Ohm's Law / Little's Law: Expanding the Analogy

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Sometimes analogies between diverse disciplines lead to important and useful results. Ford Whitman Harris (1913) observed that inventory related costs were similar to the costs for transmitting electricity as they were described by Kelvin's Law(1881). Whitman's analogy led him to formulate the Total Cost function and the Economic Order Quantity (EOQ). This paper observes the analogy between Ohm's Law in electrical engineering and Little's Law in queuing theory and attempts to explore that analogy. The analogy is particularly intriguing because Little's Law is at the heart of several metrics used in Balanced Scorecard analysis developed by Kaplan and Norton (1992, 1996).

KELVIN'S LAW

William Thomson, Lord Kelvin, derived Kelvin's Law in 1881 in the context of designing a system for distributing electric power. Kelvin's law used to be a standard element of the electrical engineering curriculum; today it is given only a passing reference. However, many engineers study the application of Kelvin's Law to inventory (EOQ) as part of an Engineering Economy course. Originally, Lord Kelvin noted that the capital cost of transmitting electricity through a wire increases with the cross sectional area of the wire, but the cost of transmission losses due to resistance decreases with the same increase in the cross sectional area of the wire. The general form of the total cost is given as.

$$TC = A * Q + \frac{B}{Q} + C$$

Q = independent variable, the cross sectional area of the wire in Kelvin's Law
A*Q = the cost that increase with the increases in the wire cross sectional area (capital costs)
B/Q = costs that decrease with increases in the wire cross sectional area (power loss due to resistance)
C = the fixed costs incurred regardless of the wire cross sectional area

The equation plots a curve just like the order quantity total cost curve, except for the fixed cost term. For the EOQ formula, the fixed cost term is the cost of the product which is ignored in many formulations of the model. The curve is concave up with a minimum that may be found by taking the first derivative (slope), setting it equal to zero, and solving.

$$Q = \sqrt{B/A}$$

Electrical engineering handbooks in the early 20th century (Kenyon,1910) show Kelvin's Law in a format that is very similar to Harris's (and the modern) exposition of the EOQ formula, that is,

1. a graph of the total cost function,
2. graphs of the components of the total cost function, that is, capital cost and power loss components,
3. setting the two components equal and solving for the optimal cross section area of the wire and /or setting the derivative of the total cost function equal to zero and solving for the optimal cross section area of the wire.

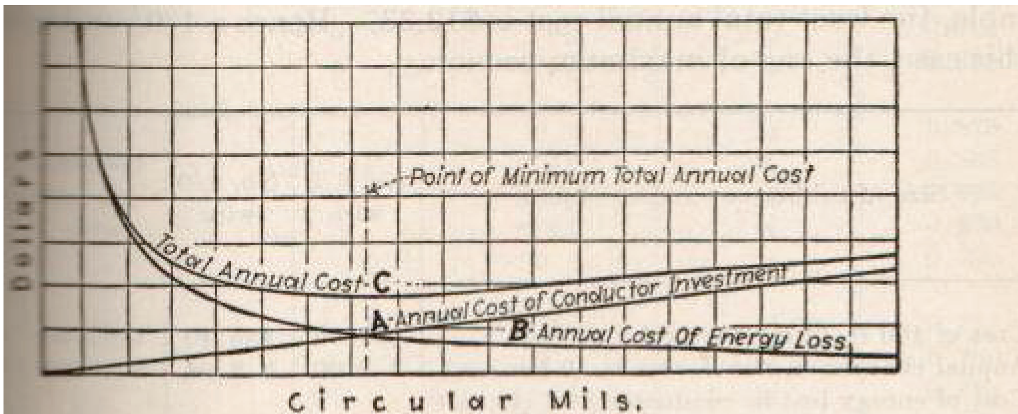


FIG. 38.—Graph illustrating Kelvin's law.

LTTLE'S LAW: AVERAGE FLOW TIME, THROUGHPUT,AND AVERAGE INVENTORY – SEZEN (2007)

The average flow time T is the average time a typical flow unit spends within process boundaries. The *throughput* R is the number of flow units that pass through the process per unit of time. The average Inventory I is the number of flow units that are within process boundaries at any given point in time. In a stable process, Little's Law is the relationship among these three performance measures.

$$I = R * T$$

Average Inventory I = throughput R * flow time T. What is the logic behind this? An arbitrary flow unit enters the process boundaries and spends *T* time units before departing. During this time, new flow units enter the process at rate *R*. Thus, during the time *T* that our marked flow unit spends in the system, *R* x *T* new flow units arrive. Thus, at the time our marked flow unit exits the system, the inventory is *R* x *T*.

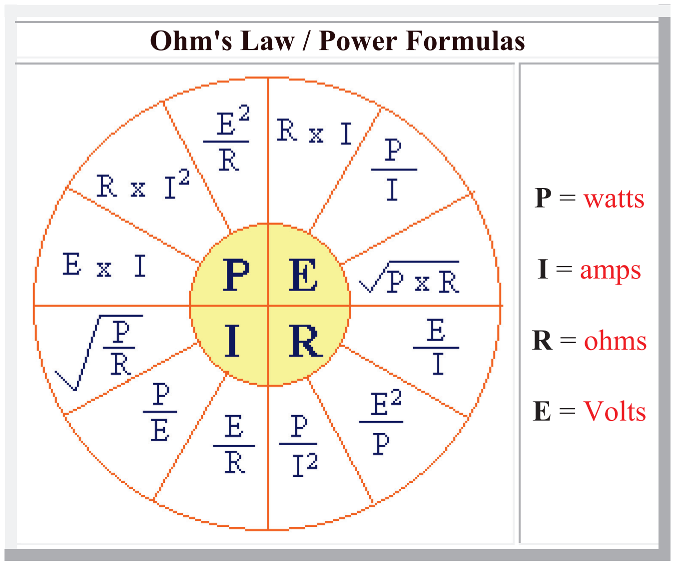
Example: An airport security checkpoint, average queue size *I* = 17.5 passengers, while throughput is *R* =600 passengers per hour = 10 passengers per minute.
To determine the average time spent by a passenger in the checkpoint queue, we use Little's law, *I* = *R* x *T* and solve for *T*:
T=I/R = 17.5 passengers / 10 passengers per minute = 1.75 minutes. On average a passenger spends 1.75 minutes at a security checkpoint.

OHM'S LAW

$$E=R*I$$

Analogy

Little's Law	I inventory (units)	R throughput (units/time)(COGS)	T flow time (reciprocal of Inventory turnover)
Ohm's Law	E volts	I amperes	R ohms



The analogies of I, R, and T (Little's Law) to E, I, and R (Ohm's Law) are clear enough although the use of Rand I in both laws in different contexts is distracting. Does the analogy to the power calculation mean anything in a queuing or inventory situation?

$$P = \frac{I^2}{T} = T * R^2 = I * R$$

In the context of inventory, the power formula would yield Inventory * COGS (units ^2 / year) squared per year. The meaning of this calculation or the units it is measured in are not obvious.

LITTLE'S LAW AND BALANCED SCORECARD

In the Balanced Scorecard methodology, performance measures are grouped in four or five categories: financial perspective, customer perspective, internal process perspective, and an innovation and learning perspective. An optional fifth perspective staff includes matters relating to recruiting appropriate staff. A Little's Law based analysis can be used as part of all of these perspectives. Thus the enrichment of the Little's Law analysis through the Ohm's Law analogy could potentially be very rewarding.

RATIO ANALYSIS AND LITTLE'S LAW

Operating efficiency ratios typically include turnover ratios and "number of days" ratios. Both of these fit into the Little's Law framework.

Inventory Turnover = COGS / Inventory

Cost of Goods Sold $400,000
Inventory $ 64,000

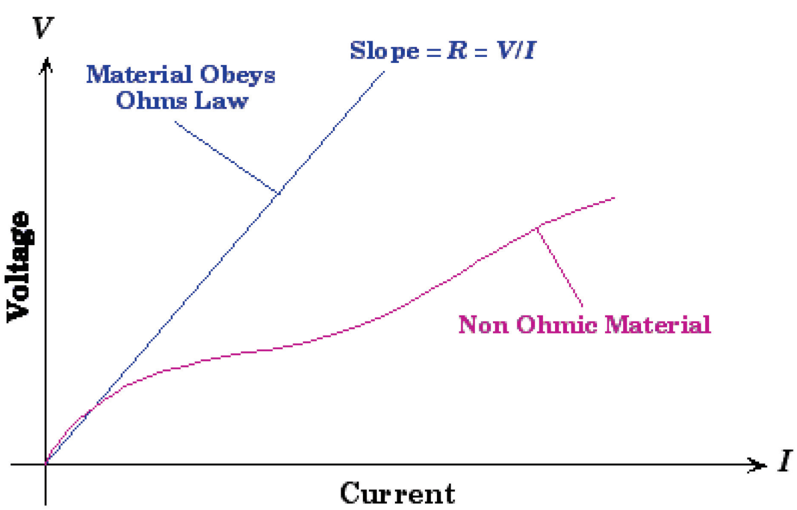
Ratio = 6.25 A business that sells fresh fish should have a much higher turnover than a jewelry shop. The reciprocal of the turnover 1/6.25 = .16 is the flow time. It takes .16 years for a typical unit of inventory (SKU) to pass through the business.

Alternatively, the Days of Sales ratio is calculated.

$64,000 /($400,000/360) =57.6 days. This is just the flow time stated in days.

NON-OHMIC MATERIALS (INVENTORIES)

Little's Law assumes that the transit time is constant at different levels of COGS. In the Ohm's Law situation, sometimes flow causes the conductor to heat up and the resistance to increase. Analogously, high levels of COGS could cause the system to heat up with a resulting increase in transit time T. When an inventory- processing system heats up, units spend relatively more time waiting in inventory (lines) and relatively less time in processing. Sometimes this waiting is due to high utilization and other times it is due to a lack of transparency in the system (Dirnberger, 2006).



The equations which describe the increase in resistance due to temperature are given below:

$$\frac{(R - R_0)}{R_0} = \alpha(T - T_0)$$

Or

$$R = R_0[1 + \alpha(T - T_0)]$$

Where α is the temperature coefficient of resistance. In a waiting line system, utilization ρ might be analogous to temperature. The critical level of utilization in a waiting line system with random arrivals and random service times is about 2/3. The critical level is significantly higher for systems with predictable arrivals and service times (Resistance and Resistivity, 2009). For financial analysis of an inventory system, the above equation could be rewritten to estimate the percent change in T the transit time in terms of the percent change in COGS:

$$\frac{T - T_0}{T_0} = \alpha(COGS - COGS_0)$$

This form of the equation should allow α to be estimated without assuming a critical value of utilization and without identifying that level of utilization. Changes in inventory processing, for example, the adoption of Vendor Managed Inventory, could make it impossible to apply this estimation technique.

If utilization is low, then transit time should be almost constant. Low utilization of plant and machine resources allows significant flexibility. Most of the day, the majority of the point-of-sale checkout stands in a supermarket are idle. Late in the afternoon, employees are transferred from stocking activities to checkout activities. The point-of-sale equipment has high utilization for only short periods of time. The scheduling of human resources assures high utilization of human resources.

ALTERNATIVE LOOK AT NON-OHMIC INVENTORIES

When the company is operating efficiently, Little's Law will give an ideal inventory level (I) for that level of Cost of Goods Sold (R) and an ideal transit time (T). The ideal transit time T_{ideal} should be almost like a physical constant. It could be estimated by adding up the times involved the various processes for inventory units or more pragmatically as the minimum of the transit times for the last 8 quarters. Observed transit times greater than T_{ideal} result in higher than ideal inventories.

$$I_{ideal} = COGS * T_{ideal}$$

Then the actual inventory I is given by

$$I = COGS * T$$

The difference between these two equations gives the unintended investment in inventory

$$I - I_{ideal} = COGS * (T - T_{ideal})$$

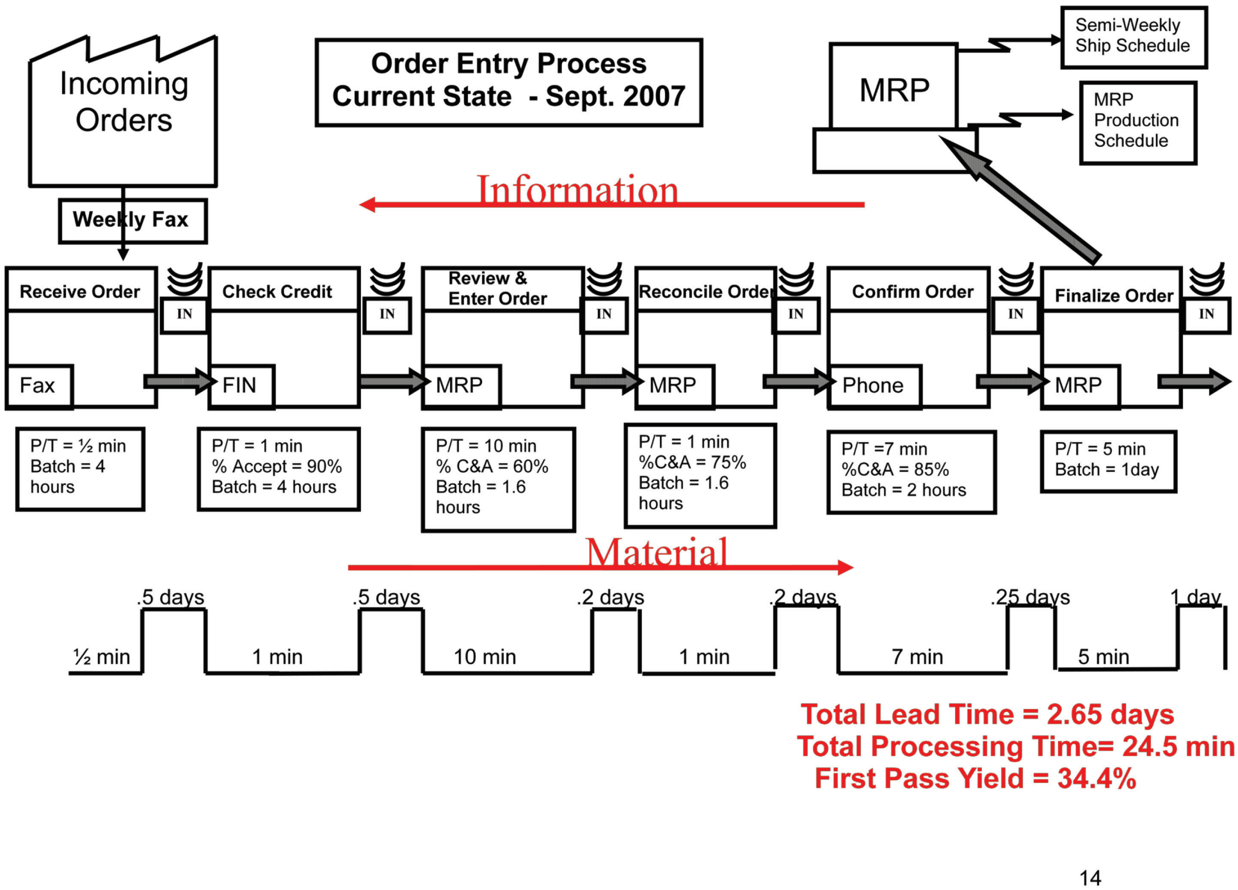
The result could be interesting on more than one level. Unintended inventory could be a useful extension of operating ratio analysis. The ratio of unintended inventory to inventory seems a likely choice. Unintended inventory would be available on a firm level and could make possible some new economic models of the business cycle which could focus on firm behavior. General Motors and Chrysler, in particular, had large unintended investments in inventory. Ford, having moved to a "pull" concept of inventory, had a much lower unintended investment in inventory.

VALUE STREAM MAPPING (VSM)

Value Stream Mapping is a technique that helps the user see and understand the flow of product and information as the product moves through the value stream.

> A value stream map will take into account not only the activity of the product, but the management and information systems that support the basic process. You will gain insight into

the decision making flow in addition to the process flow. The basic idea is to first map your process, then above it map the information flow that enables the process to occur. (Business 901, 2009)



(Bolesta, 2007)

MicroSoft® Office Visio Professional 2007 includes a feature to facilitate the drawing of value stream maps. Value stream mapping is an integral part of the Lean Six Sigma methodology. Value stream mapping was part of the original lean manufacturing methodology, the Toyota Production System, developed at Toyota by Shigeo Shingo and Taiichi Ohno.

The basic VSM methodology is to draw a value stream map for the current process. The value stream mapping of the current process allows planner to identify waste in the current system: over production, waiting, transport, inappropriate processing, unnecessary inventory, unnecessary motion, and defects. A future value stream mapping is drawn with the waste removed. Then the analysts figure out how to move from the current VSM to the future VSM.

UNDERSTATED INVENTORIES

For the quarter ending January 29, 2010, Dell Computer showed an inventory turnover figure of

Inventory Turnover = COGS / Inventory = 43641 / 1051 =41.5

Dell's inventory practices, Vendor Managed Inventory (VMI), could be interpreted as understating inventory and overstating inventory turnover. Taking a more normal inventory turnover figure 12.0, one can calculate an imputed inventory figure of

Inventory = COGS / Turnover = 43641 / 12.0 = 3622

The increase in current assets 3622 – 1051 = 2571 would be an increase in Accounts Payable. There would be a corresponding decrease in asset turnover and a decrease in return on assets.

The average days of supply in inventory would go from 8 to 30. Days in Accounts Payable would go from 82 to 101.

SUMMARY AND CONCLUSIONS

The result of this exploration of the analogy between Ohm's Law and Little's Law is the suggestion of a way to calculate unintended investment in inventory at the firm level:

$$I - I_{ideal} = COGS * (T - T_{ideal})$$

T is the value from the current VSM and T_{ideal} is from the future VSM. The value of the Little's Law analysis is to quantify some of the gains associated with VSM and Lean Six Sigma.

The same model can be used when practices like VMI make figures from firms in the same industry difficult to compare, for example, Dell and HP computes.

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