Investor Sentiment and the Asset Pricing Process – Extension of an Existing Model

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Common aspects of human behavior, like overconfidence or misconceptions in updating beliefs, might influence market behavior. We extend an existing single-asset overlapping generation model by considering two correlated risky assets for which noise traders may form different beliefs. When the second correlated risky asset was introduced in the model, the asymmetric effect from positive/negative shifts in sentiment was consistently reduced. Thus, a theoretical contribution that this study brings to the existing literature is that good/bad news has an asymmetric effect on the asset pricing process.

INTRODUCTION

Many researchers have attempted to examine theoretically and empirically the Efficient Market Hypothesis (EMH) and perhaps the most well known advocate of different levels of market efficiency is Fama (1970, 1991). However, some economists have tried to solve some of the inexplicable phenomena like the January Effect or the Day-of-the-Week Effect. Because researchers have been unable to explain different scenarios of market inefficiency, they seem to accept (at least in a tacit way) the existence of some irrational forces affecting the market.

This paper tries to explain how common aspects of misconceptions might influence the market behavior. As Schiller (2000) mentioned, departures from market efficiency can have negative effects for future generations; any misallocation of resources through market overvaluation takes resources away from other needy sectors of the economy. The goal is to pinpoint some of the hidden factors that guide the market in a specific direction, to analyze them, and to infer possible judgments for future market behavior.

Recent literature combines all these irrational, unusual influences into an aggregate term, “investor sentiment”, that should be able to explain some of the observed anomalies in asset returns or market volatility. For example, De Bondt and Thaler (1985) found that people overreacted to news; their choices did not satisfy Bayes' rule: "individuals tend to overweight recent information and underweight prior (or base rate) data" (p. 793) and they tested the predictive power of the overreaction phenomenon.

Summers (1986) analyzed whether the stock market reflects fundamental values rationally, and he concluded that it was hard to identify some types of market inefficiency by using standard methods.

Fama and French (1996) argued that the anomalies are related and proposed a three-factor model to explain them. The model appeared to account for the cross-sectional volatility in returns and the long-run returns' reversals, but could not explain the short-term returns' continuation.

An interesting issue that has received a lot of attention is the extent to which speculators' actions affect the asset prices. In the past, researchers were confident that any destabilizing action by speculators could be offset by an opposite action from rational investors. However, more recent research shows that rational investors work to bring asset prices closer to fundamentals, but they cannot succeed due to risk aversion to noise traders' unpredictable actions. DeLong, Shleifer, Summers, and Waldmann (1990) developed an overlapping generation model and showed that noise traders ("irrational investors") do affect asset prices, and in certain conditions can earn higher expected returns.

One psychological characteristic of individuals in general and of investors in particular is overconfidence (Daniel, Hirshleifer, & Subrahmanyam, 1998). This characteristic presumably generates high forecast errors through overestimation of private information. Using psychological support in explaining people's behavior, Shleifer, Barberis, and Vishny (1998) built a model of investor sentiment that could explain both underreaction and overreaction phenomena.

Recent studies in financial behavior recognize the limits of arbitrage, and Daniel, Hirshleifer, and Teoh (2002) pointed out some psychological factors that may affect decision-making processes: self-attribution (attribute the success and blame the bad luck when failure), self-deceptions, emotion-based judgments, framing effects, and mental accounting.

Brown and Cliff (2002) separated investor sentiment into an institutional and an individual one. In addition, using the Sentiment Index data, Lee et.al (2002) showed that "sentiment is a priced risk factor" (p. 2281).

The purpose of this paper is to extend the single-asset theoretical analysis in DeLong et. al. (1990) model to allow for two correlated risky assets for which noise traders may have different beliefs. The theoretical model shows that the presence of a second risky asset reduces the asymmetric effect from positive/negative shifts in sentiment.

THE THEORETICAL FRAMEWORK

We extend the two-period single-asset overlapping generation model (DeLong et. al., 1990) by considering two-correlated risky assets for which investors may form different beliefs.

Assets

- The safe asset (s), in perfect elastic supply; its price is normalized at 1
- Two risky assets (u_1 and u_2), not in elastic supply; their prices are p_{t_1} and p_{t_2} respectively.
- Each asset pays the same dividends: r (no fundamental risk is assumed).

Investors

- Rational investors (i) have rational expectations (have an accurate perception of the distribution of returns from holding the risky asset).
- Irrational investors or noise traders (n) misperceive the expected price of the risky asset by an independent identically distributed random variable:

$$\rho_t \approx N(\rho^*, \sigma_\rho^2), \tag{1}$$

where ρ_t is noise trader's misperception, $\rho*$ is the mean (an average between optimism and pessimism), and σ_ρ^2 is the variance of noise traders' misperceptions per unit of the risky asset.
- Irrational investors have different misconceptions of the expected prices of the risky assets: $\beta_1\rho_t$ for asset 1 and $\beta_2\rho_t$ for asset 2 (β_1 and β_2 are parameters).
- The number of irrational investors is μ and the number of rational investors is $(1-\mu)$.

Assumptions
- The resources to be invested are exogenously given, so there is no first-period consumption and no labor supply decision.
- The only decision that investors have to make is to choose a portfolio when young (first period), based on their beliefs.
Each agent's utility function is a CARA of wealth when old:

$$U = -e^{-(2\gamma)\omega}, \tag{2}$$

where γ is the coefficient of absolute risk aversion and w is the expected final wealth.
Assuming normally distributed returns, maximizing U is equivalent to maximizing:

$$\bar{w} - \gamma\sigma_w^2, \tag{3}$$

where $\bar{w}$ is the average of wealth and σ_w^2 is one period ahead variance of wealth.

a) Rational investors choose the amount $\lambda_{t_1}{}^i$ of the risky asset u_1 and the amount $\lambda_{t_2}{}^i$ of the risky asset u_2 and maximize:

$$\begin{aligned} E(U_i) &= c_0 + \lambda_{t_1}{}^i[r + E_t p_{t_1+1} - p_{t_1}(1+r)] + c_0 + \lambda_{t_2}{}^i[r + E_t p_{t_2+1} - p_{t_2}(1+r)] - \\ &-\gamma[(\lambda_{t_1}{}^i)^2\sigma^2{}_{t_1,pt_1+1} + (\lambda_{t_2}{}^i)^2\sigma^2{}_{t_2,p_{t_2+1}} + 2\rho_{12}\lambda_{t_1}{}^i\sigma_{t_1,p_{t_1+1}}\lambda_{t_2}\sigma_{t_2}{}^i{}_{,pt_2+1}] \end{aligned} \tag{4}$$

where $\sigma^2{}_{t_1,p_{t_1+1}} = E_t\{[p_{t_1+1} - E_t(p_{t_1+1})]^2\}$ (5.1.)

and $\sigma^2{}_{t_2,p_{t_2+1}} = E_t\{[p_{t_2+1} - E_t(p_{t_2+1})]^2\}$ (5.2.)

are the expected variance of p_{t_1+1} and p_{t_2+1}. ρ_{12} is the correlation coefficient between p_{t_1+1} and p_{t_2+1}.

b) Noise traders choose the amount $\lambda_{t_1}{}^n$ of the risky asset u_1 and the amount $\lambda_{t_2}{}^n$ of the risky asset u_2 and maximize:

$$\begin{aligned} E(U_n) &= c_0 + \lambda_{t_1}{}^n[r + E_t p_{t_1+1} - (p_{t_1} - \beta_1\rho_t)(1+r)] + c_0 + \lambda_{t_2}{}^n[r + E_t p_{t_2+1} - \\ &-(p_{t_2} - \beta_2\rho_t)(1+r)] - \gamma[(\lambda_{t_1}{}^n)^2\sigma^2{}_{t_1,p_{t_1+1}} + (\lambda_{t_2}{}^n)^2\sigma^2{}_{t_2,p_{t_2+1}} + 2\rho_{12}\lambda_{t_1}{}^n\sigma_{t_1,p_{t_1+1}}\lambda_{t_2}\sigma_{t_2}{}^n{}_{,p_{t_2+1}}] \end{aligned} \tag{6}$$

Equilibrium prices
In the second generation, agents sell the safe asset for consumption goods to the new young. They also sell risky assets: u_1 for $p_{t_{1+1}}$ and u_2 for p_{t_2+1}.

Market Clearing Conditions

$$\mu\lambda_{t_1}{}^{n} + (1-\mu)\lambda_{t_1}{}^{i} = R_1$$
$$\mu\lambda_{t_2}{}^{n} + (1-\mu)\lambda_{t_2}{}^{i} = R_2$$

where R_1 is the supply of asset u_1 and R_2 is the supply of asset u_2.
Substitute for agents' demands for the risky assets and solve for p_{t_1} and p_{t_2}:

$$p_{t_1} = \frac{\rho_{12}\sigma_1}{\sigma_2} p_{t_2} + \mu(\beta_1 - \frac{\rho_{12}\sigma_1}{\sigma_2}\beta_2)\rho_t + \\ + \frac{2\gamma\sigma_1{}^2\sigma_2(\rho_{12}{}^2-1)R_1 - (\rho_{12}\sigma_1 - \sigma_2)r + \sigma_2\tilde{p}_1 - \rho_{12}\sigma_1\tilde{p}_2}{\sigma_2(1+r)} \quad (7.1.)$$

$$p_{t_2} = \frac{\rho_{12}\sigma_2}{\sigma_1} p_{t_1} + \mu(\beta_2 - \frac{\rho_{12}\sigma_2}{\sigma_1}\beta_1)\rho_t + \\ + \frac{2\gamma\sigma_1\sigma_2{}^2(\rho_{12}{}^2-1)R_2 - (\rho_{12}\sigma_2 - \sigma_1)r + \sigma_1\tilde{p}_2 - \rho_{12}\sigma_2\tilde{p}_1}{\sigma_1(1+r)} \quad (7.2.)$$

where $\sigma_1 = \sigma_{t_1}, p_{t_{1+1}}$; $\sigma_2 = \sigma_{t_2}, p_{t_{2+1}}$; $\tilde{p}_1 = E_t p_{t_{1+1}}$; $\tilde{p}_2 = E_t p_{t_2+1}$.
Consider only steady state equilibrium by imposing that unconditional distributions of p_{t_1+1}, $p_{t_{2+1}}$ be equal to the distributions of p_{t_1} and p_{t_2} respectively:

$$E_t p_{t_1+1} = p_{t_1} + \mu(\rho* - \beta_1\rho_t) \quad (8.1.)$$
$$E_t p_{t_2+1} = p_{t_2} + \mu(\rho* - \beta_2\rho_t) \quad (8.2.)$$

Then solve for p_{t_1} and p_{t_2} explicitly:

$$p_{t_1} = 1 + \mu\beta_1\rho_t + \frac{\mu\rho*}{r} - \frac{2\gamma\sigma_1}{r}[\sigma_1 R_1 + \rho_{12}\sigma_2 R_2] \quad (9.1.)$$

$$p_{t_2} = 1 + \mu\beta_2\rho_t + \frac{\mu\rho*}{r} - \frac{2\gamma\sigma_2}{r}[\rho_{12}\sigma_1 R_1 + \sigma_2 R_2] \quad (9.2.)$$

In both p_{t_1} and p_{t_2} equations, only the second term is variable, so one-step-ahead variance of p_{t_1} and p_{t_2} are functions of the constant variance of one-generation of noise traders' misconceptions, ρ_t:

$$\sigma_1{}^2 = \sigma_t{}^2,_{pt_1+1} = \sigma^2{}_{pt_1+1} = \frac{\beta_1^2\mu^2\sigma_\rho{}^2}{(1+r)^2} \quad (10.1.)$$

$$\sigma_2{}^2 = \sigma_t{}^2{}_{,pt_2+1} = \sigma^2{}_{pt_2+1} = \frac{\beta_2{}^2\mu^2\sigma_\rho{}^2}{(1+r)^2} \qquad (10.2.)$$

Consequently, the final equations for p_{t_1} and p_{t_2} are as follows:

$$p_{t_1} = 1 + \mu\beta_1\rho_t + \frac{\mu\rho*}{r} - \frac{2\gamma\mu^2\sigma_\rho{}^2}{r(1+r)^2}[\beta_1{}^2 R_1 + \rho_{12}\beta_1\beta_2 R_2] \qquad (11.1.)$$

$$p_{t_2} = 1 + \mu\beta_2\rho_t + \frac{\mu\rho*}{r} - \frac{2\gamma\mu^2\sigma_\rho{}^2}{r(1+r)^2}[\rho_{12}\beta_1\beta_2 R_1 + \beta_2{}^2 R_2] \qquad (11.2.)$$

Alternatively, in an economy with only one risky asset, the solution for single price p_t is:

$$p_t = 1 + \mu\rho_t + \frac{\mu\rho*}{r} - \frac{2\gamma\mu^2\sigma_\rho{}^2}{r(1+r)^2} \qquad (11.3.)$$

The first term in each of the equations 11.1. and 11.2. gives the fundamental value of one, and the remaining three terms explain the effect of noise traders' misconceptions on the prices of the two risky assets. Consequently, when ρ_t converges to zero, asset prices converge to their fundamentals.

The fluctuations in the two prices of the risky assets are revealed by the second term in the equations 11.1. and 11.2.: one generation of optimistic noise traders will increase the asset's price; the higher the number of noise traders, the higher the equilibrium price will be. The opposite situation is true for the pessimistic noise traders' case.

For the third term in the equations 11.1. and 11.2., since $\rho*$ is not zero, one generation of noise traders will be, on average, optimistic or pessimistic. This, in turn, will deviate the equilibrium prices above/below their fundamentals. These findings are similar to DeLong, Shleifer, Summers, and Waldmann's (1990) findings: "The Price Pressure Effect", because it puts upward/downward pressure on the risky assets' price more than it would otherwise be in absence of misconceptions.

My contribution to the existing research comes from analyzing the last term of equations 11.1. and 11.2. The last term in the pricing formula (equations 11.1. and 11.2.) proves that arbitrage cannot eliminate the price volatility, due to unpredictability in the noise traders' behavior in the near future. Consequently, rational investors are willing to hold the risky asset only if they are compensated for the risk that future pessimistic noise traders could induce. This is the Space Effect that the noise traders create: noise traders' future beliefs are uncertain and uncertainty makes the risky asset riskier and increases its return.

Overall, the introduction of the second correlated risky asset in the model makes a difference in the final price equations. For example, when investors are optimistic about future returns on both assets, and assuming that future prices are positively correlated (β_1, β_2, and $\rho_{12} \geq 0$ in equations 11.1. and 11.2.), the deviation of price above each asset's fundamental value is smaller than that in a single-asset case (equation 11.3). For the opposite situation, when investors are pessimistic about future returns on both assets, and assuming that future prices are positively correlated (β_1, $\beta_2 \leq 0$, $\rho_{12} \geq 0$ in equations 11.1. and 11.2), the deviation of price below each asset's fundamental value is greater than that in single-asset case. This outcome is consistent with some empirical findings about the asymmetric effect that good/bad news has on market volatility.

CONCLUSIONS

In the theoretical part of this paper I extended a single-asset overlapping-generation model by considering two correlated risky assets for which noise traders might form different beliefs. Consistent with DeLong et. al (1990), I found that investors' misconceptions have a long-run effect in the asset pricing process.

A theoretical contribution that this study brings to the existing literature is that good/bad news has an asymmetric effect on the asset prices process. This theoretical result is consistent with previous empirical findings that bad news creates higher volatilities than good news of the same magnitude.

In conclusion, there is theoretical evidence in favor of sentiment measures as significant variables to explain the stock's excess returns; this result is inconsistent with the Capital Asset Pricing Model's prediction. Based on empirical evidence about market anomalies, common sense suggests considering some less than fully rational explanations to account for returns' pattern.

On the other hand, models considering imperfect rationality approaches are not very likely to be generally accepted. Consequently, the issue is to find some out of sample predictive explanations with several empirical applications.

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