Portfolio Allocations and Benefits using Various Treasury Inflation Strategies

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The objectives of this study included a brief review of TIPS or TIIS securities that have been issued since January 1997, identification of some of the important characteristics of TIPS that make them desirable for inclusion in a risky optimal portfolio of assets, acquisition of securities data over the period August 1998 to June 2004, construction of a model using MS Excel® and the Solver® add-in, the computation of an efficient frontier, and finally an evaluation of the proportion of TIPS included in optimal risky portfolios at various levels of rates of return and standard deviations. The results provide a well-behaved efficient frontier. TIPS are included in portfolio with low required rates of rates of return. TIPS were then found to be replaced first by increased allocations to the 30-year Treasury bond fund, then by other securities such as bonds, equities, and indices. The study results are consistent with similar results but there is concern that given the study period, the diversification benefits of TIPS were not fully observed at higher minimum portfolio rates of return.

INTRODUCTION

Investors have a multitude of choices for constructing an investment portfolio that matches his/her risk and return requirements over a specific investment horizon. A new class of securities becomes available almost daily to entice investors to purchase traditional and engineered financial assets. Although there has been a significant amount of research conducted using portfolio theory to determine the optimal risk-return portfolio considering many assets, few studies have discussed the optimal asset allocation strategies when one considers including U.S. or international TIPS (Treasury Inflation-Protection Securities) or TIIS (Treasury Inflation-Indexed Securities). A common example is the Series I Bonds issued by the U.S. Treasury that are guaranteed to offer a real rate of return. A specific example would be the 5-Year Treasury Inflation-Indexed Note that was due 7/15/2002.

There has been a significant amount of research conducted describing the market, characteristics, benefits, and drawbacks of TIIS. Deacon, Derry, and Mirfendereski recently published a book on the subject: Inflation-Indexed Securities: Bonds, Swaps, and other Derivatives (Deacon, March 2004), and TIIS have been investigated by governments, securities analysts, professional financial services advisors, and academics. The literature contains several detailed investigations of specific attributes of these securities and some address the effects of adding these securities to one's portfolio. Chen and Terrien of Ibbotson Associates (Chen, November 1999) looked at "TIPS as an Asset Class". AARP, TIAA-CREF, the financial news, and investigative sources have also investigated the topic.

The research presented will include a brief review of relevant literature concerning the relationships among TIIS and other securities, domestically and internationally. A Microsoft (MS) Excel® empirical model was developed to generate an efficient frontier for selected equities, mutual funds that include corporate bonds, government bonds, municipal bonds, various market indices, and TIIS or TIPS. Finally, the results of the computations will include reporting how TIPS enter into the optimal risky asset portfolios that lie along the efficient frontier.

BRIEF HISTORY OF TIPS OR TIIS

The concept of indexation of debt instruments to protect the bondholder's real purchasing value and rate of return if the instrument is held to maturity is not new. Fisher (Fisher, 1922) reported that in 1707, Bishop William Fleetwood produced a study that considered the erosion of purchasing power of money. Indeed, Irving Fisher, Alfred Marshall, John Maynard Keynes, Richard Musgrave, Milton Friedman, and Robert Barro have all supported the concept of indexation. The probable first issue dated back to 1742 when the State of Massachusetts first issued bills of public credit linked to the cost of silver on the London Stock Exchange (Deacon, Derry, and Mirfendereski, 2004 and Fisher, 1913).

An indexed bond or inflation indexed security (IIS) is different from other bonds such as U.S. corporate bonds in that the coupon or interest payments and/or the principal or face amount change with the change in the indexation bundle of goods or services. A key issue concerning indexation is what is the appropriate measure of change in purchasing value since each individual's consumption and investment in goods and services differs. The indexation bundle must be stable and consistent and reported in a timely manner.

Indexed public sector bonds have been issued by a number of countries (Deacon, Derry, and Mirfendereski, 2004, p.5). This is particularly true of a country that has high and volatile inflation and corresponding interest rates. Recently, Brazil, Argentina, and Mexico have issued public debt with indexation due to inflationary concerns. However, countries may issue public debt that is indexed even when faced with stable to declining inflation and interest rate environment. The most germane example for this study is the Treasury Inflation Indexed Securities (TIPS) that were issued in January 1997. Although the U.S. issued similar securities in 1742 and 1780, the most recent issues have drawn much attention from the general public as well as researchers. The primary source of specific detailed information on U.S. government issued TIPS is from Treasury Direct, maintained by the U.S. Bureau of the Public Debt (http://publicdebt.treas.gov/).

Although the first issues of TIPS in the U.S. were relatively illiquid and demand did not materialize as quickly as initially thought, the market for these securities has increased. Since the U.S. Treasury offered $7 billion of TIPS with a real yield of 3.45% in January 1997, TIPS

have been offered by other governmental agencies such as the Tennessee Valley Authority, the Federal Home Loan Bank, Federal Farm Credit Bureau, Student Loan Marketing Agency, and the Federal National Mortgage Agency. Financial firms such as Salmon Brothers, Toyota Motor Credit, Nationsbank, and Merrill Lynch also issued TIPS to increase the market capitalization to $2 billion (Lamm, Jr. and McFall, R., 1998). A second Treasury auction of $8 billion in 10-year TIPS with a real yield of 3.65% was conducted on April 9, 1997 and an additional $8 billion in 5-year notes with a real yield of 3.74% followed on July 8, 1997. Other issues have also been auctioned. Some issued TIPS have retired while others are still outstanding.

In February 2004, the Chicago Mercantile Exchange initiated a Consumer Price Index (CPI) futures contract that was the first linked to a major economic indicator (Wood, 2005). Wood also reported that the CPI Futures would create synthetic inflation-protected income and that there is a growing market for over-the-counter (OTC) derivatives. Hence, there are increasing opportunities for flexible, interconnected markets.

IMPORTANT CHARACTERISTICS OF U.S. TIPS

An overview of the characteristics of U.S. TIPS is available at www.treasurydirect.gov /indiv/products/tips_glance.htm . TIPS pay a fixed rate of interest specified at time of issue. The value of a TIPS issue is adjusted semiannually based on changes in the CPI-U (Consumer Price Index-All Urban Consumers), the broadest measure of the purchasing value of the U.S. dollar that reflects the general rate of inflation. The observed CPI-U is applied to the inflation-adjusted principal, not the original face value. If the CPI-U increases, the principal increases. If the CPI-U decreases, principal decreases with a floor of the original face value. TIPS are issued at various coupon rates and are issued with original terms to maturity of 5, 10, and 20 years. At maturity, the inflation-adjusted principal is deposited into the bank account of the owner.

The Public Debt site indicates that one uses TIPS to:

- Diversify a portfolio
- Supplement retirement income

With respect to taxation of TIPS:

- Interest income is exempt from state and local income taxes
- Interest income is subject to Federal income tax

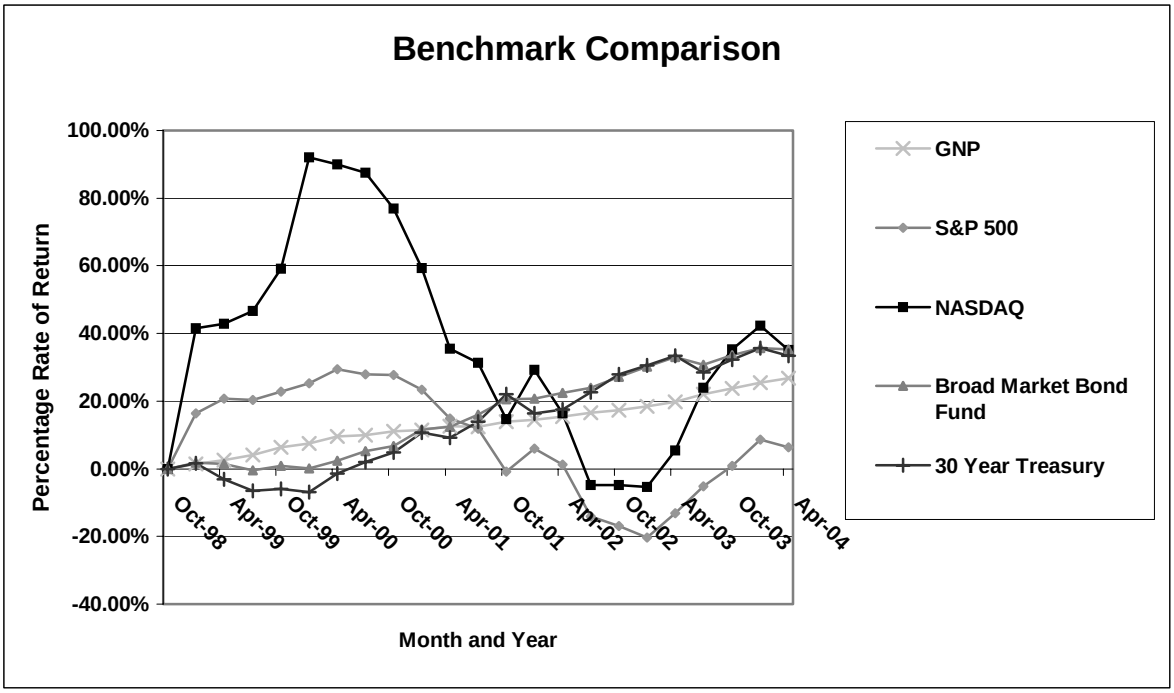
TIPS provide investors with a new, U.S. dollar denominated asset class for inclusion in their diversified portfolio. Dalio lists several advantages (Dalio, 1996):

1. a U.S. dollar real return that is known and fixed to maturity [although the realized nominal cash flows and rate of return is not known]
2. an expected return that is equal to or marginally higher than conventional nominal bonds [in the same risk class and with similar indenture characteristics]
3. a risk which is considerably lower than conventional bonds of the same duration, and
4. correlations with conventional bonds and stocks that are low or negative (depending on the time horizon)

Figure 1 presents an illustration of the relationship among various select groups of assets, markets, indices, and U.S. GNP derived from the data set constructed for this study. Illustrated are the computed annualized rates of return using monthly data for a TIPS series

that was one of the first offered and that is still outstanding, a represented corporate bond portfolio using a mutual fund product as an example, and the S&P 500 index that represents a portfolio of large cap common stocks or equities. Note the data for the TIPS is for one security where the bond and equities shown are from somewhat diversified portfolios that reduce variability and rates of return. Of particular importance to this study is point 4 above. TIPS do offer negative correlation with most bonds and equities hence help reduce portfolio risk. Also note that the data used to generate Figure 1 uses nominal prices of the securities. It is assumed that market price will imbed the market's rate of return based on the bundle of characteristics associated with each of the securities shown. The data are not adjusted for inflation or taxation differences that are inherent in the TIPS in particular. An example of the yields and cash flows differences among TIPS, I Bonds, and fixed coupon securities and their sensitivities to changes in interest rates, inflation rates, and marginal tax rates is provided in Carlson, Haskins, and Hanson (Carlson, 2005).

FIGURE 1



The broad market bond fund and 30-year Treasury fund used in Figure 1 are described as follows:

HABDX – The Harbor Bond Fund is an income fund that invests at least 80% of its assets in income securities. At least 80% of total assets are invested in dollar denominates securities. The fund primarily invests in investment grade securities. The purpose of this asset is to serve as a proxy of a broad market bond fund to offer perspective as to the relative performance of bonds over the study period.

VUSTX – The Vanguard Long-Term Treasury Fund serves as a proxy for the returns to the 30-year Treasury bond. The fund normally invests at least 80% of its assets in long-term U.S. government securities.

METHODOLOGY

An objective of this study is to estimate an efficient frontier, using modern portfolio theory and MS Excel® as the analytical application to evaluate randomly and quasi-randomly selected daily and monthly price and index data representing equities, bonds, mutual funds, and TIPS. The Solver® add-in function was used to determine an optimal set of portfolio weights that minimizes standard deviation subject to a minimum expected rate of return. To be consistent with capital market theory, the efficient frontier must be concave to the origin.

Once the efficient frontier is constructed that represents the efficient set of portfolios of risky assets, the question is how much of the efficient portfolio's investment or weight is comprised of TIPS. One would expect that since TIPS are generally negatively to slightly positively correlated with other securities and have realized returns that are less volatile compared to other securities, they should represent a significant portion of low risk portfolios. As the standard deviation of portfolios increases, one would expect the contribution of TIPS to the efficient risky portfolios to decline.

Selection of Data

The main concern for selecting equity data from the set of available publicly traded equities was to ensure random selection thereby reducing or eliminating bias. The first step was to analyze the equity population as defined in Yahoo!® Finance and Reuters®.

Company selection was the first step. A list of two-digit SIC codes were obtained from the U.S. Department of Labor Occupational Safety & Health Administration's (OSHA) web site (http://www.osha.gov/pls/imis/sic_manual.html). The two-digit SIC classification, in its entirety, basically accounts for all possible business endeavors a company can possibly undertake. However, the specific assignment of an SIC classification number is based on the company's major commercial activities. Therefore, for an accurate and unbiased equity market simulation, a representative company from each of these two-digits SIC categories were selected and added to the equity portfolio.

Yahoo!® Finance offers an industry index (http://biz.Yahoo!.com/ic/ind_index.html) that mirrors the two-digit SIC classification. Each industry is represented by a three-digit classification and the sub-listing represents the two-digit classification. Each of the sub-listings was in the form of hyperlinks. In each of the links is a description, summary, and miscellaneous data much like OSHA's website. On this web page there is a link called company index that provides a list of all companies represented under each classification. There is an option to segregate publicly-traded companies from the list.

Publicly-traded equity was obtained by importing the company lists from each of the sub-listings into MS Excel® through web queries. This list was categorized by sub-industry. When the data were compiled the process of company selection began. It is impractical to include statistical characteristics for all publicly-traded companies so one company was chosen from each category as a representative of that industry. Given the concept of construction of a well-diversified portfolio of securities, one does not have to invest in the population of available securities. Even random selection of securities greater than 50 should create a portfolio essentially void of event risk and represent only non-diversifiable or systematic risk according to capital market theory.

A randomize selection process was used to eliminate potential biasness. A randomly selected sample set specifically should not consider company characteristics such as capitalization, brand recognition, volatility of price or earnings, or any other factor.

A pseudo-random number generator add-in available in MS Excel® was used to select companies for analysis. This add-in is called ZRandom. ZRandom uses the Mersenne Twister algorithm to generate better pseudo-random numbers than the RAND( ) formula in Excel® and Rnd( ) function in VBA[1].

After a sample of publicly-traded companies had been selected, historical price data was obtained from Yahoo!® Finance. Monthly data was extracted for each company in the sample. The data are adjusted for stock splits and dividends. The study period was defined as August 15, 1998 to June 1, 2004. Securities data were downloaded to a 2003 MS Excel® workbook. The data for each sample company were subsequently moved to and manipulated in a master workbook. The list of specific equities and funds included in the study is excluded in the interest of brevity.

The TIPS data were obtained from the Federal Reserve historical data series and from the U.S. Treasury, Public Debt website and entered into the master workbook. Each data series had to be imported to the workbook separately.

Mutual funds were added in a non-random fashion to represent fixed income, real estate, municipal securities, and the utilities industry. The following are these mutual funds:

LMSFX - Federated Municipal Securities Fund seeks current income exempt from federal income tax. The fund invests at least 65% of assets in municipal bonds; at least 80% of annual interest income is exempt from Federal income tax. The fund may invest in certain Alternative Minimum Tax (AMT)-subject bonds

CABTX – Alliance Bernstein Municipal Income Fund Insured National Portfolio seeks income exempt from Federal income tax. The fund normally invests at least 65% of its assets in insured municipal securities that generate income not subject to the AMT. The fund may invest up to 25% of assets in municipal securities whose issuers are located in the same state.

MDHQX - Merrill Lynch Bond Core Bond Fund seeks current income. The fund normally invests at least 80% of assets in high-quality corporate debt. The fund expects to hold debt with longer maturities. It may also shift credit quality and maturity to mitigate adverse price changes.

MGFIX - Managers Bond Fund seeks income. The fund normally invests at least 65% of assets in fixed income securities. It may also invest up to 10% of assets in non-U.S. dollar-denominated fixed income securities. The fund may not invest in securities rated lower than BB.

PBSMX - Dryden Short-Term Corporate Bond Fund seeks income consistent with the preservation of principal. The fund normally invests at least 80% of assets in bonds of corporations with maturities of six years or less. Up to 35% of assets may be invested in dollar-denominated obligations issued in the U.S. by foreign corporations and governments. The advisor may also invest up to 20% of assets in debt obligations issued by the U.S. Government or government related entities, and up to 10% of assets in below investment-grade debt obligations.

[1]This statement was derived from the ZRandom website (http://www.zrandom.com/zrand/

The fund also engages in active trading in order to take advantage of new investment opportunities or yield differentials.

ACREX - Dryden Short-Term Corporate Bond Fund seeks income consistent with the preservation of principal. The fund normally invests at least 80% of assets in bonds of corporations with maturities of six years or less. Up to 35% of assets may be invested in dollar-denominated obligations issued in the U.S. by foreign corporations and governments. The advisor may also invest up to 20% of assets in debt obligations issued by the U.S. Government or government related entities, and up to 10% of assets in below investment-grade debt obligations. The fund also engages in active trading in order to take advantage of new investment opportunities or yield differentials.

PUGIX - Putnam Utilities Growth & Income Fund seeks long-term capital appreciation. The fund normally invests in a combination of bonds and stocks of U.S. companies in the public utilities industries, with a significantly greater focus on value stocks. It typically invests in bonds that are mostly investment grade in quality with maturities of at least three years. The fund tends to invest in large-capitalization companies.

Together these funds represent long, intermediate, and short maturity corporate bonds. The above funds also include long and intermediate maturity municipal bonds. Utilities and real estate agencies are also included.

Investors interpret a company's stock price as the intrinsic value of expected discounted future cash flows. Investors similarly interpret Bond prices where the expected cash flows are generally the periodic coupon payments and redemption value at maturity, call, or conversion. Direct comparisons of rates of return on equities and yields on debt instruments are not possible without some adjustment. Stock performance is typically measured in rates of return based on annualized capital gains/losses plus any dividend payments over the period. Bond performance is measured by the yield. The yield calculation includes current price and the expected present value of coupon payments and value received at redemption. Because a stock has no maturity it is inappropriate to measure its performance in terms of a yield. Bonds can be measured in terms of rate of return if three variables are known. The first is the coupon payment; it is treated like a stock dividend when adjusted. The second variable is the prices observed over equal time intervals during the life of the debt instrument. The third is the redemption value received at maturity, call, or conversion. Once this information is acquired, bonds can be looked at in the same terms as stocks. For the purposes of this analysis an adjustment for comparability was employed. The raw bond data was collected from Reuters® and entered into a MS Excel® workbook. Each time a coupon was paid the value was cumulatively subtracted from the closing prices. The end result was an adjusted price that reflected periodic cash payments equivalent to those of an equity.

CALCULATION PROCEDURE

To determine the efficient risky portfolios, a table of all data was constructed and the average of rates of return on each security calculated. The rates of return were calculated using the following.

$$\left(\frac{P_1 - P_0}{P_0} \right) \qquad \textbf{(1)}$$

The average of each of the securities' rates of return was calculated and added to the data set was used to identify the efficient portfolios. The next step was to create a variance-covariance matrix. This was done by using variance or VAR( ) and covariance or COVAR( ) functions. Once the variance-covariance matrix was constructed, Solver® parameters were specified to perform the estimation of each point along the efficient frontier.

There were 132 decision variables representing the optimal weights that were assigned to each security. The target cell used to find a solution was the total portfolio variance which is calculated by using the formula: =SUMPRODUCT(MMULT(COVARMATRIX,DECISION VARIABLES),DECISION VARIABLES). This model is shown in (Barlow, 1998, p.67). The constraints were:

- the sum of decision variables (portfolio weights) must equal 1,
- the calculated rate of return (=SUMPRODUCT(DECISION VARIABLES,AVERAGE RETURN)) must equal the selected required rate of return, and
- short sales were not permitted so non-negativity applied to all portfolio weights.

A non-linear model option was selected. The precision was set to the 100 millionth with a tolerance of 0%. Convergence was set to the thousandth. Specific required rates of return were used to estimate points along an efficient frontier. The solutions were estimated in rate of return increments of 0.5% increments. The percentage of TIPS contained in each estimated efficient portfolio was then identified. From these results, one can assess the extent of influence that TIPS have in an efficient and optimized portfolio at a specific rate of return and standard deviation.

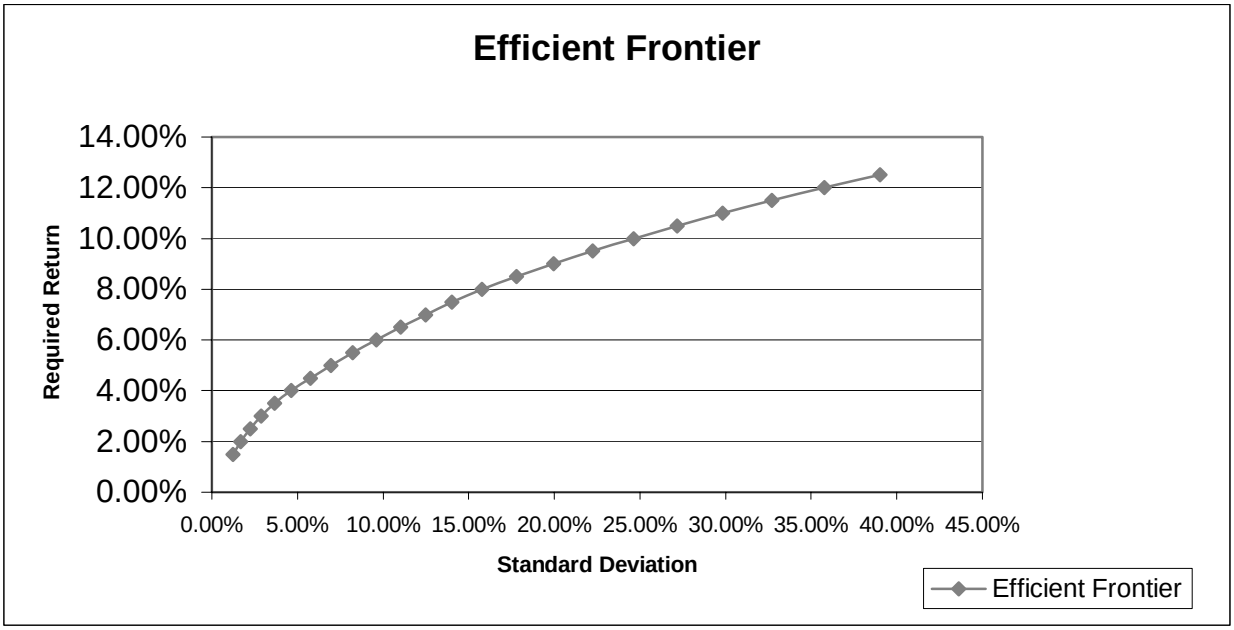
RESULTS

A first important result of the analysis is the construction of an efficient frontier based on the data collected and analyzed. The efficient frontier information is presented in Table 1 and illustrated in Figure 2. The possible range of expected rates of return is 0.00% to 13.00%. As theory would indicate, portfolio rates of return must be positively correlated with portfolio standard deviation. This reflects efficient capital markets in that investors who expect higher rates of return must assume greater risk. Note that the efficient frontier described in Table 1 and represented in Figure 1 is very well behaved and illustrates that as the minimum rate of return on risky portfolios increases, the standard deviation or measure of portfolio risk must increase.

TABLE 1
POINTS ALONG THE EFFICENT FRONTIER

Rate of Return	Standard Deviation	% TIPS in Portfolio	% 30-Year Bonds in Portfolio
12.50%	39.03%	0.00%	0.00%
12.00%	35.77%	0.00%	0.00%
11.50%	32.69%	0.00%	0.00%
11.00%	29.83%	0.00%	0.00%
10.50%	27.16%	0.00%	0.00%
10.00%	24.64%	0.00%	0.00%
9.50%	22.24%	0.00%	0.00%
9.00%	19.95%	0.00%	0.00%
8.50%	17.78%	0.00%	0.00%
8.00%	15.78%	0.00%	0.00%
7.50%	14.01%	0.00%	0.00%
7.00%	12.48%	0.00%	0.00%
6.50%	11.01%	0.00%	0.00%
6.00%	9.60%	0.00%	0.00%
5.50%	8.23%	0.00%	0.00%
5.00%	6.94%	0.00%	0.00%
4.50%	5.74%	0.00%	5.21%
4.00%	4.63%	0.00%	11.05%
3.50%	3.66%	0.00%	15.15%
3.00%	2.88%	8.46%	15.66%
2.50%	2.23%	13.22%	14.73%
2.00%	1.68%	10.97%	11.42%
1.50%	1.22%	6.84%	7.38%

FIGURE 2



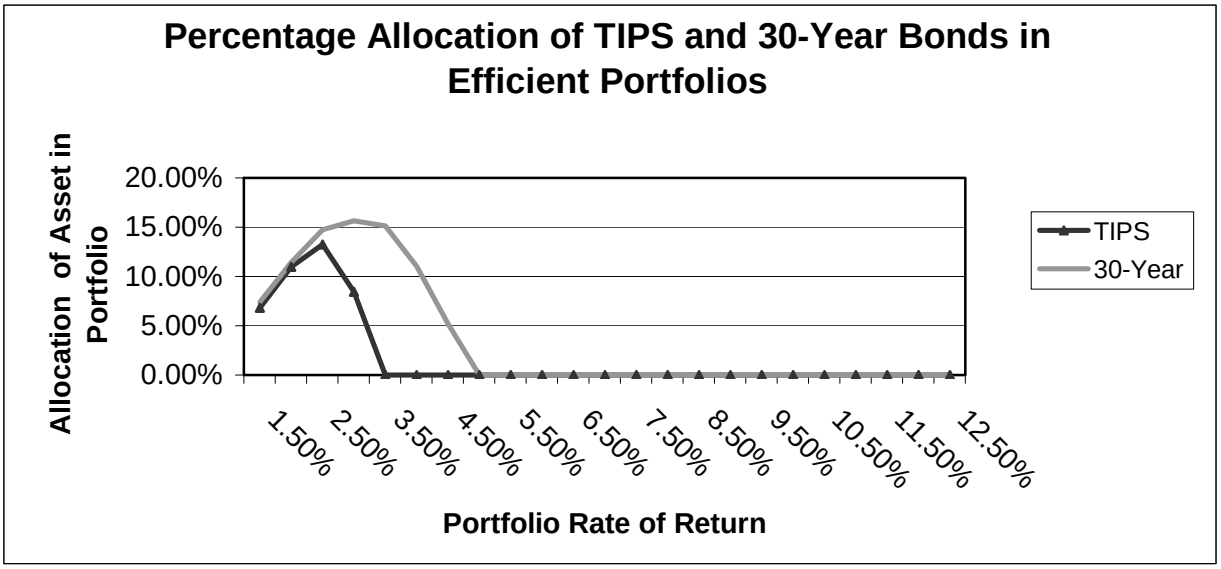
TIPS AND U.S. 30-YEAR TREASURY BOND BEHAVIOR

The range of the highest to lowest average rate of returns for the securities included in this study is 13.00%. So the highest possible rate of return would be 13.00% but the allocation would end up as 100% investment in the one asset providing the 13.00% rate of return. The analysis was conducted in one-half of one percentage rate of return intervals with a minimum of 1.5% and maximum of 12.5% portfolio rate of return. The minimum of 1.50% was used as the rate where an investor could purchase short-term Treasury Bills and have a risk-free rate of return.

Starting with a portfolio minimum rate of return of 1.50% and increasing the rates in one-half of one percentage intervals, equities, bonds, indices, and TIPS all enter in the optimal risky portfolio of assets. As the required minimum rate of return increases beyond 2.50%, the 30-year Treasury fund begins to replace the TIPS in the portfolio. This continues with the 30-year Treasury fund allocation peaking at 3.00% rate of return for the portfolio then declining to zero allocation at 5.00%. TIPS no longer enter the portfolio allocation beyond 3.50%. Above these minimum portfolio rates of returns, other bonds, equities, and indices dominate the portfolio allocation.

The results are as one would expect. Given the investor is willing to accept a low minimum rate of return on a portfolio, the investor would seek to add those securities that have lower variability of returns yet still assist in diversifying the risk of the total portfolio. Since the rates of return on TIPS are relatively low although the real rate of return is essentially guaranteed, they must be replaced with higher rate of return securities that still have low volatility and contribute diversification of risk. Eventually, to achieve the higher minimum rates of return on the portfolio, equities, and indices must enter and dominate the portfolio. There is a limitation in this analysis to the extent other bond funds, corporate and municipal funds were not evaluated in terms of their contributions at higher levels of minimum portfolio rates of return. However, the focus of this paper was to construct risky optimal asset portfolios using sample data and evaluate the contribution of TIPS as an asset class to these efficient portfolios.

FIGURE 3



The optimal risky portfolios calculated generally provide the anticipated results concerning how much of the portfolio would be comprised of TIPS. As expected, TIPS enter at low levels of risk where investors would want to not only diversify their portfolio but also use the TIPS as an inflationary hedge. In fact, TIPS represents a significant portion of the portfolios up to a 3.00% rate of return and accounts for a significant portion of the portfolio from 3.50% to 5.00%. At rates higher than 5.00%, TIPS are not selected for the optimal risky portfolio to any significant degree.

CONCLUSIONS

Using random and quasi-random selection of equity, bond, indices, mutual funds, the TIPS available over the study period, TIPS entered the portfolio of risky assets only at low minimum acceptable portfolio rates of return. This result is consistent with what other studies have found with respect to TIPS inclusion in investor portfolios. However, one would have expected that due to the diversification contributions of TIPS, particularly since they offer a guaranteed risk-free real rate of return that adjusts for the effect of inflation, TIPS should have been included in a fully diversified portfolio even at higher minimum portfolio rates of return. In a normative sense, TIPS should replace precious metals securities as a hedge against inflation even at higher rates of return than indicated in this study.

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