to understand the learning of *others*, as is required for skilled teaching (Feiman-Nemser & Buchman, 1983). Therefore, to achieve both innovation and efficiency, some learning behaviors must be unlearned. Sole focus on one's own thinking is a primary example of a behavior that must be unlearned in order to enact adaptive expertise. In order to unlearn this sole focus, preservice teachers must engage in observing, interpreting, and responding to student thinking. This means the practice of eliciting student thinking could serve as an "on ramp" for the acquisition of adaptive expertise.

Learning to Respond to and Elicit Incorrect Student Answers During Instruction. Adaptive expertise also includes the ability to respond to new or unforeseen classroom events with flexibility. As a teacher develops adaptive expertise, so too develops their experience responding to the wide-array of student contributions. One of the most difficult challenges a teacher faces is responding to student misconceptions or errors. As mentioned in the findings section, it is no surprise that candidates failed to elicit students' incorrect responses to their initial elicitation. This work is challenging and requires expert knowledge of content and students. In responding to student's incorrect answers, teachers face what they perceive as criticizing students, when encouragement and affirmation are preferred. D. L. Ball (1997) explains that,

> Teachers want students to understand... Being a teacher is centrally about helping others learn, and feeling successful is wrapped up in accomplishing this. Teachers are disappointed when they confront their students' confusions, missing pieces, distorted understandings. They care about their students. After investing time and effort in a particular student, a teacher wants to hear right answers ... Teachers also want students to feel good about their own accomplishments and may shy away from pushing students for fear of being seen as critical. (p. 800-801)

As teacher candidates develop relationships with students and focus on understanding student thinking, care and concern for students simultaneously develops. As Ball writes, teachers' feelings of success and accomplishment are tied to student successes and student feelings of accomplishment. It makes sense that as teacher candidates develop relationships with students, they struggle to confront student errors, not wanting to erode the trust and social capital they have worked so hard to create. Eliciting student thinking is one practice teachers can use to inquire about student ideas, correct or incorrect (or somewhere in between) without presuming judgement. Using this practice as an information-seeing endeavor, situating students as bearers of knowledge whose thinking is valued at all levels of understanding and misunderstanding, supports teacher candidates in developing the knowledge base that is necessary for adaptive expertise.

Focused Work on Mathematics Teaching Practices in Methods Coursework

Given that the treatment group, Cohort 2, experienced focused instruction on the practice of eliciting student thinking, their higher levels of enactment of this practice is not surprising. Purposefully exposing teacher candidates to the practice of eliciting student thinking in gradual and sustained ways over the course of a four-month semester, allowed for Cohort 2 to develop skill in enacting the practice, as observed in the video records of their instruction. By reading about eliciting, viewing videos of experts engaged in eliciting student thinking, planning for eliciting during instruction, and rehearsing eliciting in small peer teaching groups, candidates built up their knowledge of this mathematics teaching practice. In purposefully engage students in receptive modalities (reading and viewing) to develop their comprehension of eliciting thinking first, and then in productive modalities (planning and rehearsing) second, candidates in Cohort 2 were supported developmentally. These findings support the need for, and utility of, a developmental approach to teaching mathematics teaching practices that we know are high-leverage.

The Need for a Developmental Trajectory for Learning High Leverage Practices

Other research in the field of education has described developmental approaches applied to learning to teach (e.g., Conway & Clark, 2003; Darling-Hammond, 1996; Hollins et al., 2004; John-Steiner, & Mahn, 1996). Other fields that prepare practitioners have also utilized developmental frameworks to describe and train candidates (e.g., Benner, 1984; Grossman et al, 2007; Jordan, 1989; Robertson, 2004; Rose, 1999). Benner (1984), a nurse educator, describes the stages of clinical competence using Dreyfus & Dreyfus' (1980) five levels of proficiency: (1) Novice, (2) Advanced beginner, (3) Competent, (4) Proficient, and (5) Expert. In this model, Brenner describes the movement from one level to the next, reflecting on two general aspects of skilled performance: (1) movement from reliance on abstract principles to the use of past, concrete experiences as paradigms, and (2) change in the perception and understanding of a situations, such that the situation is seen less as a complication of equally relevant parts and more as a complete whole in which only certain parts are relevant (Benner, 1984). Benner's inclusion of both the developmental levels, as well as descriptors of the quality of performance, nicely mirror the finding of this study. In thinking about how to categorize the kinds of elicitations candidates produced from this data, one might apply Benner's five levels of proficiency. For example, the field of teacher education could describe "Novice" practice, as candidates who include elicitations as part of their teaching practice. "Advanced Beginner" could be applied to candidates who include academic language in their elicitations.

Teaching high leverage practices using a developmental approach means providing candidates with (1) sustained exposure to the practice, increasing the cognitive demand over time, and (2) clear descriptions of the levels of development candidates will move through as they develop expertise. An initial conceptual framework for this developmental approach is provided in Figure 2.

FIGURE 2
CONCEPTUALIZING A DEVELOPMENTAL APPROACH FOR LEARNING TO ELICIT STUDENT THINKING

	Pre-service Teachers			In-Service Teachers	
	Novice	**Advanced Beginner**	**Competent**	**Proficient**	**Expert**
Observable in written lesson plans	scripted elicitations	elicitations for method and/or reasoning	• elicitations for method and reasoning • likely student errors/ misconceptions and elicitations to uncover and address these misconceptions		
Observable in teaching rehearsals	2 or more elicitations of any quality	elicitations that: • include academic language • focus on student method (procedural thinking) • focus on student reasoning (conceptual thinking)	elicitations that: • include academic language • focus on student method (procedural thinking) • focus on student reasoning (conceptual thinking) • respond to incorrect (or incomplete) student answers		

Figure 2 displays five levels of proficiency, using the same level descriptors as Benner (1984). Levels that are assigned to pre-service teachers (or teacher candidates) are Novice, Advanced Beginner and Competent. For the purposes of this model, year of preparation are not specified, but one might conclude that "Novice" and "Advance beginner" are stages for candidates engage in introductory or methods courses, whereas "Competent" could be used to describe candidates during student teaching. The descriptors for "Proficient" and "Expert" have not yet been populated, as this study focuses on pre-service teachers, and did not analyze or produce findings specific to in-service teachers. What this study, and this model do, is provide teacher educators with a description of the practice of eliciting student thinking as it is being learned by teacher candidates. In doing so it provides a road map, or guide, for (1) how teacher candidates may be taught the practice of eliciting student thinking, (2) in what order information may be presented to candidates, and (3) the qualities against which candidates' performance can be assessed. The developmental approach provided here is meant to be utilized and shared with researchers, teacher educators, in-service teachers who mentor early career teachers, as well as teacher candidates. Although this model and the research described in this paper focus on the practice of eliciting student thinking, it stands to reason, that other high leverage practices may be conceptualized in a similar way. Employing gradual, developmentally-focused exposure to a teaching practice as well as defining levels of proficiency as modeled in Figure 2, could be applied to the mathematics teaching practices identified by the National Council of Teachers of Mathematics (2014), such as "use and connect mathematical representations," or "facilitate meaningful mathematical discourse" (p. 10).

Conceptualizing preservice teacher learning is not a simple task, but one that must be undertaken by the field of teacher education, specifically mathematics teacher education, in order to insure high-quality, equitable mathematical classrooms and learning for all students. A focus on a practice, like eliciting student thinking, that highlights students and the assets they bring to the classroom is a good start, but there is more work to be done.

REFERENCES

Anfara, V.A., Jr., Brown, K.M., & Mangione, T.L. (2002). Qualitative analysis on stage: Making the research process more public. *Educational Researcher*, *31*(7), 28-38.

Anthony, G., Hunter, J., & Hunter, R. (2015). Prospective teachers' development of adaptive expertise. *Teaching and Teacher Education*, *49*, 108-117.

Ball, D.L. (1988). *Research on teaching mathematics: Making subject matter knowledge part of the equation*. National Center for Research on Teacher Education.

Ball, D.L. (1997). What do students know? facing challenges of distance, context, and desire in trying to hear children. In B. Biddle, T. Good, & I. Goodson (Eds.), *International handbook of teachers and teaching*, (Vol. 2, p. 769-818). Kluwer Press.

Ball, D.L., & Forzani, F.M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, *60(5*), 497-511.

Ball, D.L., Sleep, L., Boerst, T., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *Elementary School Journal*, *109*(5), 458-474.

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education*, *59*(5), 389-407.

Baroody, A.J., & Dowker, A. (Eds.). (2013). *The development of arithmetic concepts and skills: Constructive adaptive expertise*. Routledge.

Baroody, A.J., Lai, M.L., & Mix, K.S. (2006). The Development of Young Children's Early Number and Operation Sense and its Implications for Early Childhood Education. In B. Spodek & O. Saracho (Eds.), *Handbook of Research on the Education of Young Children* (pp. 187-222). Lawrence Erlbaum Associates.

Bastable, V., & Russell, S.J. (1999). *Number and Operations: Building a system of tens. Part 1*. Dale Seymour Publication.

Benner, P. (1984). *From novice to expert*. Menlo Park.

Bennett, C.A. (2010). "It's Hard Getting Kids to Talk about Math": Helping New Teachers Improve Mathematical Discourse. *Action in Teacher Education*, *32*(3), 79-89.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education*, *29*(1), 41-6.

Boaler, J., & Brodie, K. (2004). The importance, nature, and impact of teacher questions. *Proceedings of the 26th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 774-782).

Borasi, R. (1994). Capitalizing on errors as" springboards for inquiry": A teaching experiment. *Journal for Research in Mathematics Education*, pp. 166-208.

Bransford, J., Derry, S., Berliner, D., & Hammerness, K. (2005). Theories of learning and their roles in teaching. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 40-87). Wiley & Sons.

Bransford, J.D., Brown, A.L., & Cocking, R.R. (Eds.). (2000). *How people learn: Bridging research and practice*. National Academies Press.

Bruner, J.S. (1960). *The process of education: A searching discussion of school education opening new paths to learning and teaching*. Vintage Books.

Bursal, M., & Paznokas, L. (2006). Mathematics Anxiety and Preservice Elementary Teachers' Confidence to Teach Mathematics and Science. *School Science & Mathematics*, *106*(4),173-179.

Caldwell, J.S., & Leslie, L. (2012). *Intervention strategies to follow informal reading inventory assessment: So what do I do now?* (3rd ed.). Pearson.

Carpenter, T.P., Franke, M.L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Heinemann.

Cazden, C.B. (1988). *Classroom discourse: The language of teaching and learning*. Heinemann.

Chapin, S.H., O'Connor, C., O'Connor, M.C., & Anderson, N.C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions.

Clift, R.T., & Brady, P. (2005). Research on methods courses and field experiences. In M. Cochran-Smith & K.M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 309-424). Lawrence Erlbaum associate, Inc.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics education*, *23*(1), 2-33.

Conway, P.F., & Clark, C.M. (2003). The journey inward and outward: a re-examination of Fuller's concerns-based model of teacher development. *Teaching and Teacher Education*, *19*(5), 465-482.

Crawford, V.M., Schlager, M., Toyama, Y., Riel, M., & Vahey, P. (2005). *Characterizing adaptive expertise in science teaching*. Report on a laboratory study of teacher reasoning. Paper presented at the Annual Meeting of the American Educational Research Association, Montreal, Canada.

Creswell, J. (2002). *Education research: Planning, conducting, and evaluation quantitative and qualitative research.* Merrill Prentice Hall.

Darling-Hammond, L. (1996) The quite revolution: Rethinking teacher development. *Educational Leadership*, *53*(6), 4-10.

Darling-Hammond, L., & Bransford, J. (Eds.). (2007). *Preparing teachers for a changing world: What teachers should learn and be able to do*. John Wiley & Sons, Inc.

Dewey, J. (1923). *Democracy and education: An introduction to the philosophy of education*. Macmillan.

Donovan, S., & Bransford, J.D. (2005). *How Students Learn: History, Mathematics, and Science in the Classroom*. The National Academies Press.

Dreyfus, S.E., & Dreyfus, H.L. (1980). *A five-stage model of the mental activities involved in directed skill acquisition* (No. ORC-80-2). California Univ Berkeley Operations Research Center.

Feiman-Nemser, S., & Buchmann, M. (1983). *Pitfalls of Experience in Teacher Preparation*. Occasional Paper No. 65.

Fouche, J. (2015). Predicting Student Misconceptions in Science. *Educational Leadership*, *73*(1),60-65.

Franke, M.L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory Into Practice*, *40*(2), 102-109.

Ginsburg, A., Leinwand, S., Noell, J., & Pollock, E. (2009). Reassessing US international mathematics performance: New findings from the 2003 TIMSS and PISA. *Colección Digital Eudoxus*, (22).

Glaser, B., & Strauss, A. (1967). Grounded theory: The discovery of grounded theory. Sociology. *The Journal of The British Sociological Association*, *12*, 27-49.

Goetz, J.P., & LeCompte, M.D. (1984). *Ethnography and qualitative designs in ethnographic research*. Academic.

Gresham G. (2007). A Study of Mathematics Anxiety in Pre-Service Teachers. *Early Childhood Education Journal*, *35*, 181–188.

Grossman, P. (2018). *Teaching Core Practices in Teacher Education*. Harvard Education Press.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*(1), 184-205.
Grossman, P., Compton, C., Shahan, E., Ronfeldt, M., Igra, D., & Shaing, J. (2007). Preparing practitioners to respond to resistance: A cross-professional view. *Teachers and Teaching: Theory and Practice*, *13*(2), 109-123.
Hatch, T., & Grossman, P. (2009). Learning to look beyond the boundaries of representation. *Journal of Teacher Education*, *60*(1), 70-85.
Hayden, H.E., Rundell, T.D., & Smyntek-Gworek, S. (2013). Adaptive expertise: A view from the top and the ascent. *Teaching Education*, *24*(4), 395-414.
Herbal-Eisenmann, B.A., & Breyfogle, M.L. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School*, *10*(9), 484-489.
Hollins, E.R. (2011) Teacher Preparation for Quality Teaching. *Journal of Teacher Education*, *62*(4) 395-407.
Hollins, E.R., McIntyre, L.R., DeBose, C., Hollins, K.S., & Towner, A. (2004). Promoting a self-sustaining learning community: Investigating an internal model for teacher development. *International Journal of Qualitative Studies in Education*, *17*(2), 247-264.
Jacobs, V.R., Lamb, L.L., & Philipp, R.A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, pp. 169-202.
John-Steiner, V., & Mahn, H. (1996). Sociocultural approaches to learning and development: A Vygotskian framework. *Educational Psychologist*, *31*(3-4), 191-206.
Jordan, B. (1989). Cosmopolitical obstetrics: Some insights from the training of traditional midwives. *Social Science Medicine*, *28*, 925- 944.
Kazemi, E., & Stipek, D. (2009). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Journal of Education*, *189*(1-2), 123-137.
Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics* (Vol. 2101). National Research Council (Ed.). National Academy Press.
Lampert, M. (2009). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education*, *61*(1-2), 21-34.
Lobato, J., Clarke, D., & Ellis, A.B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, *36*(2*),*101-136.
Lortie, D. (1975). *Schoolteacher: A sociological study*. University of Chicago.
Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates.
McMillian, J.H., & Schumacher, S. (2001). *Research in education: A conceptual introduction*. New York, NY: Longman.
Merriam, S.B. (2009). *Qualitative Research: a guide to design and interpretation*. Jossey-Bass.
Miles, M.B., & Huberman, A.M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.
National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. NCTM.
National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.
National Research Council. (2000). *How people learn: Brain, mind, experience, and school* (Expanded edition). National Academies Press.
New Hampshire Institutes of Higher Education Network. (2013). New Hampshire Teacher Candidate Assessment of Performance (NH TCAP) Phase II. Concord, NH.
Nuthall, G. (1997). Understanding student thinking and learning in the classroom. In *International handbook of teachers and teaching* (pp. 681-768). Springer, Dordrecht.
Peterson, P.L., Clark, C.M., & Dickson, W.P. (1990). Educational psychology as a foundation in teacher education: Reforming an old notion. *Teachers College Record*, *91*(3), 322-346.
Piaget, J. (1964). Part I: Cognitive development in children: Piaget development and learning. *Journal of Research in Science Teaching*, *2*(3), 176-186.

Quinn, R.J. (1997). Effects of mathematics methods courses on the mathematical attitudes and content knowledge of preservice teachers. *Journal of Educational Research*, *91*(2), 108-114.

Robertson, C.L. (2004). Teaching pilots judgment, decision-making, & critical thinking *International Journal of Applied Aviation Studies*, *4*(2), 203-220.

Rogers, C.R. (1979). The foundations of the person-centered approach. *Education*, *100*(2), 98-107.

Rogers, C.R., Lyon, H.C., & Tausch, R. (2013). *On becoming an effective teacher: Person-centered teaching, psychology, philosophy, and dialogues with Carl R. Rogers and Harold Lyon*. Routledge.

Rose, M. (1999). "Our hands will know": the development of tactile diagnostic skill - teaching, learning, and situated cognition in a physical therapy program. *Anthropology & Education Quarterly*, *30*(2), 133-160.

Rougeé, A.O.T. (2017). *How Do Mathematics Teachers Manage Students' Responses In-The-Moment?* [Unpublished doctoral dissertation]. University of Michigan – Ann Arbor.

Schraw, G., Olafson, L., & Lunn Brownlee, J. (2017). *Teachers' Personal Epistemologies: Evolving Models for Transforming Practice*. Information Age Publishing.

Shaughnessy, M., & Boerst, T.A. (2017). Uncovering the Skills That Preservice Teachers Bring to Teacher Education: The Practice of Eliciting a Student's Thinking. *Journal of Teacher Education*, *68*(3), 40-55.

Sherman, D. (2016). *Preservice Elementary Teachers' Early Practice of Eliciting and Responding to Students' Mathematical Thinking* [Unpublished doctoral dissertation]. University of Michigan.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23.

Sleep, L., & Boerst, T.A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education*, *28*(7), 1038-1048.

Sleep, L., Boerst, T., & Ball, D. (2007). *Learning to do the work of teaching in a practice-based methods course*. Atlanta, GA: NCTM Research Pre-session.

Soslau, E. (2012). Opportunities to develop adaptive teaching expertise during supervisory conferences. *Teaching and Teacher Education*, *28*(5), 768-779.

Stake, R.E. (2010). *Qualitative research: Studying how things work*. Guilford Press.

Stein, M.K., & Smith, M. (2011). *Practices for Orchestrating Productive Mathematics Discussions*. National Council of Teachers of Mathematics, NCTM.

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational Psychology*, *38*(3), 196-210.

TeachingWorks (2020). *Eliciting and Interpreting Student Thinking*. TeachingWorks Resource Library. https://library.teachingworks.org/curriculum-resources/teaching-practices/eliciting-and-interpreting/

Thompson, F., & Logue, S. (2006). An exploration of common student misconceptions in science. *International Education Journal*, *7*(4), 553-559.

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, *24*(3), 335.

Vygotsky, L.S. (1978). *Mind in society: The development of higher mental process*. Harvard University Press.

Weiss, R.S. (1994). *Learning from Strangers: The Art and Method of Qualitative Research*. New York: Free Press.

Yin, R.K. (2006). Case study methods. In J.L. Green, G. Camilli, & P.B. Elmore (Eds.), *Handbook of complementary methods in education*. Lawrence Erlbaum Associates.

Yin, R.K. (2009). *Case study research and applications: Design and methods* (4th ed.). Sage.

Zumwalt, K., & Craig, E. (2005). *Teachers' characteristics: Research on the demographic profile*. Lawrence Erlbaum Associates.